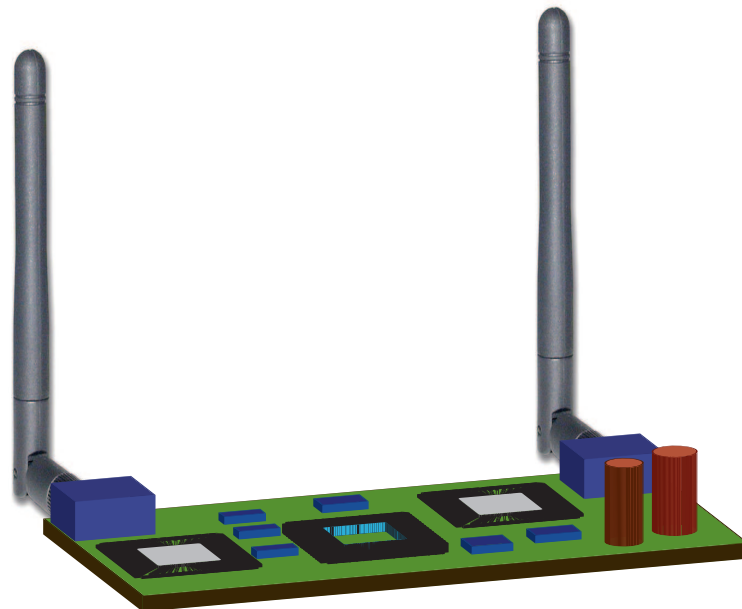


## Lab course Analog Part of a State-of-the-Art Mobile Radio Receiver

Exercise KT 43

Small: September 17, 2010



*The theoretical questions in Chapter 2 must be solved before the start of the lab course. The practical part of Chapter 3 will be solved during the lab course.*

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# Chapter 1

## Introduction

Nowadays, daily life without mobile voice communication is not to be thought of anymore, and since universal mobile telecommunications system (UMTS) has been introduced a couple of years ago new applications such as multi-media streaming have further increased the impact of mobile radio. Still a lot of research is concerned with further increasing data rates in mobile communications. This research is mainly concerned with the digital part of transmitters and receivers. At the same time, the basic structure of the analog part of a modern receiver did not undergo many modifications over the last years.

In this lab course the standard receiver structure for demodulating radio-frequency (RF) signals to baseband is explained by means of Matlab experiments. The considered down-conversion structure is commonly denoted as **super heterodyne** receiver (also known as *superhet*). The lab course is subdivided into two parts. You will explore the receiver in a first theoretical part, before Matlab based experiments shall give more insights in a second practical part.

Focus is set on the understanding of the essential operations occurring in the receiver, and on practicing basic engineering skills like basic filter and amplifier design.

The prepared Matlab code allows for monitoring the signal shapes on their way through the receiver.

Explanations are in tutorial style. The student is expected to be familiar with basic engineering tools from Signal & System theory. Teaching assistants are at the students' disposal at any time.

# Chapter 2

## Theory Part

### 2.1 Modulation and Demodulation Basics

This section provides a short repetition on the theory of modulation and demodulation. The material presented here is not supposed to be self-contained. For a more comprehensive treatment of the addressed topics, the reader is referred to the book “Communication Systems” by S. Haykin.

#### 2.1.1 Passband Modulation

In order to transmit data over a communication channel, the bit sequence  $\{b_k\}_{k=-\infty}^{\infty}$  is mapped onto the complex symbol sequence  $\{s_k\}_{k=-\infty}^{\infty}$ . This symbol sequence is modulated onto a form-invariant baseband pulse  $\phi(\cdot)$  of bandwidth  $B$  with inter-symbol spacing  $T$ . In mathematical terms the continuous baseband signal is written as:

$$s(t) = \sum_{k=-\infty}^{\infty} s_k \cdot \phi(t - kT)$$

Before transmission, this signal is modulated onto a sinusoidal carrier, the frequency of which is much larger than the bandwidth of the baseband pulse. More precisely, real and complex part of the symbol  $s_k$  are carried by two sinusoids shifted by  $\pi/2$  in phase and the modulated signal  $s_c(t)$  is:

$$s_c(t) = \sum_{k=-\infty}^{\infty} [\Re\{s_k\} \cdot \phi(t - kT) \cos(2\pi f_c t) + \Im\{s_k\} \cdot \phi(t - kT) \sin(2\pi f_c t)]$$

where  $\Re\{s_k\}$  and  $\Im\{s_k\}$  represent the real part and the imaginary part of  $s_k$  respectively.

