Blind Channel Identification and Equalization in OFDM-Based Multiantenna Systems

Helmut Bölcskei, Member, IEEE, Robert W. Heath, Jr., Student Member, IEEE, and Arogyaswami J. Paulraj, Fellow, IEEE

Abstract—Wireless systems employing multiple antennas at the transmitter and the receiver have recently been shown to have the potential of achieving extraordinary bit rates. Orthogonal frequency division multiplexing (OFDM) significantly reduces receiver complexity in multiantenna broadband systems. In this paper, we introduce an algorithm for blind channel identification and equalization in OFDM-based multiantenna systems. Our approach uses second-order cyclostationary statistics, employs antenna precoding, and yields unique channel estimates (up to a phase rotation for each transmit antenna). Furthermore, it requires only an upper bound on the channel order, it does not impose restrictions on channel zeros, and it exhibits low sensitivity to stationary noise. We present simulation results demonstrating the channel estimator and the corresponding multichannel equalizer performance.

Index Terms—Blind equalization, cyclostationarity, MIMO, multiantenna systems, OFDM.

I. INTRODUCTION

Deploying multiple antennas at both the transmitter and the receiver of a wireless system has recently been shown to yield extraordinary bit rates [1]–[5]. The corresponding technology, known as spatial multiplexing [1] or BLAST [2], [6], allows an impressive increase in data rate in a wireless radio link without additional power or bandwidth consumption. Orthogonal frequency division multiplexing (OFDM) [7]–[9] significantly reduces receiver complexity in wireless broadband systems and has recently been proposed for use in wireless broadband multiantenna systems [4], [5], [10]. In practice, in order to get the promised increase in data rate, accurate channel state information is required in the receiver. This information can be obtained by sending training data and estimating the channel [10]–[12]. The training overhead required, unfortunately, is more significant in estimating multiple-input multiple-output (MIMO) channels. To avoid this problem, we propose an algorithm for blind channel identification and equalization in OFDM-based MIMO systems.

Blind MIMO Channel Estimation: Blind identification and equalization of MIMO channels has been a very active area of research during the past few years. Due to lack of space, we will not summarize all existing ideas and algorithms; rather, we refer the interested reader to [13], which contains an excellent overview of the subject and an extensive reference list until 1996. More recent references can be found, for example, in [14]–[17]. To the best of our knowledge, previous work on blind MIMO channel estimation was restricted to single-carrier systems. The use of cyclostationary statistics to accomplish blind MIMO channel estimation has first been proposed for frequency-flat fading channels in [18] and [19]. More recently, the use of conjugate cyclostationary statistics in combination with constant-modulus antenna precoding has been suggested in [20] to accomplish blind MIMO channel estimation in the single-carrier case. We finally note that since we are dealing with OFDM signals, which, in practice, have a high number of subcarriers (typically between 512 and 8192), the transmit signals will be Gaussian. This makes the application of source separation approaches relying on the assumption that the different sources have different distribution functions and using higher order statistics difficult (see e.g., [21] and references therein).

Contributions: In this paper, using a periodic nonconstant-modulus precoding scheme, we introduce an algorithm for the blind identification and equalization of OFDM-based MIMO systems. Our method uses second-order cyclostationary statistics and identifies the matrix channel on a subchannel by subchannel basis, i.e., each scalar subchannel is identified individually. Important aspects of the proposed algorithm include the following.

• It requires only an upper bound on the channel order.
• It does not impose restrictions on channel zeros.
• It exhibits low sensitivity to stationary noise.

Our equalization method recovers the transmitted symbol streams up to a phase rotation (which will in general be different for different symbol streams). This remaining ambiguity can be resolved using short training sequences.

Relation to Previous Work: Blind MIMO channel identification algorithms based on second-order statistics are, in general, able to identify the MIMO channel up to a unitary mixing matrix only and suffer from common zeros problems. Our approach overcomes both of these drawbacks and is an extension of an idea first proposed by the authors for the single-carrier case in [22]. The algorithm introduced in [18] and [19] is restricted to single-carrier modulation and frequency-flat fading (i.e., no delay spread). Besides applying to OFDM, our algo-

Manuscript received January 26, 2000; revised October 11, 2001. This work was supported in part by FWF under Grants J1629-TEC and J1868-TEC and by funding from Ericsson Inc. through the Networking Research Center at Stanford University. The associate editor coordinating the review of this paper and approving it for publication was Prof. Lang Tong.

H. Bölcskei is with the Coordinated Science Laboratory and Department of Electrical Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA (e-mail: bolcskei@comm.cs.uiuc.edu).

R. W. Heath, Jr. and A. J. Paulraj are with the Information Systems Laboratory, Department of Electrical Engineering, Stanford University, Stanford, CA 94305 USA.

Publisher Item Identifier S 1053-587X(02)00414-2.
OFDM-BASED SPATIAL MULTIPLYING SYSTEMS

In this section, we briefly review OFDM-based spatial multiplexing, and we provide some preparation for the rest of the paper. In the following, \( M_T \) and \( M_R \) denote the number of transmit and receive antennas, respectively.

OFDM-Based Spatial Multiplexing: An apparent disadvantage of single-carrier based spatial multiplexing systems is the fact that the computational complexity of the receiver (either a vector-MLSE or a multichannel equalizer) will, in general, be very high. The use of OFDM alleviates this problem by turning the frequency-selective MIMO channel into a set of parallel narrowband MIMO channels. This makes equalization very simple. In fact, only a constant matrix has to be inverted for each OFDM tone \([4], [5], [10]\).

In an OFDM-based spatial multiplexing system, the individual data streams are first OFDM-modulated and then transmitted simultaneously from the \( M_T \) antennas. Fig. 1 illustrates a baseband discrete-time single antenna OFDM transceiver. The modulator applies an \( N \)-point IFFT to \( N \) data symbols and prepends the cyclic prefix (CP) \([7]\) of length \( L \), which is a copy of the last \( L \) samples of the IFFT output. The overall OFDM symbol length is, therefore, given by \( M = N + L \). Throughout this paper, we assume that \( L \leq M/2 \), which is usually satisfied in practice. The CP acts as a guard space between consecutive OFDM symbols and avoids intersymbol-interference (ISI) if the channel impulse response length is less than or equal to the length of the CP. In the receiver, the CP is first removed, and then, an \( N \)-point FFT is applied. The signals received by the individual antennas are first passed through OFDM demodulators, separated, and demultiplexed (and potentially decoded). Fig. 2 shows an OFDM-based spatial multiplexing system.

Let us next introduce some notation and some basic results needed later in the paper. In the following, the transmitted signal corresponding to the \( \ell \)th \(( \ell = 0, 1, \ldots, M_T-1 \) antenna is given by

\[
s_{\ell}[n] = \sum_{l=-\infty}^{\infty} g[n-lM] \sum_{k=0}^{N-1} e^{j(2\pi/N)k(n-lM)} I_{\ell,k,l}
\]

where \( g[n] = \text{rect}_{\{0, M-1\}}[n] \) with

\[
\text{rect}_{\{T_1, \ldots, T_2\}}[n] = \begin{cases} 1, & n = T_1, T_1+1, \ldots, T_2 \\ 0, & \text{else} \end{cases}
\]

Furthermore, \( e^{j(2\pi/N)k(n-lM)} \) denotes the (complex) data symbol transmitted on the \( \ell \)th tone in the \( \ell \)th OFDM symbol from the \( \ell \)th antenna.

Now, denote the continuous-time impulse response between the \( \ell \)th transmitter and the \( \eta \)th receiver as \( h_{\text{trans}}^{(\ell)}(t) = h_{\text{trans}}^{(\eta)}(t) = h_{\text{trans}}^{(\ell)}(t) \times h_{\text{trans}}^{(\eta)}(t) \times h_{\text{trans}}^{(\ell)}(t) \times h_{\text{trans}}^{(\eta)}(t) \) are the transmit

---

**Fig. 1.** Baseband OFDM transceiver. (a) Modulator. (b) Demodulator.

**Fig. 2.** OFDM-based spatial multiplexing system. (OMOD and ODEMOL denote an OFDM-modulator and demodulator, respectively.)
and the receive pulse shaping filters, \( h_{\text{rsh}}(\cdot)(t) \) is the impulse response of the subcarrier channel, and \( * \) is the convolution operator. Taking samples at a rate of \( T_s = 1/(NF_s) \), where \( F_s \) is the subcarrier bandwidth, we obtain the discrete-time impulse response \( h_{\text{m,}\ell}(nT_s) \).

