Performance of Space-Time Codes in the Presence of Spatial Fading Correlation *

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Abstract

Previous work on space-time coding has been restricted to the idealistic case of uncorrelated spatial fading. In practice, however, insufficient antenna spacing or lack of scattering cause the individual antennas to be correlated. In this paper, we study the impact of spatial fading correlation on the performance of space-time codes. In particular, we quantify the loss in diversity gain and coding gain as a function of angle spread and antenna spacing. We furthermore show that if a space-time code achieves full diversity in the uncorrelated case, the diversity order achieved in the correlated case is given by the product of the rank of the transmit correlation matrix and the rank of the receive correlation matrix. Finally, we provide simulation results demonstrating the impact of spatial fading correlation on the symbol error rate of space-time codes.

1 Introduction and Outline

Diversity in wireless systems is a powerful means to combat fading. In recent years the use of spatial (or antenna) diversity has become increasingly popular, which is mostly due to the fact that it can be provided without loss in spectral efficiency. Receive diversity, i.e., the use of multiple antennas on the receive side of a wireless radio link is a well-studied subject [1]. Driven by mobile wireless applications, where it is difficult to deploy multiple antennas in the handset, transmit diversity or equivalently the use of multiple antennas on the transmit side has become an active area of research [2]-[8]. Space-time coding evolved as one of the most promising transmit diversity techniques [9]-[13]. Previous work on space-time coding has been restricted to the idealistic case of i.i.d. channels, i.e., uncorrelated fading. In practice, however, insufficient antenna spacing and lack of scattering cause the individual antennas to be correlated. Therefore, the i.i.d. model will in general not be an accurate description of real-world multi-antenna channels. Whilst the impact of spatial fading correlation on the capacity of multiple-input multiple-output (MIMO) systems has been studied to some extent in [14], the impact of correlated fading on the performance of space-time codes is less understood. Some results along these lines for the OFDM-MIMO case have been reported in [15].

Contributions. For a physically motivated real-world channel model, we study the impact of spatial fading correlation on space-time codes by quantifying the loss in diversity gain and coding gain as a function of angle spread and antenna spacing. We express the diversity order achieved by a space-time code in the correlated fading case in terms of the rank of the transmit correlation matrix, the rank of the receive correlation matrix, and the diversity order achieved by this code in the i.i.d. case. In particular, we prove that if a space-time code achieves full diversity in the i.i.d. case, its diversity order in the correlated fading case is given by the product of the rank of the transmit correlation matrix and the rank of the receive correlation matrix.

Organization of the paper. The rest of this paper is organized as follows. In Section 2, we introduce the channel model and briefly review space-time coding. In Section 3, we derive the average pairwise error probability of space-time codes as a function of the transmit and the receive correlation matrix. In Section 4, we discuss the impact of spatial fading correlation on the performance of space-time codes by quantifying the loss in diversity gain and coding gain. Section 5 contains some simulation results, and Section 6 concludes the paper.

2 Channel Model and Space-Time Coding

In this section, we shall describe our channel model and then briefly review the basics of space-time coding.
2.1 Channel Model

In the following $M_T$ and $M_R$ denote the number of transmit and receive antennas, respectively. We restrict ourselves to a purely Rayleigh fading scenario, where the elements of the $M_R \times M_T$ channel matrix $[H]_{m,n} = h_{m,n} (m = 0, 1, ..., M_R - 1, n = 0, 1, ..., M_T - 1)$ are (possibly correlated) circularly symmetric $1$ zero mean complex Gaussian random variables. We employ the block-fading model used in [11] wherein the channel remains constant over $N \geq M_T$ symbol periods and then changes in an independent fashion to a new realization. We furthermore assume that spatial fading correlation occurs both at the transmitter and the receiver and that the distance between transmitter and receiver is small or the scattering radii at the transmitter and the receiver are sufficiently large. Under these assumptions the MIMO channel model recently proposed in [16] reduces to