This signal is passed through the wireless communication channel. The channel is typically determined by the propagation environment, the movements of the mobile user and the antenna characteristics at the transmitter and the receiver. We assume this channel to be linear and time-invariant. The latter assumption is well justified only in the case that a symbol duration is short compared to the typical time the mobile user takes for moving over the distance of a wavelength. The channel is then characterized by the impulse response  $h(\cdot)$ . For simplicity, we set this channel impulse response to  $h(t) = \delta(t)$  in the following.

In mathematical terms the receiver extracts the bit sequence from the received continuous waveform by down-converting the signal back to baseband, which is done by

multiplying it by a sine and cosine waveform

filtering it by the ideal low-pass filter  $h_{lp}$  with cutoff frequency larger than  $B$  that rejects the mixing products at  $\pm 4\pi f_c t$ .

Hence, the desired signal is converted to baseband as follows:

In-phase component:

$$\begin{aligned} r_I(t) &= (s_c(t) \cdot 2 \cdot \cos(2\pi f_c t)) * h_{lp}(t) \\ &= \sum_{k=-\infty}^{\infty} [2 \cdot \Re\{s_k\} \cdot \phi(t - kT) \cos(2\pi f_c t)^2 \\ &\quad + 2 \cdot \Im\{s_k\} \cdot \phi(t - kT) \sin(2\pi f_c t) \cos(2\pi f_c t)] * h_{lp}(t) \\ &= \sum_{k=-\infty}^{\infty} \Re\{s_k\} \cdot \phi(t - kT) \end{aligned}$$

Quadrature component:

$$\begin{aligned} r_Q(t) &= (s_c(t) \cdot 2 \cdot \sin(2\pi f_c t)) * h_{lp}(t) \\ &= \sum_{k=-\infty}^{\infty} [2 \cdot \Im\{s_k\} \cdot \phi(t - kT) \sin(2 \cdot \pi f_c t)^2 \\ &\quad + 2 \cdot \Re\{s_k\} \cdot \phi(t - kT) \sin(2\pi f_c t) \cos(2\pi f_c t)] * h_{lp}(t) \\ &= \sum_{k=-\infty}^{\infty} \Im\{s_k\} \cdot \phi(t - kT) \end{aligned}$$

### 2.1.2 Matched Filter & Nyquist Criterion

In the previous discussion we have neglected the impact of noise effects. In practice, the demodulated received signals  $r_I(t)$  and  $r_Q(t)$  are distorted by additive noise which is modeled as a spectrally white Gaussian random process. For a certain class of baseband pulses  $\phi(\cdot)$  the symbol  $s_k$  can be optimally estimated based on a single discrete sample. This is done by a *matched filter*, the output of which is sampled at time instances  $k \cdot T$ . The impulse response of the matched filter is given by<sup>1</sup>

$$h(t) = a \cdot \phi(-t),$$

where  $a$  is an arbitrary constant.

The sample  $Y_k$  obtained at time instance  $k \cdot T$  at the matched filter output is used for the estimation of symbol  $s_k$ . The matched filter is optimal in the sense that it maximizes the signal to noise ratio (SNR), which, for  $\int |\phi(t)|^2 dt = 1$  and  $a = 1$ , is given by

$$\text{SNR} = \frac{\lim_{N \rightarrow \infty} \frac{1}{2N} \sum_{k=-N}^N |s_k|^2}{\text{E}[|W_k|^2]},$$

where  $W_k$  is the (random) difference between observation  $Y_k$  and symbol  $s_k$ .