The signal received at the \( m \)-th antenna can now be written as

\[
\mathbf{r}_m[n] = \sum_{i=0}^{M_T-1} \left[ \sum_{l=-\infty}^{\infty} h_{\text{m,}\ell}[l] \mathbf{s}_i[n-l] \right] + \boldsymbol{\rho}_m[n]
\]

(1)

where \( \boldsymbol{\rho}_m[n]|_{m=0,1,\ldots,M_R-1} \) is stationary additive (potentially colored) noise observed at the \( m \)-th receive antenna. Using the notation

\[
\begin{align*}
\mathbf{r}[n] &= [r_0[n] \quad r_1[n] \quad \cdots \quad r_{M_R-1}[n]]^T \\
\mathbf{s}[n] &= [s_0[n] \quad s_1[n] \quad \cdots \quad s_{M_T-1}[n]]^T \\
\mathbf{\rho}[n] &= [\boldsymbol{\rho}_0[n] \quad \boldsymbol{\rho}_1[n] \quad \cdots \quad \boldsymbol{\rho}_{M_R-1}[n]]^T
\end{align*}
\]

we can rewrite the input–output relation (1) in vector–matrix form as

\[
\mathbf{r}[n] = \sum_{l=-\infty}^{\infty} \mathbf{H}_l \mathbf{s}[n-l] + \mathbf{\rho}[n]
\]

where \( \mathbf{H}_l = h_{\text{m,}\ell}[l]|_{m=0,1,\ldots,M_R-1, \ell=0,1,\ldots,M_T-1} \). In the following, we assume that the delay spread of the \( M_R \times M_T \) matrix channel is \( L \) taps, i.e.,

\[
\mathbf{H}(e^{j2\pi\theta}) = \sum_{l=0}^{L-1} \mathbf{H}_l e^{-j2\pi l \theta}.
\]

Denoting the reconstructed data symbol corresponding to \( c_{k,l}^{(i)} \) as \( \hat{c}_{k,l}^{(i)} \) and organizing the data symbols into frequency vectors according to \( \hat{\mathbf{c}}_{k,l} = [\hat{c}_{k,l}^{(0)} \quad \hat{c}_{k,l}^{(1)} \quad \cdots \quad \hat{c}_{k,l}^{(M_T-1)}] \) and \( \mathbf{c}_{k,l} = [c_{k,l}^{(0)} \quad c_{k,l}^{(1)} \quad \cdots \quad c_{k,l}^{(M_T-1)}] \) for \( k=0,1,\ldots,N-1, \ell \in \mathbb{Z} \), assuming perfect synchronization, it can be shown that

\[
\begin{align*}
\hat{\mathbf{c}}_{k,l} &= \mathbf{H}(e^{j2\pi\theta/N}) \mathbf{c}_{k,l} + \mathbf{\rho}_{k,l} \\
&= \mathbf{H}(e^{j2\pi\theta/N})^{-1} \mathbf{\rho}_{k,l}
\end{align*}
\]

(2)

where \( \mathbf{\rho}_{k,l} \) is stationary additive Gaussian noise. Thanks to (2), the convolutive mixtures observed at the receive antennas can be separated and equalized by computing

\[
\mathbf{y}_{k,l} = \left[ \mathbf{H}(e^{j2\pi\theta/N}) \right]^{-1} \mathbf{\rho}_{k,l}
\]

(3)

The existence of a zero-forcing (ZF) equalizer assumes that the matrices \( \mathbf{H}(e^{j2\pi\theta/N}) \) for \( k=0,1,\ldots,N-1 \) have full rank. Note that usually, in OFDM systems, forward error correction (FEC) spreads out the information bits across frequency. Therefore, if one or several of the \( \mathbf{H}(e^{j2\pi\theta/N}) \) are rank-deficient or ill-conditioned, the corresponding data vectors will not be decoded. Rather, the receiver will attempt to make use of the FEC in the system and extract this information from the data vectors corresponding to the “well-conditioned tones.” An alternative to ZF equalization is to use a multichannel MMSE equalizer, which combats the problem of rank-deficiency of \( \mathbf{H}(e^{j2\pi\theta/N}) \) at the cost of degraded signal separation performance. In practice, the receiver estimates the \( \mathbf{H}(e^{j2\pi\theta/N}) \) using pilot tones inserted in the transmitted data vectors [4], [10]. In this paper, we will be concerned with the blind estimation of the matrices \( \mathbf{H}(e^{j2\pi\theta/N}) \) for \( k=0,1,\ldots,N-1 \). Note, however, that a direct estimation of the \( \mathbf{H}(e^{j2\pi\theta/N}) \) would be very inefficient since it requires the estimation of \( N M_R M_T \) parameters, which can be a significant number in practical systems. Rather, we will provide an algorithm for blindly estimating the channel impulse response matrix taps \( \mathbf{H}_l(l=0,1,\ldots,L-1) \) from which we can compute estimates of the \( \mathbf{H}(e^{j2\pi\theta/N}) \) using the FFT. This approach requires the estimation of \( L M_R M_T \) parameters. We emphasize that our algorithm requires knowledge of an upper bound on the channel order for appropriate selection of the CP length.

III. BLIND CHANNEL IDENTIFICATION ALGORITHM

In this section, we will introduce the novel channel identification algorithm, and we demonstrate the basic idea using a simple example with two transmit and two receive antennas. The generalization to an arbitrary number of antennas will be discussed in Section IV.

A. Preparation

The basic idea of our algorithm is to perform periodic nonconstant-modulus precoding in the transmitter such that the cyclostationary statistics allow a separate identification of the individual scalar subchannels \( h_{\text{m,}\ell}[n]|_{m=0,1,\ldots,M_R-1,i=0,1,\ldots,M_T-1} \). This is achieved by providing each transmit antenna with a different signature in the cyclostationary domain with the signatures chosen such that for a given cycle, all but one transmit antennas are nulled out. It is therefore possible to identify the matrix channel on a column-by-column basis up to a constant diagonal matrix of phase rotations. After equalization according to (3), the individual symbol streams will be decoded up to a phase rotation, which will, in general, be different for different symbol streams. This remaining ambiguity can be resolved using short training sequences. Alternatively, if differential detection is employed, the phase rotation can be ignored. We note that periodic precoding serves to separate the MIMO channel identification problem into scalar problems, and the redundancy introduced by the CP is used to blindly identify the scalar subchannels.

Periodic Precoding: Our approach is based on periodic nonconstant-modulus precoding, which consists of multiplying the individual data streams by \( P \)-periodic precoding sequences prior to transmission. The precoding sequences have to be different for different transmit antennas. More specifically, the precoded transmit signal corresponding to the \( i \)-th transmit antenna is given by

\[
s_i[n] = \sum_{k=0}^{\infty} c_{k,l}^{(i)} \delta[n-iM_R] \sum_{l=0}^{N-1} c_{k,l}^{(i)} e^{j2\pi\theta/N}(n-lM_R)
\]

(4)
where \( a_i^{(i)} = a_{k+P} \) is the \( i \)th \( P \)-periodic precoding sequence. In order to keep the average transmit SNR constant, the precoding sequences have to be normalized such that \( \sum_{i=0}^{P-1} |a_i^{(i)}|^2 = P(0, 1, \ldots, M_T-1). \) From (4), it follows that all the (complex) data symbols in the \( l \)th OFDM symbol transmitted from the \( i \)th antenna are multiplied by \( a_i^{(i)} \) before the IFFT is applied. Equivalently, this multiplication can be performed on the time-domain samples after the IFFT and parallel-to-serial conversion. We note that this form of periodic precoding has previously been suggested by Serpedin and Giannakis in [24] to introduce cyclostationarity in the transmit signal, thereby making blind channel identification based on second-order statistics in symbol-rate sampled single-carrier systems possible. The more general idea of transmitter-induced cyclostationarity has been suggested previously in [25] and [26].

We emphasize that nonconstant-modulus periodic precoding will incur a loss in spectral efficiency. The reason for this is that for a given average transmit power, the minimum distance between symbols decreases, and hence, in order to achieve a given bit error rate, a higher average transmit power is required. Therefore, even though the transmission rate is kept constant, there is a loss in spectral efficiency. In this sense, it is desirable to keep the amplitude variation in the precoding sequences as small as possible. Finally, because of the time-varying transmit SNR, the power amplifiers in the system need a higher dynamic range, which increases hardware complexity. The impact of nonconstant-modulus precoding on bit error rate (BER) performance of multiantenna OFDM systems is further investigated by means of computer simulations in Section V.

Cyclostationary Statistics: Now, defining the \( M_R \times M_R \) correlation matrix of the vector random process \( \mathbf{r}[\tau] \) as\(^1\)
\[
\mathbf{c}_r[n, \tau] = \mathbb{E}\{\mathbf{r}[\tau]\mathbf{r}^H[n - \tau]\}
\]
and assuming that the data sequences \( k_d^{(i)} \) are statistically independent of the noise and satisfy
\[
\mathbb{E}\{k_d^{(i)}, k_d^{(i')}, \ell'\} = \sigma^2 \delta[k - k'] \delta[l - \ell'] \delta[i - \ell]
\]
it is shown in the Appendix that
\[
\mathbf{c}_r[n, \tau] = \sum_{l=0}^{\infty} \mathbf{H}_r \sum_{i=0}^{M_T-1} \mathbf{c}_r[i, \tau] \mathbf{H}_r^H + \mathbf{c}_r[\tau]
\]
(5)
where \( \mathbf{c}_r[\tau] = \mathbb{E}\{\mathbf{r}[\tau]\mathbf{r}^H[\tau]\} \), and \( \mathbf{c}_r[n, \tau] = \mathbb{E}\{\mathbf{s}[n]\mathbf{s}^H[n - \tau]\} = \mathbb{E}\{\mathbf{s}[n]\mathbf{s}^H[\tau]\} \) (recall that the different transmit signals were assumed to be uncorrelated). The scalar correlation functions \( \phi_{\tau}[n, \tau] \) are given by
\[
\phi_{\tau}[n, \tau] = \begin{cases} 
\sigma^2 \sum_{k=-\infty}^{\infty} |a_k^{(i)}|^2 \text{rect}_{[0,M-1]}[n - LM], & \tau = 0 \\
\sigma^2 \sum_{k=-\infty}^{\infty} |a_k^{(i)}|^2 \text{rect}_{[N,M-1]}[n - LM], & \tau = N \\
\sigma^2 \sum_{k=-\infty}^{\infty} |a_k^{(i)}|^2 \text{rect}_{[N,M-1]}[n - LM], & \tau = -N \\
0, & \text{else}
\end{cases}
\]
\(^1\mathbb{E}\) stands for the expectation operator, and the superscript \( H \) denotes conjugate transposition.