$$H = R^{1/2}H_{\omega}S^{1/2},$$ (1)

where $H_{\omega}$ is an $M_R \times M_T$ i.i.d. complex Gaussian matrix with zero mean unit variance entries, and $S = S^{1/2}S^{1/2}$ and $R = R^{1/2}R^{1/2}$ are the transmit and the receive correlation matrix, respectively. We note that the channel model in (1) has also been used in [17] to analyze the asymptotic (in the number of antennas) capacity behavior of MIMO channels in the presence of correlated fading. We assume a uniform linear array at both the transmitter and the receiver. The relative antenna spacing is denoted as $\Delta = \frac{d_t}{\lambda}$ at the transmitter and $\Delta_r = \frac{d_r}{\lambda}$ at the receiver. Here, $d_t$ and $d_r$ stand for the absolute antenna spacing and $\lambda = c/f_c$ is the wavelength of a narrowband signal with center frequency $f_c$. Defining $\rho(\Delta, \bar{\theta}, \bar{\delta}) = E[|h_{r,k}h_{r,k+m}|^2]$ for $k = 0, 1, ..., M_T - 1$ to be the fading correlation between two receive antenna elements spaced $\Delta$ wavelengths apart, the correlation matrix $R$ can be written as

$$[R]_{m,n} = \rho((n - m)\Delta, \bar{\theta}, \bar{\delta}),$$ (2)

where $\bar{\theta}$ denotes the mean angle of arrival at the receiver and $\bar{\delta}$ is the angle spread. The transmit antenna spacing $S$ is defined similarly. Assuming that the angles of arrival at the transmitter and the receiver are Gaussian distributed around the mean angle of arrival $\bar{\theta}$, i.e., the actual angle of arrival is given by $\theta = \bar{\theta} + \bar{\delta}$ with $\bar{\delta} \sim N(0, \sigma^2_\delta)$, where the variance $\sigma^2_\delta$ is proportional to the angle spread $\delta$, it is shown in [18] that for small angle spread

$$\rho(\Delta, \bar{\theta}, \bar{\delta}) \approx e^{-\frac{2\pi \Delta \cos(\bar{\theta})}{\sigma^2_\delta}}e^{-\frac{1}{2}(2\pi \Delta \sin(\bar{\theta})\sigma_\delta)^2}.$$ (3)

Although this approximation is accurate only for small angle spread, it does provide the correct trend for large angle spread, namely uncorrelated spatial fading. Note that in the limiting case $\sigma_\delta = 0$, the correlation matrix $R$ collapses to a rank-$1$ matrix and can be written as $R = a(\bar{\theta})a^H(\bar{\theta})$ with the array response vector of the uniform linear array given by

$$a(\theta) = [1, e^{2\pi \Delta \cos(\theta)}, ..., e^{2\pi ((M_T-1)\Delta \cos(\theta))}]^T.$$ (4)

The rank of the correlation matrices $R$ and $S$ is driven by the angle spread and the antenna spacing.

2.2 Space-Time Coding

The bit stream to be transmitted is encoded by the space-time encoder into a codeword $C = [c_0, c_1, ..., c_{N-1}]$ of size $M_T \times N$ with the individual data symbols being taken from a finite complex constellation chosen such that the average energy of the constellation elements is 1. The $M_R \times 1$ received data vector for the $k$-th symbol period is given by

$$r_k = \sqrt{E_s}Hc_k + n_k, \quad k = 0, 1, ..., N - 1$$

where $n_k$ is complex-valued additive spatially and temporally white Gaussian noise satisfying

$$E[|n_k|^2] = \sigma^2_nI_{M_R} \delta[k - l]$$

with $I_{M_R}$ denoting the identity matrix of size $M_R$. Assuming perfect channel state information in the receiver, the maximum likelihood (ML) receiver computes the vector sequence $\hat{c}_k (k = 0, 1, ..., N - 1)$ according to

$$\hat{c}_k = \arg\min_{c_k} \sum_{k=0}^{N-1} \|r_k - \sqrt{E_s}Hc_k\|^2,$$

where the minimization is performed over all possible code-matrix sequences $C$.