In order to make sure that the  $k$ th sample contains no contribution of symbols other than  $s_k$  the *Nyquist criterion* needs to be fulfilled. It demands that shifted versions of the signal at the matched filter output  $r_{mf}(\cdot)$  sum up to a constant in Fourier domain:

$$\sum_{n=-\infty}^{\infty} \mathcal{F}\{r_{mf}(\cdot)\} \left(f - \frac{n}{T}\right) = \text{const.}$$

<sup>1</sup>Note that this impulse response might be non-causal. In practice the impulse response is shifted, i.e.  $h(t) = \phi(c - t)$  for a  $c$  large enough such that the non-causal part can be truncated. The sampling instance for the  $k$ th symbol is then  $kT + c$  instead of  $kT$ .

If this condition is fulfilled, no inter-symbol interference (ISI) occurs, and the matched filter is optimum.

A simple (ideal) pulse shape fulfilling the Nyquist criterion is the sinc-pulse corresponding to a rectangle in Fourier domain.

## 2.2 Receiver Design & Architecture

### 2.2.1 The Heterodyne Down-Conversion Concept

In Section 2.1.1 we have seen that a baseband pulse is demodulated from a sinusoidal carrier of frequency  $f_c$  by extracting in-phase and quadrature components through multiplications by two sinusoids of the same frequency  $f_c$  shifted by 90 degrees in phase. In practice, this process is performed in a two step procedure. Rather than converting the signal to baseband immediately, it is down-converted to an intermediate frequency (IF)  $f_{if}$  in a first step. Then, the IF signal is down-converted to baseband in a separate second step. This approach is known as the *super-heterodyne receiver* and avoids certain hardware problems coming with the single step (*homodyne*) receiver.

However, the heterodyne receiver structure comes with another problem. Consider the case that we wish to down-convert a UMTS signal from  $f_c = 2.1$  GHz to  $f_{if} = 21$  MHz. Restricting our attention to the in-phase component, this corresponds to

$$\begin{aligned} s_{if}(t) &= (s_c(t) \cdot \cos(2\pi f_{lo}t)) * h_{lp}(t) \\ &= \sum_{k=-\infty}^{\infty} (\Re\{s_k\} \cdot \phi(t - kT) \cos(2\pi f_c t) \cos(2\pi f_{lo}t)) * h_{lp}(t), \end{aligned}$$

where  $h_{lp}(t)$  is the impulse response of some low-pass filter and  $f_{lo} = f_c - f_{if}$ .

#### Task 1

- a.) Besides the frequency band occupied by the desired signal around  $[f_c - B, f_c + B]$  there is a second frequency band that must be free of interference in order to ensure that the down-converted signal is also free of interference in the band  $[f_{if} - B, f_{if} + B]$ . Identify this band. Hint: This band is called the *image band* of the desired signal. A sketch in frequency domain might be helpful.

Interference in this band must be suppressed by an appropriate filter, commonly known as the *image rejection filter*, which is placed in front of the mixer.

- b.) It is quite common to introduce another mixing stage, i.e. to use two intermediate frequencies, e.g. a first one at 300 MHz and the second one at 21 MHz. Why is such an approach beneficial in terms of image rejection?

### 2.2.2 Non-Ideal Amplifiers

This section briefly describes the impact of non-linearities in amplifiers in the context of demodulation. Low noise amplifier (LNA) is an essential building block in a receiver. They are used for amplifying the received signal to the sensitivity level of the succeeding components.