Now, using the \( P \)-periodicity of the precoding sequences \( a_i^{(i)} \), it can be verified that \( \phi_{\tau}[n, \tau] = \phi_{\tau}[n + PM, \tau] \) for \( i = 0, 1, \ldots, M_T - 1 \). Consequently, we have
\[
\phi_{\tau}[n, \tau] = \phi_{\tau}[n + PM, \tau]
\]
(6)
which shows that \( \phi_{\tau}[n] \) is a cyclostationary random process with period \( PM \). By cyclostationary vector random process with period \( PM \), we mean that each of the entries in the vector \( \mathbf{s}[n] \) is a scalar cyclostationary random process with cyclostationarity period \( PM \). Using (6), it follows from (5) that \( \phi_{\tau}[n, \tau] = \phi_{\tau}[n + PM, \tau] \), and hence, \( \phi_{\tau}[n] \) is a cyclostationary vector random process with period \( PM \) as well.

Since \( \phi_{\tau}[n, \tau] \) is \( PM \)-periodic in \( n \), we can expand it into a Fourier series with respect to \( \tau \) with the Fourier series coefficient matrices given by
\[
\mathbf{C}_r[k, \tau] = \frac{1}{PM} \sum_{n=0}^{PM-1} \phi_{\tau}[n, \tau] e^{-j(2\pi/nk)\tau} \]
k = 0, 1, \ldots, PM - 1.
\]
Next, applying a \( z \)-transform with respect to \( \tau \), we obtain the cyclic power spectral matrices
\[
\mathbf{S}_r[k, z] = \sum_{\tau=-\infty}^{\infty} \mathbf{C}_r[k, \tau] z^{-\tau}
\]
(7)
which are shown in the Appendix to be given by
\[
\mathbf{S}_r[k, z] = \mathbf{H}(z) \mathbf{S}_n[k, z] \mathbf{H}(z) + \mathbf{S}_n(z) \delta[k]
\]
where \( \mathbf{H}(z) = \mathbf{H}^H(1/z) \), \( \mathbf{S}_n(z) = \sum_{\tau=-\infty}^{\infty} \mathbf{c}_n[\tau] z^{-\tau} \), and
\[
\mathbf{S}_n[k, z] = \mathbf{S}_n[k, z] - \mathcal{D}_N[\tau] \delta[k]
\]
(8)
where \( \mathcal{D}_N[\tau] = \sum_{\tau=-\infty}^{\infty} \delta[\tau - \tau N] \delta[k]
\]
In the Appendix, it is shown that the Fourier series coefficient matrices \( \mathbf{C}_s[k, \tau] \) are given by
\[
\mathbf{C}_s[k, \tau] = \text{diag}\{\phi_{\tau}[n, \tau]\}_{n=0}^{M_T-1} \frac{N}{PM} \delta_N[\tau] \mathcal{A}_0 \mathcal{D}_0 \left[ \tau, \frac{k}{PM} \right]
\]
(9)
\[ A(g) = \frac{k}{PM} \left[ \begin{array}{c} \frac{1}{N} e^{-j2\pi(k/PM)\tau} \\ \frac{\sin\left(\frac{\pi}{PM}(M-\tau)\right)}{\sin\left(\frac{\pi}{PM}\right)} \\ \frac{\sin\left(\frac{\pi}{PM}(M+\tau)\right)}{\sin\left(\frac{\pi}{PM}\right)} \end{array} \right], \quad 0 \leq \tau \leq M-1 \]

\[ \frac{1}{N} e^{-j2\pi(k/PM)(M+\tau)} \left[ \begin{array}{c} \frac{1}{N} e^{-j2\pi(k/PM)\tau} \\ \frac{\sin\left(\frac{\pi}{PM}(M-\tau)\right)}{\sin\left(\frac{\pi}{PM}\right)} \\ \frac{\sin\left(\frac{\pi}{PM}(M+\tau)\right)}{\sin\left(\frac{\pi}{PM}\right)} \end{array} \right], \quad -M+1 \leq \tau < 0. \]

**B. Basic Identification Algorithm**

We will next explain our algorithm using a simple example with two transmit and two receive antennas. Considering the 2 \times 2 case simplifies the presentation and conveys the basic ideas underlying the algorithm. The more general case with an arbitrary number of transmit and receive antennas will be discussed in Section IV.

We use the normalized four-periodic precoding sequences \( \alpha^{(0)} = \{0.9513, 1.0464, 0.9513, 1.0464\} \) and \( \alpha^{(1)} = \{-1.0464, 0.9513, 0.9513, 1.0464\} \). From (9), it follows that \( \Phi_P^{(0)} = \{4.0000, 0.0000, -0.3801, 0.0000\} \) and \( \Phi_P^{(1)} = \{-4.0000, 0.1900, 0.0000, 0.1900 - j0.1900\} \). In the following, in order to keep the discussion more general, we will stick to the notation \( P \) for the period of the precoding sequences instead of specializing to \( P = 4 \). From (7), we obtain

\[
\begin{align*}
[S_t[k, z]]_{k_00} &= H_{000} \left( ze^{j(2\pi/kPM)k} \right) S_s^{(0)}[k, z] H_{000}(z) \\
&+ H_{010} \left( ze^{j(2\pi/kPM)k} \right) S_s^{(1)}[k, z] H_{010}(z) \\
[S_t[k, z]]_{k_11} &= H_{100} \left( ze^{j(2\pi/kPM)k} \right) S_s^{(0)}[k, z] H_{100}(z) \\
&+ H_{110} \left( ze^{j(2\pi/kPM)k} \right) S_s^{(1)}[k, z] H_{110}(z)
\end{align*}
\]

(11)

where \( H(z) = H^*(1/z^*) \) and \( k \neq 0 \). The basic idea of our channel identification algorithm now is to find cycles \( k_1 \neq 0 \) and \( k_2 \neq 0 \) such that

\[
\begin{align*}
S_s^{(0)}[k_1, z] &= 0, \quad S_s^{(1)}[k_1, z] \neq 0 \\
S_s^{(0)}[k_2, z] &= 0, \quad S_s^{(1)}[k_2, z] \neq 0.
\end{align*}
\]

(13)

Now, since \( S_s^{(0)}[k, z] = \sum_{\tau = -\infty}^{\infty} C_s^{(0)}[k, \tau] z^{-\tau} \), it follows that \( S_s^{(0)}[k, z] = 0 \forall z \in C \) if and only if \( C_s^{(0)}[k, \tau] = 0 \forall \tau \in \mathbb{Z} \). From (8), it follows that the \( C_s^{(0)}[k, \tau] \) differ only in the \( \Phi_P^{(0)}[k] \). Consequently, (13) can be satisfied by picking \( k_1 \) and \( k_2 \) such that

\[
\Phi_P^{(0)}[k_1] = 0, \quad \Phi_P^{(0)}[k_2] \neq 0 \\
\Phi_P^{(1)}[k_2] = 0, \quad \Phi_P^{(1)}[k_2] \neq 0.
\]

(14)

Clearly, setting \( k_1 = 1 \) and \( k_2 = 2 \) satisfies (14). In the following, in order to keep the discussion more general, we will stick to the notation \( k_1 \) and \( k_2 \) for the cycles. Specializing (11) and (12) and using \( \Phi_P^{(1)}[k] = \Phi_P^{(0)}[k] \), we obtain

\[
[S_t[k, z]]_{k_00} = H_{000} \left( ze^{j(2\pi/kPM)k_1} \right) S_s^{(0)}[k_1, z] H_{000}(z) \\
[S_t[k, z]]_{k_11} = H_{100} \left( ze^{j(2\pi/kPM)k_1} \right) S_s^{(1)}[k_1, z] H_{100}(z)
\]

(15)

\[
[S_t[k, z]]_{k_00} = H_{000} \left( ze^{j(2\pi/kPM)k_2} \right) S_s^{(0)}[k_2, z] H_{000}(z) \\
[S_t[k, z]]_{k_11} = H_{110} \left( ze^{j(2\pi/kPM)k_2} \right) S_s^{(1)}[k_2, z] H_{110}(z)
\]

(16)

Now, we can use a modified version of an algorithm first proposed by Tong et al. in [27] and later extended by Serpedin and Giannakis [24] to identify the scalar subchannels \( H_{000}(z) \) and \( H_{110}(z) \) from \( [S_t[k, z]]_{k_00} \) and \( [S_t[k, z]]_{k_11} \), respectively, and the subchannels \( H_{010}(z) \) and \( H_{100}(z) \) from \( [S_t[k, z]]_{k_00} \) and \( [S_t[k, z]]_{k_11} \), respectively. By proper design of the periodic precoding sequences, we have broken up the \( 2 \times 2 \) matrix channel identification problem into the identification of four scalar subchannels.