3 Average Pairwise Error Probability

We shall next provide an expression for the average pairwise error probability (PEP) taking into account the channel model in (1).

Let $C$ and $E$ be two different codeword matrices of size $M_T \times N$ and assume that $C$ was transmitted. For a given channel realization $H$, the probability that the receiver decides erroneously in favor of the signal $E$ can be upper bounded by [19]

$$P(C \rightarrow E|H) \leq e^{-\frac{d(C,E|H)}{2\sigma_n^2}},$$ (5)

where $d(C,E|H)$ is the average pairwise error probability of $C$ with respect to $E$ for a given channel realization $H$.
where
\[ d^2(C, E|H) = \sum_{k=0}^{N-1} \|H(c_k - e_k)\|^2. \]

Next, we compute the expected PEP by averaging (5) over all channel realizations taking into account (1). For this we define \( y_k = H(c_k - e_k) \) for \( k = 0, 1, ..., N - 1 \) and
\[ Y = [y_0^T \quad y_1^T \quad ... \quad y_{N-1}^T]^T. \]

With this notation we get \( d^2(C, E|H) = \|Y\|^2 \) and hence (5) can be rewritten as
\[ P(C \rightarrow E|H) \leq e^{-\frac{\|Y\|^2}{2\sigma^2}}, \] (6)
The average over all channel realizations of the right-hand-side (RHS) in (6) is fully characterized by the eigenvalues of the covariance matrix of \( Y \) [20] defined as \( C_Y = E\{Y Y^H\} \). Denoting the eigenvalues of \( C_Y \) as \( \lambda_i(C_Y) \) \( (i = 0, 1, ..., r(C_Y) - 1) \) the following result can be established
\[ P(C \rightarrow E) \leq \prod_{i=0}^{r(C_Y) - 1} \left( 1 + \lambda_i(C_Y) \frac{E_s}{4\sigma^2} \right)^{-1}, \] (7)
where \( P(C \rightarrow E) = E\{P(C \rightarrow E|H)\} \) is the PEP averaged over all channel realizations. Taking into account (1) it can be shown that
\[ C_Y = [(C - E)^T S^T(C - E)^*] \otimes R, \] (8)
where the superscripts \( T \) and \( * \) stand for transposition and element-wise conjugation, respectively, and \( A \otimes B \) denotes the Kronecker product of the matrices \( A \) and \( B \). In the following, we shall restrict ourselves to the high-SNR case where (7) takes the form
\[ P(C \rightarrow E) \leq \left( \frac{E_s}{4\sigma^2} \right)^{-r(C_Y)} \prod_{i=0}^{r(C_Y) - 1} \lambda_i^{-1}(C_Y). \] (9)

4 Spatial Fading Correlation and PEP

Based on (8) and (9) we shall next study the impact of transmit and receive correlation on the performance of specific space-time codes.

4.1 Diversity Order

In the following, we assume that a space-time code achieving \( sM_R \)-th order diversity in the i.i.d. case is employed, i.e., the minimum rank of \( (C - E) \) over the set of all pairs of codeword matrices is \( s \). Using (8) and (9) we shall next study the degradation in performance in terms of average PEP for the cases of receive correlation only, transmit correlation only, and joint transmit and receive correlation. In the following \( \lambda_i(C, E) \) denotes the eigenvalues of \( (C - E)^T(C - E)^* \).