For the purpose of discussing the impact of non-linearities in amplifiers, it suffices to consider a sine-wave impinging on the receive antenna. We approximate the input-output relation (i.e. between input signal  $s_{in}(t)$  and output signal  $s_o(t)$ ) of the amplifier by a Taylor series:

$$s_o(t) = a_0 + a_1 \cdot s_{in}(t) + a_2 \cdot s_{in}^2(t) + a_3 \cdot s_{in}^3(t) + o(s_{in}^3(t))$$

We neglect the even terms, since typically a *balanced* structure is used for practical implementations. Balanced means, that the amplifier output is the difference between the outputs of two

identical amplifiers, where one is fed with (half) the input signal and the other one with (half) the input signal multiplied by  $-1$ :

$$\begin{aligned} s_o(t) &= \frac{1}{2} (a_0 + a_1 \cdot s_{\text{in}}(t) + a_2 \cdot s_{\text{in}}^2(t) + a_3 \cdot s_{\text{in}}^3(t) + o(s_{\text{in}}^3(t))) \\ &\quad - \frac{1}{2} (a_0 - a_1 \cdot s_{\text{in}}(t) + a_2 \cdot s_{\text{in}}^2(t) - a_3 \cdot s_{\text{in}}^3(t) + o(s_{\text{in}}^3(t))) \\ &= a_1 \cdot s_{\text{in}}(t) + a_3 \cdot s_{\text{in}}^3(t) + o(s_{\text{in}}^3(t)) \end{aligned}$$

The biggest impact results from the third order non-linearity ( $s_{\text{in}}(t) \ll 1$ ) such that we can neglect the residual term to obtain a third-order approximation. Consider now the case, when not only the desired signal (amplitude  $A$ ) but also two sinusoidal interference signals (amplitude  $\frac{A}{\sqrt{2}}$  each) are present at the input. The frequencies  $f_{i_1}$  and  $f_{i_2}$  of the interference signals have the following relation to the frequency of the desired signal  $f_d$ :

$$f_d = 2 \cdot f_{i_1} - f_{i_2}.$$

In this particular case the third order term maps parts of the interference signal onto the desired frequency at the LNA output. The third order intercept point (IP) is an important measure for the linearity of an amplifier. It is defined as the value  $A$  yielding an interference sinusoidal at frequency  $f_d$ , the amplitude of which is equal to the amplitude of the linearly amplified desired signal amplitude  $a_1 \cdot A$  at the amplifier output.

### Task 2

Verify that the amplifier output component

$$\left( \frac{A}{\sqrt{2}} [\sin(2\pi \cdot f_{i_1} \cdot t) + \sin(2\pi \cdot f_{i_2} \cdot t)] \right)^3$$

contains a sinusoidal of frequency  $f_d$  and identify its amplitude to be  $3/(8\sqrt{2}) \cdot A^3$ .

Conclude that

$$\text{IP3}^2 = \frac{8\sqrt{2}}{3} \frac{a_1}{a_3}$$

## 2.3 Full Receiver Structure

Fig. 2.1 depicts the configuration of the key components of a classical super-heterodyne receiver. In the course of the afternoon you will work through all these components in detail. At this point

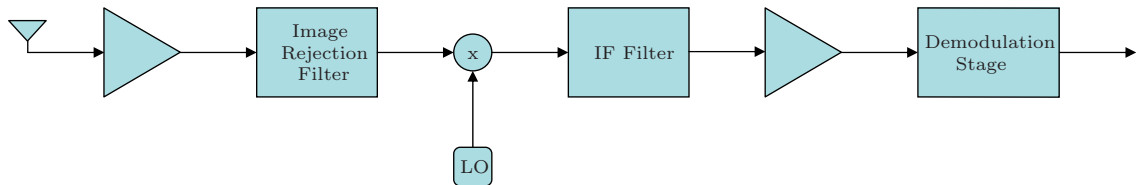


Figure 2.1: Super-Heterodyne Receiver

a brief overview is provided. The RF signal is received by the antenna and passed through a first (rather broadband) bandpass filter stage ensuring that interfering signals outside the frequency band of interest are sufficiently attenuated. The filtered RF signal is then amplified by a LNA and down-converted to the IF. Thereby, the local oscillator frequency is chosen such that the desired signal is centered around  $f_{\text{if}}$ . The image rejection filter addressed in the previous paragraph

is prepended in front of the mixer. The IF filter finally “selects” the desired channel. Before being passed to the demodulation stage the signal is again amplified by the IF amplifier. The demodulation stage performs down-conversion to baseband and (in this experiment) matched filtering.