The algorithm we are using in the following to identify the individual scalar subchannels has been used previously in CP OFDM systems [28] and in pulse shaping OFDM systems [29], [30]. We will therefore keep the presentation of the identification algorithm short. Furthermore, we restrict the discussion to the identification of \( H_{000}(z) \). The remaining subchannel filters can be identified using the same procedure. Starting from (17), we get

\[
[S_t[k_2, z]]_{k_00} = S_s^{(0)}[k_2, z] H_{000}(z) \left( ze^{j(2\pi/kPM)k_2} \right) \\
[S_t[k_2, z]]_{k_11} = S_s^{(0)}[k_2, z] H_{110}(z) \left( ze^{j(2\pi/kPM)k_2} \right)
\]

(17)

Straightforward manipulations show that (19) can be rewritten as

\[
\sum_{n=0}^{L_{k_00}-1} \left[ \Phi_{s_{k_00}}[k_2, z] \right]_{n} z^{-n} = 0, \quad n \in \mathbb{Z}
\]

(20)

where

\[
\Phi_{s_{k_00}}[k_2, z] = \left[ C_{f_000} k_00 C_{s_{000}}[z, z-n] \right]
\]

(21)

and \( L_{k_00} \) is the length of the impulse response of the subchannel filter \( H_{k_00}(z) \). When the exact value of \( L_{k_00} \) is not known, it can be shown from [24] that our algorithm is not sensitive to channel-order overestimation (and likewise fails with channel order underestimation). Since OFDM systems require an upper bound on the channel order to correctly choose the CP length, a safe estimate of \( L_{k_00} \) is always available and known to the receiver. The form of the solution when the channel is overestimated is omitted. See [24] for further discussion on this point. From (8) and (10), it follows that \( C_s^{(0)}[k, \tau] \neq 0 \) for \( \tau = -N, 0, N, \) and \( C_s^{(0)}[k, \tau] = 0 \) else. This implies that

\[
\Phi_{s_{k_00}}[k_2, z] = \left[ C_{f_000} k_00 C_{s_{000}}[z, 0] + C_{f_000} k_00 C_{s_{000}}[z, N] + [C_{f_000} k_00 + C_{f_000} k_00 C_{s_{000}}[z, -N].
\]

Authorized licensed use limited to: ETH BIBLIOTHEK ZURICH. Downloaded on February 1, 2010 at 04:32 from IEEE Xplore. Restrictions apply.
Furthermore, \(e^{j(k_0^0)\theta_0[0]} = 0\) for \(|n| \geq 2M + L_{k_0,0} - 2\). From (7), it follows that the influence of stationary noise (with arbitrary correlation function) can be minimized by considering nonzero cycles \(k \in [1, PM - 1]\) only. In order to solve (20) for the channel, we rewrite the equation in vector-matrix form as

\[
\begin{bmatrix}
T_{r_0}^{k_0^0,k_0^0} D^{k_0^0} - T_{r_0}^{k_0^0,k_0^0} D^{k_0^0} \\
G_{r_0}^{k_0^0,k_0^0}
\end{bmatrix}
\begin{bmatrix}
\mathbf{h}_{0,0} \\
\mathbf{h}_{0,0}
\end{bmatrix}
= \mathbf{0}
\tag{22}
\]

with the \((4M + 3L_{k_0,0} - 6) \times L_{k_0,0}\) Toeplitz matrices \(T_{r_0}^{k_0^0, k_0^0}\) with first row

\[
\begin{bmatrix}
\alpha_{r_0}^{k_0^0} \left[-2M - L_{k_0,0} + 3 \right] & 0 & \ldots & 0
\end{bmatrix}
\]

and first column shown at the bottom of the page, and the \(L_{k_0,0} \times L_{k_0,0}\) diagonal matrix \( \mathbf{D} = \text{diag} \{ e^{-j(2\pi/N)k_0} \}_{k_0,0=0}^{M-1} \). Furthermore

\[
\mathbf{h}_{0,0} = [h_{0,0}[0] \quad h_{0,0}[1] \quad \ldots \quad h_{0,0}[L_{k_0,0} - 1]]^T
\]

and \( \mathbf{q} \) denotes the \((4M + 3L_{k_0,0} - 6) \times 1\) all-zero vector. Using (17) and (19) and [24, th. 1], it can be shown that the channel \( \mathbf{h}_{0,0} \) is uniquely identifiable within a phase ambiguity (irrespective of channel zeros) if and only if there is no \( I \in [1, L_{k_0,0} - 1] \) such that \( e^{j(k_0^0)\theta_0[I]} = 1 \). This implies that identifiability is guaranteed for \( L_{k_0,0} \leq (PM/2)_{k_0} \). Since OFDM symbols are all-zero vector. Using

\[
\left[ \alpha_{r_0}^{k_0^0} \left[-2M - L_{k_0,0} + 3 \right] & 0 & \ldots & 0
\right]
\]

as the perfect estimate of \( \hat{\mathbf{h}}_{0,0} \) as

\[
\hat{\mathbf{h}}_{0,0} = \frac{1}{|I|} \sum_{l \in I} \left[ \mathbf{C}_r[k, \tau], \mathbf{h}_{1,0}[n] \right]^T
\]

and then solving the following optimization problem:

\[
\hat{\mathbf{h}}_{0,0} = \arg \min_{\mathbf{h}_{0,0}} \left\| \mathbf{G}_{r_0}^{k_0^0,k_0^0} \mathbf{h}_{0,0} \right\|^2
\tag{24}
\]

where \( \mathbf{G}_{r_0}^{k_0^0,k_0^0} \) is an estimate of \( \mathbf{G}_{r_0}^{k_0^0,k_0^0} \) obtained by replacing \( \mathbf{C}_r[k, \tau], \mathbf{h}_{1,0}[n] \) in (24) by the estimates \( \mathbf{C}_r[k, \tau], \mathbf{h}_{1,0}[n] \). The solution of (24) is found as the eigenvector of \( \mathbf{G}_{r_0}^{k_0^0,k_0^0} \mathbf{G}_{r_0}^{k_0^0,k_0^0} \) associated with the smallest eigenvalue. The remaining subchannel filters \( H_{0,1}(z), H_{1,1}(z), \) and \( H_{1,0}(z) \) can be identified by performing the same procedure as above for (15), (16), and (18), respectively. For a discussion of further aspects of the algorithm and an extension to the multicycle case (which potentially improves the estimator performance), see [27] and [28].

We finally note that periodic precoding increases the cyclostationarity period by a factor of \( P \), and hence, longer data records are required to arrive at good estimates of \( \mathbf{C}_r[k, \tau] \). Therefore, in practice, one should aim at precoding sequences with small \( P \). Unfortunately, to guarantee at least \( M_0 \) zero cycles, we need \( P \geq M_0 + 1 \). (Recall that the cycle \( k = 0 \) is not admissible.) Larger \( P \) allows for more zero cycles and, hence, makes the application of multicycle approaches possible, which potentially improves estimator performance.

C. Resolving the Complex Scale Ambiguity

From (20), it can be seen that each of the scalar subchannels has now been identified up to a complex constant, which, in general, will be different for different subchannels. We will next describe a two-step procedure for resolving this remaining ambiguity up to a diagonal matrix of phase rotations. Assume that the scalar subchannel filters have been identified up to a complex constant denoted as \( r_m e^{j\phi_m} \) for the filter \( H_{m,0}(z) \). Denoting \( H_{m,0}(z) \) as the perfect estimate of \( H_{m,0}(z) \) up to the scaling factor \( r_m e^{j\phi_m} \), we get from (17)

\[
[S_r[k_2, z]]_{1,0} = \sum_m H_{m,0}(z) e^{j(2\pi/M)k_2} S_0[k_2, z] \hat{H}_{0,0}(z)
\]

This suggests to form an estimate of \( r_0 \) as

\[
\hat{r}_0 = \left[ \begin{array}{c}
\mathbf{C}_r[k_2, 0] \\
\mathbf{C}_h[k_2, 0]
\end{array} \right]_{1,0}, n \in \mathbb{I}
\]

where

\[
\hat{r}_0 \left[ n \right] = \sum_m \alpha_{r_0}^{k_0^0} [k_2 - n - n] \sum_m \hat{h}_{m,0} \left[ n + n \right] e^{j(2\pi/M)k_2} S_0[k_2, z] \hat{H}_{0,0}(z)
\]

The estimator performance can potentially be improved by averaging over the interval \( \mathbb{I} \) according to

\[
\hat{r}_0 = \frac{1}{|\mathbb{I}|} \sum_{l \in \mathbb{I}} \left[ \mathbf{C}_r[k_2, 0], \mathbf{h}_{1,0}[n] \right]
\]