Receive correlation only. In this case \( S = I_{M_R} \) and (8) specializes to
\[ C_Y = [(C - E)^T(C - E)^*] \otimes R. \]

Letting \( (C - E)^T(C - E)^* = U\Sigma U^H \) and \( R = VDV^H \), where \( U \) and \( V \) are unitary and \( \Sigma \) and \( D \) are diagonal, and using the following property of Kronecker products \( (A \otimes B)\{F \otimes G\} = (AF) \otimes (BG) \), we get
\[ C_Y = (U \otimes V)(\Sigma \otimes D)(U^H \otimes V^H). \] (10)

Since \( U \) and \( V \) are unitary, it follows that \((U \otimes V)\) is unitary as well. Hence (10) is an eigendecomposition of \( C_Y \) and the diagonal matrix \( \Sigma \otimes D \) contains the eigenvalues of \( C_Y \). Clearly, \( r(C_Y) = r(C - E)r(R) \) and hence a minimum-rank error event yields \( r(C_Y)^s = s r(R) \leq sM_R \). In the following, we shall focus on pairs of \( C \) and \( E \) where \( r(C - E) = s \). It follows from (10) that the nonzero eigenvalues of \( C_Y \) are given by
\[ \sigma(C_Y) = \{ \lambda_0(C, E), \lambda_0(C, E) \lambda_1(R), \lambda_0(C, E) \lambda_1(R), ..., \lambda_s(C, E) \lambda_{s-1}(R), \lambda_s(C, E) \lambda_{s-1}(R), ..., \lambda_s(C, E) \lambda_1(R), \lambda_s(C, E) \lambda_1(R), ..., \lambda_s(C, E) \lambda_{s-1}(R) \}. \]

Using (9) the average PEP in the case of receive correlation only is given by
\[ P(C \rightarrow E) \leq \left( \frac{E_s}{4\sigma^2} \right)^{-r(R)} \prod_{i=0}^{s-1} \lambda_i^{-1}(C, E). \] (11)

The diversity order achieved by this code in a correlated fading environment is hence given by
\[ d = s r(R) \leq sM_R. \] (12)

Note that if the rank of \( R \) drops by 1 the diversity order is reduced by \( s \) or equivalently we lose \( s \) degrees of freedom. Loosely speaking, a reduction in the rank of \( R \) by 1 (due to increased spatial fading correlation) amounts to a reduction of the effective number of receive antennas by 1. Furthermore, the coding gain depends on \( r(R) \) and \( \lambda_i(R) \) through the two products on the RHS of (11).
Transmit correlation only. In the case of transmit correlation only $R = I_{M_R}$ and (8) reduces to

$$C_Y = [(C - E)^T S^T (C - E)^*] \otimes I_{M_R}.$$  

Denoting $\alpha = r((C - E)^T S^T (C - E)^*)$ and again focusing on a minimum-rank codeword pair $C$ and $E$, the average PEP can be upper-bounded by

$$P(C \to E) \leq \left( \frac{E_s}{4\sigma_n^2} \right)^{-\alpha M_R} \prod_{i=0}^{a-1} \lambda_i^{-M_R}((C - E)^T S^T (C - E)^*).$$

For nonsingular $S$, the following lemma allows to make the dependence of the average PEP on the eigenvalues of $S$ explicit.

Lemma 1. Let the eigenvalues of $(C - E)^T S^T (C - E)^*$ and $S$ be arranged in increasing order. For each $k = 1, 2, ..., a$, there exists a positive real number $\theta_k$ such that $\lambda_0(S) \leq \theta_k \leq \lambda_{a-1}(S)$ and

$$\lambda_k((C - E)^T S^T (C - E)^*) = \theta_k \lambda_k(C, E).$$

Proof: Note that $(C - E)^T S^T (C - E)^*$ can be written as $(C - E)^T S^T S^{t/2}(C - E)^*$ and the nonzero eigenvalues of $(C - E)^T S^T S^{t/2}(C - E)^*$ are equal to the nonzero eigenvalues of $S^{t/2}(C - E)^*(C - E)^T S^{t/2}$. Now, applying Ostrowski’s theorem [21] to $S^{t/2}(C - E)^*(C - E)^T S^{t/2}$, and noting that the nonzero eigenvalues of $(C - E)^*(C - E)^T$ are equal to the nonzero eigenvalues of $(C - E)^T (C - E)^*$, the result is established. \qed

Using Lemma 1 we get the following bound on PEP for nonsingular $S$

$$P(C \to E) \leq \left( \frac{E_s}{4\sigma_n^2} \right)^{-s M_R} \prod_{i=0}^{s-1} \left(\theta_i \lambda_i(C, E)\right)^{-M_R},$$

where we have used the fact that $\alpha = s$ for nonsingular $S$.