# Chapter 3

## Practical Part

### 3.1 Overview

In this practical part you shall work through the receiver structure discussed in the theory part. It is crucial that you have understood the concepts discussed therein. If there are questions left please ask the teaching assistant *before* starting the practical part.

We will use UMTS as a reference system. We use a root raised cosine signal as baseband pulse  $\phi(\cdot)$ . The output of the corresponding matched filter is a raised cosine, which fulfills the Nyquist criterion. For sake of simplicity, we restrict our considerations to the in-phase component.

### 3.2 Practical tasks

Use your Tardis username and password to login on the computer. Copy then the required files on your home directory using following commands:

```
...> tar -xvf /home/iktprak/KT43/KT43.tar
...> cd KT43
```

Start Matlab.

```
...> matlab &
```

You can find all parameters that are supposed to be modified at the beginning of the Matlab script.

#### Task 1 - Raised cosine

First observe the raised cosine pulse, by executing the Matlab file once and looking at Fig. 1 on your screen. The pulse rate is  $T = 0.3 \mu\text{s}$  which corresponds to 1.5 MHz bandwidth (in baseband) in case of an ideal sinc-pulse. A root raised cosine pulse occupies significantly more bandwidth for the same pulse rate depending on the roll-off factor  $\alpha$ , which is between 0 and 1. A raised cosine with  $\alpha = 0$  corresponds to a sinc-pulse.

- a.) Run the Matlab script for  $\alpha = 0$ .
- b.) Compare the spectrum of the raised cosine pulse depicted on your screen with your expected result. Explain the observed difference.
- c.) Run the script for different values of  $\alpha$  in order to visualize its influence.

- d.) Finally, set the roll-off factor to  $\alpha = 1/2$ .
- e.) The lower two plots in Fig. 1 on your screen show the corresponding root raised cosine pulse in time and frequency domain. Is this pulse free of inter symbol interference?

*For convenience you might want to suppress this figure in the remaining experiments.*

### Task 2 - Noisy signal

- a.) Run the script.

We consider now the reception of a signal that is corrupted by additive white Gaussian noise (AWGN) with zero mean and unit variance. If you have a look at the upper left plot in Fig. 2 on your screen (maximize the figure to see more details), you can see how the signal looks like after demodulation in baseband before it enters the matched filter. Ideally, we should see a root raised cosine in time domain. However, noise makes “visual” detection almost impossible. UMTS waveforms in fact are similarly hidden in the noise floor. Reasonable SNRs are then obtained by collecting energy over multiple symbol durations at the receiver.

- b.) In order to be able to track the signal visually, set the transmit amplitude such that the SNR is 20 dB. The current SNR is 0 dB. What is the corresponding signal amplitude?

### Task 3 - Nyquist sampling theorem

Run the simulation and you will notice that some of the graphs do not show the expected results.

While a simulation would ideally be continuous, Matlab forces the engineer to work with a discrete model with small time steps. The smaller these time steps are for a given simulation duration, the longer does the simulation take. Thus, one should choose the sampling rate large enough such that all relevant physical effects are captured by the simulation. As mentioned in the theory part, UMTS uses the spectrum around 2.1 GHz.

- a.) In the simulated setting, there are two interference signals present. Look at the code in order to identify the corresponding carrier frequencies.
- b.) Apply Nyquist’s sampling theorem in order to find the minimum sampling rate that avoids aliasing in frequency domain. Adapt the “timestep”-parameter in the script accordingly. Take into account that the signal has a bandwidth of 5 MHz (in passband).

### Task 4 - LNA and IP3

The first component the signal enters after the receive antenna is a LNA. Practical amplifiers always show a certain level of non-linearity. While only a minimum of thermal noise is added by this device, interference can be a problem. It is common in mobile communication that many parallel frequency channels are used simultaneously. Consider the case where the signal is interfered by two signals as introduced in Task 3.