The remaining subchannel gains \( r_{0,1}, r_{1,1}, \) and \( r_{1,0} \) can be estimated by performing the same procedure as above for (15), (16), and (18), respectively. Let us next show how the phase ambiguity due to \( e^{j\phi_{0,1}} \) can be reduced by considering the cross terms of \( S_r[k_2, z] \). Now, denoting \( H_{0,0}(z) \) as the perfect estimate of \( H_{0,0}(z) \) including the channel gain \( r_{0,0} \), it follows from (7) that

\[
[S_r[k, z]]_{1,0} = H_{1,0}(z) e^{j(2\pi/M)k} k
\]

\[
\times S_0[k, z] \hat{H}_{0,0}(z) e^{j\phi_{0,0} - \phi_{0,0}}
\]

\[
+ H_{1,0}(z) e^{j(2\pi/M)k} k
\]

\[
\times S_0[k, z] \hat{H}_{0,0}(z) e^{j\phi_{0,0} - \phi_{0,0}}
\]

\[
[S_r[k, z]]_{0,1} = H_{0,0}(z) e^{j(2\pi/M)k} k
\]

\[
\times S_0[k, z] \hat{H}_{1,0}(z) e^{j\phi_{0,0} - \phi_{0,0}}
\]

\[
+ H_{0,0}(z) e^{j(2\pi/M)k} k
\]

\[
\times S_0[k, z] \hat{H}_{1,0}(z) e^{j\phi_{0,0} - \phi_{0,0}}
\]

\[
[S_r[k, z]]_{0,1} = H_{0,0}(z) e^{j(2\pi/M)k} k
\]

\[
\times S_0[k, z] \hat{H}_{1,0}(z) e^{j\phi_{0,0} - \phi_{0,0}}
\]

\[
+ H_{0,0}(z) e^{j(2\pi/M)k} k
\]

\[
\times S_0[k, z] \hat{H}_{1,0}(z) e^{j\phi_{0,0} - \phi_{0,0}}
\]

\[
[S_r[k, z]]_{0,1} = H_{0,0}(z) e^{j(2\pi/M)k} k
\]

\[
\times S_0[k, z] \hat{H}_{1,0}(z) e^{j\phi_{0,0} - \phi_{0,0}}
\]

\[
+ H_{0,0}(z) e^{j(2\pi/M)k} k
\]

\[
\times S_0[k, z] \hat{H}_{1,0}(z) e^{j\phi_{0,0} - \phi_{0,0}}
\]

\[
[S_r[k, z]]_{0,1} = H_{0,0}(z) e^{j(2\pi/M)k} k
\]

\[
\times S_0[k, z] \hat{H}_{1,0}(z) e^{j\phi_{0,0} - \phi_{0,0}}
\]

\[
+ H_{0,0}(z) e^{j(2\pi/M)k} k
\]

\[
\times S_0[k, z] \hat{H}_{1,0}(z) e^{j\phi_{0,0} - \phi_{0,0}}
\]
Using $S_s^{(1)}[k_2,z] = 0$, this suggests an estimation of the phase difference $\phi_k = \phi_{k,0} - \phi_{k,0}$ as

$$\hat{\phi}_k = \arg \left\{ \left[ \hat{C}_r[k_2,n] \right]_{1,0} \right\}, \quad n \in \mathcal{I}$$

where $\hat{C}_r[k_2,n] = \sum_{u=0}^{\infty} C_s^{(0)}[k_2,n-u] \sum_{n=0}^{\infty} \hat{h}_{k,0}[u + l_k e^{j2\pi/PM}k_0(u+\eta)]_n$ with $\hat{h}_{k,0}[n]$ denoting the estimate of $h_{k,0}[n]$, including the estimated scale factor $\eta_{k,0}$ and $\mathcal{I} := \left\{ n \mid \hat{h}_{k,0}[n] \neq 0 \right\}$. Again, the estimator performance can potentially be improved by averaging over the interval $\mathcal{I}$ according to

$$\hat{\phi}_k = \frac{1}{|\mathcal{I}|} \sum_{n \in \mathcal{I}} \arg \left\{ \left[ \hat{C}_r[k_2,n] \right]_{1,0} \right\}.$$

Further averaging can be done by considering

$$\hat{S}_r[k_2,z]_{0,1} = \hat{H}_k^{(0)}(e^{j2\pi/PM}k_2) \times S_s^{(0)}[k_2,z] \hat{H}_k^{(1)}(z) e^{-j\phi_{k,0} - \phi_{k,0}}$$

(25)

and performing the same procedure as above. The phase difference $\phi_{k,0} - \phi_{k,0}$ can be estimated similarly from $\hat{C}_r[k_2,n]$ and $\left[ \hat{C}_r[k_2,n] \right]_{1,0}$. Now, since we have estimated the channel gains $\hat{h}_{m,i}(m,i) = 0,1$ and the phase differences $\phi_{k,0} - \phi_{k,0}$ and $\phi_{k,1} - \phi_{k,1}$, we have identified the $2 \times 2$ channel transfer matrix $\hat{H}(z)$ up to a diagonal matrix of phase rotations. The only ambiguities left are the $\phi_{k,i}$ ($i = 0,1$) or, equivalently, any matrix

$$\hat{H}'(z) = \hat{H}(z) \text{ diag } \left\{ e^{j\phi_{k,i}} \right\}_{i=0,1}$$

with arbitrary $\phi_{k,i}$ ($i = 0,1$) is a valid solution of our algorithm. Now, since

$$\hat{H}'(e^{j2\pi/PM}k) = \hat{H}(e^{j2\pi/PM}k) \text{ diag } \left\{ e^{j\phi_{k,i}} \right\}_{i=0,1}$$

we can see that equalization according to (3) recovers the individual data streams up to a phase rotation.

IV. EXTENSION OF THE ALGORITHM AND DESIGN OF THE PRECODING SEQUENCES

In this section, we describe how the new algorithm can be generalized to an arbitrary number of transmit and receive antennas, and we discuss the design of the precoding sequences.

A. Arbitrary Number of Antennas

Channel Identification: Starting from (7), we obtain

$$S_r[k,z] = \hat{H}(2e^{j2\pi/PM}k) \begin{bmatrix} S_s^{(0)}[k,z] & 0 & \ldots & 0 \\ 0 & S_s^{(1)}[k,z] & \ldots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & S_s^{(M-1)}[k,z] \end{bmatrix} \times \hat{H}(z) + S_n(z)e^{j\phi_k}$$

(26)

Assume that the periodic precoding sequences $a_k^{(i)}$ have been designed such that for each $i \in [0,M-1]$, there is at least one $k' \in [1,P-1]$ that satisfies $\phi_k^{(i)}[k'] \neq 0$ and $\phi_k^{(i)}[k'] = 0$ for $k' \neq i$. This implies that $S_s^{(i)}[k',z] \neq 0$ and $S_s^{(i)}[k',z] = 0$ for $k' \neq i$. In the cyclostationary domain, this causes the $i$th transmit antenna to be “active” at cycle $k'$, whereas all other antennas are “inactive.” From (26), it then follows that

$$S_r[k',z]_{m,n} = S_s^{(k')}[k',z]H_{m,n}(e^{j2\pi/PM}k') \hat{H}_{m,n}(z)$$

$m = 0,1,\ldots,M-1$.

(27)

The filters $H_{m,n}(z)(m = 0,1,\ldots,M-1)$ or, equivalently, the $i$th column of the channel transfer matrix $\hat{H}(z)$ can now be identified by performing a scalar subchannel by subchannel identification on the $S_r[k',z]_{m,n}$ using the subspace-based algorithm discussed in Section III. This process has to be repeated for all columns $i \in [0,M-1]$.

Resolving the Complex Scale Ambiguity: Once all the scalar subchannel filters have been identified, we can resolve the remaining channel gain and phase ambiguities by proceeding as follows. An estimate of $\hat{r}_{m,i}$ can be obtained as

$$\hat{r}_{m,i} = \frac{1}{|\mathcal{J}|} \sum_{n \in \mathcal{J}} \left\{ \frac{\hat{C}_r[k',n]}{\hat{h}_{k,i}[n]} \right\}_{m,i}, \quad n \in \mathcal{I}$$

where $\hat{h}_{k,i}[n] = \sum_{u=0}^{\infty} C_s^{(0)}[k',n-u] \sum_{n=0}^{\infty} \hat{h}_{k,i}[u + l_k e^{j2\pi/PM}k_0(u+\eta)]_n$ with $\hat{h}_{k,i}[n]$ denoting the estimate of $h_{k,i}[n]$ obtained from (24) and $\mathcal{J} := \left\{ n \mid \hat{h}_{k,i}[n] \neq 0 \right\}$. This procedure has to be repeated for all subchannel gains. The phase differences $\phi_{k,i}(m,n) = \phi_{k,i} - \phi_{k,i}$ can be estimated according to

$$\hat{\gamma}_{k,i}(m,n) = \frac{1}{|\mathcal{J}_{m,n}|} \sum_{n \in \mathcal{J}_{m,n}} \left\{ \arg \left\{ \hat{C}_r[k',n] \right\}_{m,n} \right\}$$

where $\hat{r}_{k,i}[n] = \sum_{u=0}^{\infty} C_s^{(0)}[k',n-u] \sum_{n=0}^{\infty} \hat{h}_{k,i}[u + l_k e^{j2\pi/PM}k_0(u+\eta)]_n$ with $\hat{h}_{k,i}[n]$ denoting the estimate of $h_{k,i}[n]$ including the estimated gain $\hat{g}_{k,i}$, and

$$\mathcal{J}_{m,n} := \left\{ n \mid \hat{h}_{k,i}[n] \neq 0 \right\}.$$  

Further averaging can be done by considering $S_r[k',z]_{m,n}$ and performing the same procedure. Now, we have identified the channel transfer matrix $\hat{H}(z)$ up to a diagonal matrix of phase rotations, i.e., any matrix

$$\hat{H}(z) = \hat{H}(z) \text{ diag } \left\{ e^{j\phi_{k,i}} \right\}_{i=0,1}$$

with arbitrary $\phi_{k,i}$ ($i = 0,1,\ldots,M-1$) is also a valid solution of our algorithm. Again, it follows that equalization according to (3) recovers the individual symbol streams up to a phase rotation.