For general $S$ the achievable diversity order is given by

$$d = \alpha M_R.$$  

Using basic results on the rank of matrix products we get [21]

$$s + r(S) - M_T \leq \alpha \leq \min\{s, r(S)\}.$$  

For a full rank transmit correlation matrix, i.e., $r(S) = M_T$, we get $d = s M_R$. For a space-time code achieving full diversity in the i.i.d. case $s = M_T$ and hence $d = r(S) M_R$. In the latter case if the rank of $S$ is reduced by 1 we lose $M_R$ degrees of freedom, which loosening speaking is equivalent to losing one effective transmit antenna. We note that (15) shows that if $S$ is singular and the space-time code does not achieve full diversity in the i.i.d. case, i.e., $s < M_T$, the diversity order can only be lower-bounded by $(s + r(S) - M_T) M_R$. In this case it is difficult to make statements on the exact diversity order since the geometry of $S$ and the geometry of the space-time code, i.e., the geometry of the code difference matrices $(C - E)$ play an important role in assessing the exact diversity order. Loosely speaking, in this case in order to obtain good performance in terms of PEP it is important that the space-time code excites the range space of $S$.

Joint transmit-receive correlation. In this case the covariance matrix $C_Y$ is given by (8) and

$$P(C \to E) \leq \left( \frac{E_s}{4\sigma_n^2} \right)^{-\alpha r(R)} \prod_{i=0}^{a-1} \lambda_i^{-r(R)}((C - E)^T S^T (C - E)^*).$$

For nonsingular $S$ it follows from Lemma 1 that the eigenvalues $\lambda_i((C - E)^T S^T (C - E)^*)$ in (16) can be replaced by $\theta_i \lambda_i(C, E)$ with the $\theta_i$ defined in Lemma 1.

For general $S$ the diversity order is given by

$$d = \alpha r(R).$$

Assuming that the space-time code is designed such that it achieves full diversity in the i.i.d. case, i.e., $s = M_T$, we have $\alpha = r(S)$ and hence the diversity order is given by

$$d = r(S) r(R).$$

In the case of high transmit and receive correlation where $r(S) = r(R) = 1$ we get $d = 1$ and hence there is no diversity gain at all. Finally, for arbitrary $s$ and nonsingular $S$ we get $\alpha = s$ and hence $d = sr(R)$.

4.2 Diversity and Propagation Parameters

We have seen in the previous subsection that the rank and the eigenvalue distribution of the correlation matrices determine the diversity gain and the coding gain achieved by a space-time code in the correlated fading case. Using the channel model introduced in Sec. 2.1, we shall now relate angle spread and antenna spacing to the eigenvalues of $S$ and $R$ and subsequently in Sec. 4.3 to the diversity gain and the coding gain of the space-time code. Let us restrict our attention to the receive correlation matrix $R$. (The same analysis applies to $S$). Since $R$ is a Töplitz matrix, we can invoke Stégou’s theorem [22] to obtain the limiting $(M_R \to \infty)$ distribution of the eigenvalues of $R$ as