- a.) Compare the (desired) linearly amplified signal power at the output of the LNA to the overall output signal power at the frequency  $f_c$ . As an approximation you may do so by simply looking at the maximum amplitude of the signal. You can find the first order Taylor coefficient in the script, and you are expected to extract the output signal power from Figure 2 on your screen.
- b.) Calculate the maximum 3rd order non-linearity of the LNA, such that the desired output signal is 20 dB stronger than the interference term (ignore the noise in your calculations). Hint: you may use the IP formula from the theory part and the relation

$$s_o(t) \approx a_1(A \sin(f_d t)) + \frac{(\sqrt{2}B)^3}{\text{IP}3^2} \sin(2f_{i_1} - f_{i_2})t,$$

where  $A$  is the time-domain amplitude of the signal and  $B$  the corresponding amplitude of the interferers.  $B$  can be extracted from Fig. 2 by looking at the amplitude ratio in the frequency domain.

The third order non-linearity also introduces some higher frequency terms. In practice, this has usually no impact, but due to the time-discrete simulation, care needs to be taken of this in order to avoid aliasing effects in the spectral domain.

- c.) Adapt the “timestep”-parameter such that Nyquist’s sampling theorem is fulfilled at the LNA output.

### Task 5 - Mixing

Next, the signal is down-converted to the intermediate frequency which is chosen to be 20 MHz in this experiment. In general, there are two options how the down-conversion can be performed: high-side and low-side injection. Other than in the theory part, we use high side injection here, meaning that the local oscillator has a higher frequency than the received signal.

- a.) Find a LO frequency higher than 2.1 GHz that down-converts the signal to 20 MHz.
- b.) What is the highest frequency component after the local oscillator?

You do not need to adjust the sampling rate, as the highest frequency has virtually no impact. Aliasing is negligible.

There can be an interfering image, 20 MHz at the opposite side of the local oscillator. Therefore an image rejection filter is needed. Here a Butterworth low-pass filter will be used.

- c.) What would be a good (approximate) cut-off frequency for the filter?
- d.) Which filter order is required for a SNR of 20 dB? Hint: For a Butterworth filter the frequency response decays 20 dB per decade for frequencies larger than the cutoff frequency. At the cutoff frequency the signal is attenuated by 3 dB.

If the filter order is large, the simulation complexity grows significantly. We can avoid this by switching to low-side injection (i.e.  $f_{lo} < f_c$ ) instead. Of course this does not change the problem in real life, but here it simulates an excellent image rejection filter.

- e.) Switch to low-side injection.
- f.) What is the local oscillator frequency now?

### Task 6 - Demodulation

Finally, the signal is demodulated. In our simple case a second mixing stage is used for bringing the signal to baseband. The mixing products are removed by an ideal low pass filter. You have access to the noisy baseband signal in the vector “tbase”. A root raised cosine pulse is available in the vector “trrcos2”.

- a.) Implement the matched filter and simulate its output. This is a single line only. Search via ctrl-f for *tsk6*, to find the position in the code. Hints: A convolution is performed by the Matlab function “conv(·)”. The convolution takes two vectors of length  $L$  as arguments, and returns a vector of length  $2L - 1$ . In order to obtain the correct output amplitude, the convolved signal has to be multiplied by the overall signal length. Therefore, an appropriate scaling of the signal is needed.
- b.) Plot the output signal both in temporal and spectral domain.
- c.) Recall that the used pulse shape perfectly fulfills the Nyquist criterion for interference-free transmission. Why does the signal amplitude slightly deviate from zero at sampling instances  $t = k \cdot T$  for  $k \neq 0$  nevertheless?

**Task 7 - Summary**

The lab course is completed by briefly explaining the observed effects for each plot in Fig. 2 to the teaching assistant. Focus on the questions:

- a.) Which signal do you observe?
- b.) To which stage in the receiver chain does it correspond to?
- c.) What are the characteristic properties of the signal?
- d.) Why has the amplitude of the signal exactly this value, are there unexpected deviations?

Study the code to clear eventual doubts.