B. Design of the Precoding Sequences

We will next discuss the design of the precoding sequences and related tradeoffs on channel estimator performance.

Simple Construction: We provide a simple construction of periodic precoding sequences for an arbitrary number of transmit antennas. This method shows the existence of proper
precoding sequences for any $M_T$. Let $P = \prod_{i=0}^{M_T-1} P_i$ be the product of prime integers $P_i$ ($i = 0, 1, \ldots, M_T - 1$), and suppose the squared precoding sequence for the $i$th transmit antenna has period $P_i$ (and, hence, period $P$ as well). Let $C^{(i)}_k (k = 0, 1, \ldots, P_i - 1)$ denote the Fourier series coefficients of this squared sequence with respect to period $P_i$, and let $C^{(i)}_k (k = 0, 1, \ldots, P - 1)$ denote the Fourier series coefficients of the same sequence with respect to period $P$. It is now easy to show that

$$C^{(i)}_k = \left\{ \begin{array}{l} P_i^{-1} C^{(i)}_{k P_i} \quad k = l P_i \forall l \in \mathbb{Z} \\
\text{otherwise,} \end{array} \right.$$

(28)

Thus, the $i$th sequence has zeros in its Fourier series, except where the cycle is a multiple of $P_i$. Since we have chosen the periods to be prime, the nonzero cycles are mutually exclusive. Now, noting that the precoding sequences are nonconstant-modulus, it follows that every sequence has at least one nonzero cycle with index $k \neq 0$. Hence, if cycle $k$ for sequence $i$ is nonzero, then cycle $k$ will be zero for all the other sequences.

As an example, suppose $M_T = 3$ with $P_0 = 2$, $P_1 = 3$, $P_2 = 5$, and hence $P = 30$. Antenna 1 is active for cycles $k = 0, 15$, antenna 2 is active for cycles $k = 0, 10, 20$, and antenna 3 is active for cycles $k = 0, 6, 12, 18, 24$. Thus, we can use cycle $k = 15$ to estimate the channels induced by the first transmit antenna, and cycles $k = 10, 20$ to estimate the channels induced by the second transmit antenna, and cycles $k = 6, 12, 18, 24$ to estimate the channels induced by the third transmit antenna.

While such a construction guarantees the presence of zero cycles, it leads to large periods, especially as $M_T$ increases. Therefore, it is of interest to design shorter precoding sequences with zeros in specific prescribed locations in the spectrum. We present one such construction in the following.

**Alternative Design Procedure**: In the following, let $a^{(i)}$ denote the $P \times 1$ vectors given by

$$a^{(i)} = \left[ \begin{array}{c} |a_0^{(i)}|^2 \ |a_1^{(i)}|^2 \ \ldots \ |a_{P-1}^{(i)}|^2 \end{array} \right]^T$$

and

$$\Phi^{(i)} = \left[ \begin{array}{c} \Phi_P^{(i)}[0] \ \Phi_P^{(i)}[1] \ \ldots \ \Phi_P^{(i)}[P-1] \end{array} \right]^T.$$

We then have

$$W_P a^{(i)} = \Phi^{(i)}, \quad i = 0, 1, \ldots, M_T - 1$$

(29)

where $[W_P]_{mn} = e^{-j2\pi mn/P}$ is the DFT matrix of size $P \times P$. Using $W_P^H W_P = P I_P$ in (29), we obtain

$$a^{(i)} = \frac{1}{P} W_P^H \Phi^{(i)} , \quad i = 0, 1, \ldots, M_T - 1.$$  

(30)

The design problem now consists of finding vectors $a^{(i)}$ with strictly positive real elements such that the corresponding $\Phi^{(i)}$ have the desired zero pattern. Let us demonstrate how this can be done.
done for the case of two transmit antennas (arbitrary number of receive antennas) and precoding sequences of period 4. Our goal is to design $\mathbf{a}^{(0)}$ and $\mathbf{a}^{(1)}$ such that $\Phi^{(0)} = [x \ 0 \ 0 \ 0]$ and $\Phi^{(1)} = [x \times 0 \ 0 \times]$ with the $x$ being nonzero. We note that two zeros in $\Phi^{(0)}$ are not required; however, their presence enables improved estimator performance using multicycle techniques (see, e.g., [24]). Therefore, we have to satisfy the following equation:

$$
\begin{bmatrix}
[1 \ 1 \ 1 \ 1] \\
[1 \ -j \ -1 \ j] \\
[1 \ -1 \ 1 \ -j] \\
[1 \ j \ -1 \ -j]
\end{bmatrix}
\begin{bmatrix}
x \\
x \\
x \\
x
\end{bmatrix}
= 0
$$

with the elements in $\mathbf{a}^{(0)}$ and $\mathbf{a}^{(1)}$ being strictly positive real. By inspection, one possible solution follows as $\Phi^{(0)} = [a \ 0 \ c \ 0]^T$ with $a, c \in \mathbb{R}$ and $a + c > 0$ and $\Phi^{(1)} = [a \ b \ 0 \ b^*]^T$ with $b \in \mathbb{C}$, $a + 2\text{Re}(\{b\}) > 0$, and $a + 2\text{Im}(\{b\}) > 0$. This choice for the $\Phi$-vectors results in $\mathbf{a}^{(0)} = [a + c \ a - c \ a + c \ a - c]^T$ and $\mathbf{a}^{(1)} = [a + 2\text{Re}(\{b\}) \ b \ a - 2\text{Re}(\{b\}) \ a + 2\text{Im}(\{b\})]^T$. Finally, the magnitude of the coefficients $\alpha_{\mathbf{r}}^{(0)}$ and $\alpha_{\mathbf{r}}^{(1)}$ can be obtained from $\mathbf{a}^{(0)}$ and $\mathbf{a}^{(1)}$ by taking the square root of the individual elements. The phases assigned to the $\alpha_{\mathbf{r}}^{(0)}$ and $\alpha_{\mathbf{r}}^{(1)}$ can be chosen arbitrarily.

**Tradeoffs on Estimator Performance:** In the above example, it is obvious that making $|\theta|$ and $|c|$ as large as possible will improve the channel estimator performance due to better separation of the individual transmit signals’ cyclostationary statistics [cf. (11) and (12)]. On the other hand, in terms of overall system performance, it is desirable to keep the degree of amplitude variation in the precoding coefficients as small as possible in order to have the transmit SNR as constant as possible over time (see the discussion in Section III-A). In the present example, this requirement translates to having $|\theta|$ and $|c|$ as small as possible. We conclude that in general, in terms of overall system performance, there will be an optimum choice for the precoding sequences, which will, however, depend on the remaining system parameters such as the symbol constellation and the specific FEC scheme. In Section V, we will provide a simulation example (see Simulation Example 3) pertaining to this issue. Finally, we emphasize that for a higher number of transmit antennas, better separation properties are required in order to avoid a degradation in estimator performance. Therefore, the choice of the precoding sequences will also depend on $M_T$.

**Examples of Precoding Sequences:** Based on the alternative design procedure discussed above, we designed simple precoding sequences with periods 4, 8, and 16 for two, three, and four transmit antennas, respectively. These precoding sequences are simple since they consist of two different elements $\alpha$ and $\beta$ only, where $\alpha$ and $\beta$ are design parameters. The sequences along with the zero patterns of the corresponding $\Phi_P^{(0)}(\mathbf{a})$ are summarized in Table I. In Tables II–IV, the functions $|\Phi_P^{(0)}(\mathbf{a})|$ are normalized to

**TABLE III**

| Absolute Values $|\Phi_P^{(0)}(\mathbf{a})|$ for $|\mathbf{a}| = 4$ and Normalized Precoding Sequences |
|---------------------------------------------------------------|
| $|\Phi^{(0)}| = [4.0000 \ 0 \ 3.5294 \ 0]^T$                      |
| $|\Phi^{(1)}| = [4.0000 \ 2.4957 \ 2.4957]^T$                      |
| $|\Phi^{(0)}| = [8.0000 \ 0 \ 7.0588 \ 0 \ 0]^T$                      |
| $|\Phi^{(1)}| = [8.0000 \ 4.6114 \ 0 \ 1.9101 \ 0 \ 1.9101 \ 4.6114]^T$ |
| $|\Phi^{(2)}| = [8.0000 \ 4.9913 \ 0 \ 0 \ 0 \ 0 \ 4.9913]^T$          |
| $|\Phi^{(0)}| = [16.0000 \ 0 \ 0 \ 0 \ 0 \ 0 \ 14.1176 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$ |
| $|\Phi^{(1)}| = [16.0000 \ 0 \ 0 \ 9.9827 \ 0 \ 0 \ 0 \ 0 \ 0 \ 9.9827 \ 0 \ 0 \ 0]^T$ |
| $|\Phi^{(2)}| = [16.0000 \ 9.2228 \ 0 \ 0 \ 0 \ 3.8202 \ 0 \ 0 \ 0 \ 3.8202 \ 0 \ 0 \ 0 \ 9.2228]^T$ |
| $|\Phi^{(3)}| = [16.0000 \ 9.0456 \ 0 \ 3.1764 \ 0 \ 2.1224 \ 0 \ 1.7993 \ 0 \ 1.7993 \ 0 \ 2.1224 \ 0 \ 3.1764 \ 0 \ 9.0456]^T$ |