$$\lambda(\nu) = \sum_{x=-\infty}^{\infty} p(s \Delta, \delta, \delta) e^{-j2\pi \nu x}, \quad 0 \leq \nu < 1,$$
which upon using (3) yields
\[
\lambda(\nu) = \theta_3 \left( \pi (\nu - \Delta \cos(\theta)), e^{-\frac{1}{2} (\theta^2 + \theta_0^2 \cos^2(\theta))} \right) \quad (17)
\]
with the third-order theta function given by [23]
\[
\theta_3(\nu, q) = \sum_{n=-\infty}^{\infty} q^{n^2} e^{2j\nu n}.
\]
Although this expression yields the exact eigenvalue distribution only in the limiting case \( M_R \to \infty \), in the finite case good approximations of the eigenvalues can be obtained by sampling \( \lambda(\nu) \) uniformly on the unit circle [22], which allows us to assume that the eigenvalue distribution in the finite case follows the distribution given by \( \lambda(\nu) \). Noting that the correlation function \( \rho(s, \Delta, \theta, \delta) \) as a function of \( s \) is essentially a modulated Gaussian function with its spread increasing for decreasing antenna spacing or decreasing angle spread and vice versa, it follows that \( \lambda(\nu) \) will be more flat in the case of large antenna spacing and/or large angle spread (i.e. low spatial fading correlation). For small antenna spacing and/or small angle spread (i.e. high spatial fading correlation) \( \lambda(\nu) \) will be peaky. Figs. 1(a) and (b) show the limiting eigenvalue distribution of \( R \) for high and low spatial fading correlation, respectively.

![Fig. 1. Limiting eigenvalue distribution of the correlation matrix \( R \) for the cases of (a) high spatial fading correlation, and (b) low spatial fading correlation.](image)

Finally, for any \( M_R \), using the fact that \( R = R^H \), the eigenvalues of \( R \) can be lower-and upper-bounded by the infimum and the supremum of \( \lambda(\nu) \). In particular, defining
\[
m = \text{ess inf } \lambda(\nu), \quad M = \text{ess sup } \lambda(\nu), \quad \nu \in [0,1]
\]
we have that [24]
\[
m \leq \lambda_i(R) \leq M \quad i = 0,1,...,M_R - 1.
\]

This result allows to provide an upper bound on the PEP in terms of the spectrum \( \lambda(\nu) \) and hence makes the dependence of the average PEP on angle spread and antenna spacing more explicit. For example, assuming that \( S \) is nonsingular and denoting the greatest lower bound of the spectrum of \( S \) as \( m \), (13) can be written as
\[
P(C \rightarrow E) \leq \left( \frac{mE_s^2}{4\sigma_n^2} \right)^{-s M_R} \prod_{i=0}^{s-1} \lambda_i^{-M_R}(C, E).
\]
Now, it follows from \( \text{Trace}(S) = M_T \) that \( m \leq 1 \) with \( m = 1 \) if and only if \( S \) is unitary or equivalently spatial fading is uncorrelated. The presence of spatial fading correlation can therefore be interpreted as a reduction in the effective SNR by a factor of \( m \).

4.3 Loss in Average PEP

In the following, we shall quantify the loss in average PEP due to spatial fading correlation. Due to the lack of space, we shall restrict ourselves to the case of receive correlation only.

We compare the performance of a given space-time code in the i.i.d. case to its performance in the correlated fading case by defining a loss function as follows
\[
\mathcal{L} = \log \frac{P_{\text{corr}}(C \rightarrow E)}{P_{\text{i.d.}}(C \rightarrow E)},
\]
where \( P_{\text{corr}} \) and \( P_{\text{i.d.}} \) denote the upper bound on the average PEP in the correlated and the uncorrelated case, respectively. Using (9) and (11), we get
\[
\mathcal{L} = s(M_R - \tau(R)) \log \left( \frac{E_s}{4\sigma_n^2} \right) + (M_R - \tau(R)) \sum_{i=0}^{s-1} \log \lambda_i(C, E)
\]
\[
- s \left( \sum_{i=0}^{\tau(R)-1} \log \lambda_i(R) \right) \quad (18)
\]
The first term on the RHS of (18) reflects the loss in diversity gain, the second term reflects the loss in coding gain, and the third term measures the deviation of the distribution of the nonzero eigenvalues of \( R \) from a uniform distribution. In the previous subsection we have seen that small antenna spacing and/or small angle spread yields a peaky eigenvalue distribution for \( R \) and hence taking into account (18) results in a significant loss in terms of average PEP. In fact, using Jensen's inequality [25], it is easily seen that the last term in (18) is minimized when the nonzero eigenvalues of \( R \) are uniformly distributed. Thus, the more \( \lambda(\nu) \) deviates from a uniform distribution, the higher \( \mathcal{L} \) will be.
This shows that spatial fading correlation always results in a performance degradation in terms of PEP.