**TABLE IV**

| Absolute Values $|\Phi_P^{(0)}(\mathbf{a})|$ for $|\mathbf{a}| = 8$ and Normalized Precoding Sequences |
|---------------------------------------------------------------|
| $|\Phi^{(0)}| = [4.0000 \ 0 \ 3.8769 \ 0]^T$                      |
| $|\Phi^{(1)}| = [4.0000 \ 2.7414 \ 0 \ 2.7414]^T$                      |
| $|\Phi^{(0)}| = [8.0000 \ 0 \ 0 \ 7.7538 \ 0 \ 0]^T$                      |
| $|\Phi^{(1)}| = [8.0000 \ 5.0565 \ 0 \ 2.0992 \ 0 \ 2.0992 \ 0 \ 5.0565]^T$ |
| $|\Phi^{(2)}| = [8.0000 \ 5.4828 \ 0 \ 0 \ 0 \ 0 \ 5.4828 \ 0]^T$          |
| $|\Phi^{(0)}| = [16.0000 \ 0 \ 0 \ 0 \ 0 \ 0 \ 15.5077 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$ |
| $|\Phi^{(1)}| = [16.0000 \ 0 \ 0 \ 10.9656 \ 0 \ 0 \ 0 \ 0 \ 0 \ 10.9656 \ 0 \ 0 \ 0 \ 0]^T$ |
| $|\Phi^{(2)}| = [16.0000 \ 0 \ 10.1309 \ 0 \ 0 \ 0 \ 4.1964 \ 0 \ 0 \ 0 \ 4.1964 \ 0 \ 0 \ 10.1309 \ 0]^T$ |
| $|\Phi^{(3)}| = [16.0000 \ 9.9362 \ 0 \ 3.4891 \ 0 \ 2.3314 \ 0 \ 1.9764 \ 0 \ 1.9764 \ 0 \ 2.3314 \ 0 \ 3.4891 \ 0 \ 9.9362]^T$ |
corresponding to the normalized precoding sequences specified in Table I have been evaluated numerically for $|\beta/\alpha| = 2, 4,$ and 8, respectively. It is clearly seen that for bigger $|\beta/\alpha|$, better separation is achieved. Note that in Tables II–IV, $[\Phi^{(\xi)}]$ stands for the vector obtained by taking elementwise absolute values in $[\Phi^{(\xi)}]$. We conclude by noting that the minimum period $P$ required using the first design approach is $P = 6$ for $M_T = 2$, $P = 30$ for $M_T = 3$, and $P = 210$ for $M_T = 4$. Although the second design procedure is less systematic, it is preferable since it yields significantly shorter precoding sequences.

V. SIMULATION RESULTS

In this section, we provide simulation results demonstrating the performance of the proposed matrix channel estimation algorithm and the corresponding multichannel equalizer. In all simulation examples, the estimator performance was measured in terms of the average bias given by the expression at the bottom of the page and the mean square error (MSE)

$$
\frac{1}{M_R M_T} \sum_{m=0}^{M_R-1} \sum_{n=0}^{M_T-1} \frac{1}{IL_{h_{m,n}}} \sum_{l=0}^{L-1} \left| \hat{h}_{m,n}^{(l)} - h_{m,n} \right|^2
$$

where $I$ denotes the number of Monte Carlo trials. We simulated a system with two transmit and two receive antennas, $N = 12$ subcarriers, and CP of length $L = 4$ (i.e., $M = N + L = 16$), and we used the length-4 precoding sequences provided in Table I. The data symbols were i.i.d. 4-PSK symbols, and the channel code employed was a convolutional code of rate $1/2$ and constraint length 5. The signal-to-noise-ratio (SNR) for the $i$th data stream was defined as $SNR_i = 10 \log_{10} \left( \frac{\sigma_p^2}{\sigma^2} \right)$, where $\sigma_p^2 = \mathbb{E}[|\Phi^{(\xi)}|^2]$, and $\sigma^2$ is the variance of the white noise process $\mathbb{A}[n]$. Since in all simulations the SNR was chosen to be the same for all data streams, in the following, we drop the index $i$ in SNR in order to simplify the notation. Unless specified otherwise, all results were obtained by averaging over $I = 500$ independent Monte Carlo trials, where each realization consisted of 8256 data symbols (i.e., 516 OFDM symbols). The sampled matrix channel we simulated is given by

$$
\mathbf{H}(z) = \begin{bmatrix}
0.4851 & 0.3200 \\
-0.3676 & 0.2182 \\
0.7276 & -0.1290 \\
0.2941 & -0.4364
\end{bmatrix} z^{-1} + \begin{bmatrix}
-0.4851 & 0.9387 \\
0.8823 & 0.8729 \\
0.1290 & -0.4364
\end{bmatrix} z^{-2}.
\tag{31}
$$

In all simulation examples, the precoding sequences were normalized.

**Simulation Example 1:** In the first simulation example, we computed the average bias and the MSE of the channel estimator as a function of SNR in decibels. Fig. 3 shows the results for $r = |\beta/\alpha| = 2, 4$, and 8. We can see that the performance of the estimator generally improves with increasing SNR and that the best estimator performance is obtained for $r = 8$. Furthermore, we can observe that going from $r = 2$ to $r = 4$ improves the estimator performance significantly, whereas a further increase to $r = 8$ yields less improvement.

**Simulation Example 2:** In the second simulation example, we investigate the effect of the length $L$ of the data record used for estimating the cyclic statistics $\mathbb{C}[k, \tau]$ on the performance of the channel estimator. For $SNR = 11$ dB, Fig. 4(a) and (b) shows the average bias and the MSE, respectively, of the channel estimator as a function of $L$ for $r = |\beta/\alpha| = 2, 4$, and 8. (Note that in Fig. 4, the length of the data record has been specified in
OFDM symbols. The actual length of the data record is therefore obtained by multiplying the number of symbols by 16.) We can observe that the performance of the estimator generally improves with increasing data record length. Similar to the previous example, increasing $r$ from 2 to 4 yields a significant improvement in estimator performance, whereas a further increase to $r = 8$ yields only a slight improvement. We can furthermore conclude that, especially if short data records have to be used (i.e., $L$ is small), it is crucial to have $r = |\beta/\alpha|$ sufficiently large.

**Simulation Example 3:** For the channel estimates obtained in Simulation Example 1, we investigate the performance of the corresponding multichannel equalizer. For each SNR value, we took the average of the channel estimates over all Monte Carlo runs and computed the corresponding $H(e^{j2\pi k/N}k) = \sum_{l=0}^{L-1} H_l e^{-j2\pi kl}$. Fig. 5 shows the BER as a function of the SNR for $r = |\beta/\alpha| = 2, 4$, and 8. We can see that $r = 4$ consistently yields the best performance in terms of BER. In order to isolate the impact of nonconstant-modulus precoding on BER performance, we also plot the BER curves for the case of perfect channel knowledge. We find that for $r = 4$ and $r = 8$, the blind algorithm comes very close to the perfect channel knowledge case. For $r = 2$, there is a significant difference between the blind approach and the perfect channel knowledge case, which is due to the fact that for $r = 2$, the channel estimator performance is very poor. We can furthermore see that for increasing $r$, the BER performance degrades consistently.

**VI. CONCLUSION**

We introduced an algorithm for blind channel identification and equalization in OFDM-based spatial multiplexing systems. Our approach uses second-order cyclostationary statistics and employs periodic precoding. The basic idea of our method is to provide each transmit antenna with a different signature in the cyclostationary domain to null out the influence of all but one transmit antenna at a time. This makes a scalar subchannel by subchannel identification of the matrix channel possible. Our algorithm yields unique channel estimates (up to a phase rotation for each transmit antenna), its performance does not degrade significantly if the number of transmit antennas is increased, it requires knowledge of an upper bound on the channel length only, it does not impose restrictions on channel zeros, and it exhibits low sensitivity to stationary noise. We discussed the design of the precoding sequences, and we provided simulation examples demonstrating the channel estimator performance, the corresponding multichannel equalizer performance, and the impact of precoding on BER.

**APPENDIX**

With the received signal vector $r[n]$ given by

$$r[n] = \sum_{l=-\infty}^{\infty} H_l s[n-l] + n[n]$$
we obtain

\[
c_r[n, \tau] = E \{ \mathbf{r}[n] \mathbf{r}^H[n - \tau] \} = E \left\{ \sum_{l=-\infty}^{\infty} \mathbf{H}_l \mathbf{s}[n - l] \sum_{l'=\infty}^{\infty} \mathbf{H}^H[n - \tau - l'] \mathbf{H}^H_l \right\} + E \{ \mathbf{f}[n] \mathbf{f}^H[n - \tau] \}
\]

\[
= \sum_{l=-\infty}^{\infty} \sum_{l'=\infty}^{\infty} \mathbf{H}_l E \{ \mathbf{s}[n - l] \times \mathbf{H}^H[n - \tau - (l - l')] \mathbf{H}^H_l + c_p[\tau] \}
\]

\[
= \sum_{l=-\infty}^{\infty} \mathbf{H}_l \sum_{l'=\infty}^{\infty} c_s[n - l, \tau] \mathbf{H}^H_{l+l'} + c_p[\tau]
\]

where \( c_s[n, \tau] = E \{ \mathbf{s}[n] \mathbf{H}^H[n - \tau] \} \). Now, computing the cyclic spectrum of \( \mathbf{r}[\tau] \) according to

\[
S_r[k, \tau] = \sum_{n=-\infty}^{\infty} C_s[k, \tau] z^{-\tau}
\]

\[
= \sum_{n=-\infty}^{\infty} \sum_{l=0}^{P-1} c_s[n, \tau] e^{-j(2\pi/N)k(n-l)} z^{-\tau}
\]

\[
= \sum_{n=-\infty}^{\infty} \sum_{l=0}^{P-1} \frac{1}{P} \mathbf{H}_l \times \sum_{r=-\infty}^{\infty} c_s[n - l, \tau] \mathbf{H}^H_{l+l'} e^{-j(2\pi/N)k(n-l)} z^{-\tau}
\]

\[
= \mathbf{H} \left( \mathbf{z} e^{j(2\pi/N)k} \right) S_p[k, \tau] \mathbf{H}(z) + S_p(\tau) \delta[k]
\]

where \( S_p(\tau) = \sum_{r=-\infty}^{\infty} c_p[r] z^{-\tau} \).

Next, we compute the correlation matrix of the transmit signal vector \( \mathbf{s}[n] \) given by

\[
\mathbf{s}[n] = \sum_{k=0}^{N-1} \sum_{l=-\infty}^{\infty} \text{diag} \left\{ \sigma_k^2 \right\}^{M_r} e^{j(2\pi/N)k(n-l)} \mathbf{H} \left( \mathbf{z} e^{j(2\pi/N)k} \right) S_p[k, \tau] \mathbf{H}(z) + S_p(\tau) \delta[k]
\]

where \( c_{k,l} = \left[ c_{k,l}^{(0)} \ c_{k,l}^{(1)} \cdots c_{k,l}^{(M_r-1)} \right]^T \), and

\[
g[n] = \begin{cases} 1, & n = 0, 1, \ldots, M - 1 \\ 0, & \text{else} \end{cases}
\]

The correlation matrix of \( \mathbf{s}[n] \) is given by

\[
c_s[n, \tau] = E \{ \mathbf{s}[n] \mathbf{s}^H[n - \tau] \}
\]

\[
= \sum_{k=0}^{N-1} \sum_{l=-\infty}^{\infty} \text{diag} \left\{ \sigma_k^2 \right\}^{M_r} c_{k,l} g[n - l] \mathbf{H}_l \times e^{j(2\pi/N)k(n-l)} g[n - l'] \mathbf{H}_l^H + c_p[\tau]
\]

\[
= \sum_{k=0}^{N-1} \sum_{l=-\infty}^{\infty} \text{diag} \left\{ \sigma_k^2 \right\}^{M_r} c_{k,l} g[n - l] \mathbf{H}_l \times e^{j(2\pi/N)k(n-l)} g[n - l'] \mathbf{H}_l^H + c_p[\tau]
\]

\[
= \sum_{k=0}^{N-1} \sum_{l=-\infty}^{\infty} \text{diag} \left\{ \sigma_k^2 \right\}^{M_r} c_{k,l} g[n - l] \mathbf{H}_l \times e^{j(2\pi/N)k(n-l)} g[n - l'] \mathbf{H}_l^H + c_p[\tau]
\]

Next, using \( E \{ \mathbf{c}_{k,l} \mathbf{H}^H_{l'} \} = \text{diag} \{ \sigma_k^2 \}^{M_r} \mathbf{e}[k - k'] \mathbf{e}^H[l - l'] \), we get

\[
c_s[n, \tau] = \sum_{l=-\infty}^{\infty} g[n - LM] g[n - LM - \tau] \times \text{diag} \{ \sigma_k^2 \}^{M_r} \mathbf{e}[k - k'] \mathbf{e}^H[l - l']
\]

where \( \delta_N[\tau] = \sum_{\tau=-\infty}^{\infty} \delta(\tau - LN) \). Let us next compute the Fourier series coefficients of \( c_s[n, \tau] \) with respect to \( n \), i.e.,

\[
C_s[k, \tau] = \frac{1}{P} \sum_{n=0}^{P-1} c_s[n, \tau] e^{-j(2\pi/N)k n}
\]

\[
= \frac{N}{P} \sum_{n=0}^{P-1} \sum_{l=-\infty}^{\infty} g[n - LM] g[n - LM - \tau] \times \text{diag} \{ \sigma_k^2 \}^{M_r} e^{j(2\pi/N)k n}
\]

Now, substituting \( l \to v + uP \), where \( v = 0, 1, \ldots, P - 1 \) and \( u \in \mathbb{Z} \), we obtain

\[
C_s[k, \tau] = \frac{N}{P} \delta_N[\tau] A(k) \sum_{v=0}^{P-1} \sum_{u=-\infty}^{\infty} g[n - vM - uPM] \times \text{diag} \{ \sigma_k^2 \}^{M_r} e^{j(2\pi/N)k n}
\]

where \( \delta_N[\tau] A^2(k) \sum_{v=0}^{P-1} \sum_{u=-\infty}^{\infty} g[n - vM - uPM] \times \text{diag} \{ \sigma_k^2 \}^{M_r} e^{j(2\pi/N)k n} \)

\[
= \sum_{n=0}^{P-1} g[n] g[n - \tau] e^{-j(2\pi/PM)k} e^{j(2\pi/N)(vM + uPM)}, \text{ and } A^2(k) \sum_{v=0}^{P-1} \sum_{u=-\infty}^{\infty} g[n - vM - uPM] \times \text{diag} \{ \sigma_k^2 \}^{M_r} e^{j(2\pi/N)k n} \]
REFERENCES


Helmut Bölcskei (M’98) was born in Mödling, Austria, on May 29, 1970. He received the Dipl.-Ing. and Dr. techn. degrees in electrical engineering/communications from Vienna University of Technology (VUT), Vienna, Austria, in 1994 and 1997, respectively. From 1994 to 2001, he was with the Institute of Communications and Radio-Frequency Engineering, VUT. Since March 2001, he has been an Assistant Professor of Electrical Engineering at the University of Illinois at Urbana-Champaign. In February 2002, he will join ETH Zurich, Zurich, Switzerland, as an Assistant Professor of communication theory. From February to May 1996, he was a Visiting Researcher at Philips Research Laboratories, Eindhoven, The Netherlands. From February to March 1998, he visited the Signal and Image Processing Department at ENST Paris, Paris, France. His research interests include communication and information theory and signal processing with special emphasis on wireless systems, multi-input multi-output (MIMO) antenna systems, space-time coding, orthogonal frequency division multiplexing (OFDM), and wireless networks. From 1999 to 2001 he was is a consultant for Iospan Wireless Inc. (formerly Gigabit Wireless Inc.), San Jose, CA.

Dr. Bölcskei was an Erwin Schrödinger fellow of the Austrian National Science Foundation (FWF) from February 1999 to February 2001, performing research in the Smart Antennas Research Group with the Information Systems Laboratory, Department of Electrical Engineering, Stanford University, Stanford, CA. He serves as an Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING.

Robert W. Heath, Jr. (S’95) received the B.S. and M.S. degrees from the University of Virginia, Charlottesville, and the Ph.D. degree from Stanford University, Stanford, CA, in 2001, all in electrical engineering.

From 1998 to 1999, he was with Iospan Wireless, Inc. (formerly Gigabit Wireless Inc.), San Jose, CA. In January 2002, he will join the University of Texas at Austin as an Assistant Professor. His research group—the Wireless Systems Engineering Laboratory—will focus on the theory, design, and practical implementation of wireless systems. His current research interests are coding, modulation, synchronization, and equalization for multiantenna wireless systems.
Arogyaswami J. Paulraj (F’91) received the Ph.D. degree from Naval Engineering College and the Indian Institute of Technology, Bangalore, in 1973.

He has been a Professor at the Department of Electrical Engineering, Stanford University, Stanford, CA, since 1993, where he supervises the Smart Antennas Research Group. This group consists of approximately 12 researchers working on applications of space-time signal processing for wireless communications networks. His research group has developed many key fundamentals of this new field and helped shape a worldwide research and development focus onto this technology. His nonacademic positions included Head, Sonar Division, Naval Oceanographic Laboratory, Cochin, India; Director, Center for Artificial Intelligence and Robotics, Bangalore; Director, Center for Development of Advanced Computing; Chief Scientist, Bharat Electronics, Bangalore, and Chief Technical Officer and Founder, Iospan Wireless Inc., San Jose, CA. He has also held visiting appointments at Indian Institute of Technology, Delhi, Loughborough University of Technology, Loughborough, U.K., and Stanford University. He sits on several boards of directors and advisory boards for companies/venture partnerships in the United States and India. His research has spanned several disciplines, emphasizing estimation theory, sensor signal processing, parallel computer architectures/algorithms, and space-time wireless communications. His engineering experience included development of sonar systems, massively parallel computers, and, more recently, broadband wireless systems.

Dr. Paulraj has won several awards for his engineering and research contributions. These include two President of India Medals, the CNS Medal, the Jain Medal, the Distinguished Service Medal, the Most Distinguished Service Medal, the VASVIK Medal, the IEEE Best Paper Award (Joint), among others. He is the author of over 250 research papers and holds eight patents. He is a Member of the Indian National Academy of Engineering.