Based on (18) we can make further interesting observations.

- For full rank \( R \), i.e., \( r(R) = R \), the loss \( L \) is given by
  \[
  L = -a \sum_{i=0}^{M_R-1} \log \lambda_i(R)
  \]
  and is hence independent of the particular codeword pair considered. In particular, since \( \text{Trace}(R) = M_R \), \( L = 0 \) for \( R \) unitary.

- For given \( R \) with \( r(R) < M_R \) and given spatial fading correlation, the loss \( L \) is minimum for the error events minimizing \( \sum_{i=0}^{r-1} \log \lambda_i(C, E) \).

5 Simulation Results

In this section, we provide simulation results demonstrating the performance of space-time codes in correlated Rayleigh fading environments. We used \( M_T = 2 \) and \( M_R = 4 \) and the 4-PSK 16-state code proposed in [11]. The signal-to-noise-ratio (SNR) was defined as \( \text{SNR} = 10 \log_{10} \left( \frac{E_b}{N_0} \right) \). All results were obtained by averaging over 1,000 independent Monte Carlo trials where each burst consisted of 80 data symbols.

Fig. 2 shows the average symbol error rate for the i.i.d. (uncorrelated) case, and the cases of transmit correlation only \((\Delta_t = 0.1, \sigma_{\theta,t} = 0.25)\), receive correlation only \((\Delta_r = 0.1, \sigma_{\theta,r} = 0.25)\), and transmit and receive correlation \((\Delta_t = \Delta_r = 0.1, \sigma_{\theta,t} = \sigma_{\theta,r} = 0.25)\), respectively. We can clearly see that the performance consistently degrades in the presence of fading correlation. It can furthermore be seen that the performance in the case of transmit correlation only is better than the performance in the case of receive correlation only. In order to explain this, note first that due to small antenna spacing the transmit and the receive correlation matrix are both low rank. For simplicity we consider the extreme case where the correlation matrices are rank 1 (cf. (4)). Since the space-time code employed achieves full diversity gain in the i.i.d. case, it follows from (14) that in the case of transmit correlation only the diversity order is \( d = r(S)M_R = 4 \), whereas in the case of receive correlation only it follows from (12) that \( d = M_T r(R) = 2 \). Hence, in the case of transmit correlation only, the diversity order achieved by the code is higher than in the case of receive correlation only. Similar arguments can be used to show that the same is true for coding gain. Finally, we note that the performance is worst in the case where transmit and receive correlation are present at the same time.

![Fig. 2. Symbol error rate as a function of SNR in the cases of uncorrelated fading (i.i.d. case), transmit correlation only, receive correlation only, and transmit and receive correlation.](image)

6 Conclusion

We studied the performance of space-time codes in correlated Rayleigh fading environments. Using a channel model incorporating both transmit and receive correlation, we expressed the diversity order achieved by a space-time code in the correlated fading case as a function of the diversity order the code achieves in the i.i.d. case and the rank of the transmit and the receive correlation matrix. In particular, we showed that if a space-time code achieves full diversity in the uncorrelated case, the diversity order achieved in the correlated case is given by the product of the rank of the transmit correlation matrix and the rank of the receive correlation matrix. We furthermore quantified the loss in average pairwise error probability and studied the impact of the eigenvalue distribution of the correlation matrices on error rate performance. Finally, we provided simulation results.

References


