Impact of Noisy Carrier Phase Synchronization on Linear Amplify-and-Forward Relaying

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***Abstract*—We consider a distributed wireless network where amplify-and-forward relays assist the communication between multiple source/destination pairs. To provide a global phase reference at the relays it suffices for each relay to know the phase difference between its own local oscillator (LO) and the LO of a global master node. Based on a scheme to achieve this, we characterize the phase synchronization inaccuracy caused by additive signal noise and LO phase noise. It turns out that relay phase noise only has a limited impact on the performance. Merely the phase uncertainty during the time between phase estimation and channel measurements at the relays affects the system. Finally, we quantify the impact of phase estimation inaccuracy on multiuser zero-forcing relaying.**

Keywords – carrier phase synchronization, cooperative relaying, phase noise, multiuser zero-forcing relaying

I. Introduction

Cooperative wireless communication is a field of research that has drawn a lot of attention lately. A number of autonomous nodes cooperate in order to increase the link reliability and/or the data rate of the whole network. In cooperative relaying systems, some of the present terminals act as relays to the data transmission between source/destination pairs. Literature mainly distinguishes between amplify-and-forward (AF) and decode-and-forward (DF) relaying. There are also other schemes like compress-and-forward, but in this work we will restrict ourselves to AF relaying. When talking about *coherent forwarding* we imply that the phase information of the channel (not the data) is used at the relays in order to achieve a spatial multiplexing or diversity gain. Examples of papers discussing coherent cooperative communication are [1], [2], [3]. In [1] the authors show that coherent relaying with a finite number of single-antenna relays can actually achieve full spatial multiplexing gain. The authors of [2] and [3] investigate several coherent cooperative relaying strategies for MIMO AF nodes that allow for spatial multiplexing. They compare schemes based on channel inversion with schemes that utilize QR decomposition and subsequent successive interference cancellation to orthogonalize several users.

However, coherent cooperation among distributed nodes requires that the carrier phases are synchronized globally. Otherwise, the arbitrary phase rotations introduced by the individual local oscillators (LOs) will lead to a loss of coherency. Synchronizing the carrier phase of multiple independent and distributed wireless nodes is a demanding and interesting problem. Connected to that, the evaluation of the impact of carrier phase estimation errors is also of great interest as estimates are never perfect in practical systems.

In [4] the authors use the theory of random arrays to assess the performance of distributed beamforming. They investigate a cluster of randomly placed nodes collaboratively transmitting signals such that they add up coherently in the target direction. A scheme to synchronize the carrier frequency and phase of a pair of two nodes is presented in [5]. In [6] the authors investigate how to provide a global carrier phase to a cluster of multiple sensor nodes that are to beamform to a remote destination. They present a simple algorithm where one master node broadcasts a sinusoidal reference signal. When knowing its distance to the master, each relay uses a phase-locked-loop (PLL) with detection circuit to adopt to the beacon. The same problem has also been addressed by [7]. The authors investigate how phase errors affect the SNR gain achieved by distributed transmit beamforming.

In this work, we revisit an alternative approach to synchronize the carrier phase of multiple autonomous relay nodes to a global master node, based on the estimation of phase differences [8]. Subsequently, we characterize the phase estimation error. Being aware of the individual phase offsets, we then calculate the relay gain factors such that all source/destination links are orthogonalized (based on [1]). Finally, we quantify the impact of phase estimation errors on the equivalent source/destination channels and show simulation results. Note, however, that we do not consider the impact of phase noise on the data transmission in this work.

Notation: $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^{\mathrm{H}}$ denote the complex conjugate, transpose, and hermitian transpose, respectively. $\mathbf{I}_x$ is the $x \times x$ unity matrix and $\mathbf{A}[x,y]$ the element in row x, column y of the matrix $\mathbf{A}$. $\mathrm{E}_x[\cdot]$ is the expectation with respect to x and we use $\angle\{x\}$ to denote the phase of x. Finally, the operator $\mathrm{diag}(\cdot)$ has two meanings: When the argument is a matrix, it takes the diagonal elements and puts them into a column vector. When the argument is a vector, it puts the elements of the vector into a diagonal matrix.

II. System Model

We consider a wireless network where N_{S} sources communicate concurrently over the same physical channel with N_{D} destinations. We assume $N_{\mathrm{S}} = N_{\mathrm{D}} := N_{\mathrm{SD}}$. N_{R} amplify-and-forward relay nodes assist the communication in a half-duplex scheme, i.e. they do not receive and transmit at the same time. The relays multiply the signals they receive from the sources with a complex gain factor before retransmitting them. The

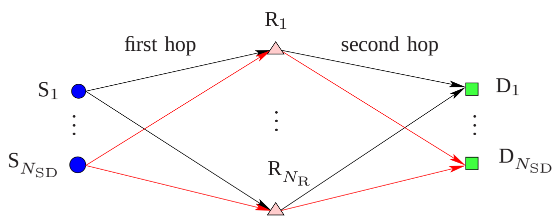

Fig. 1. System configuration for a two-hop relay network

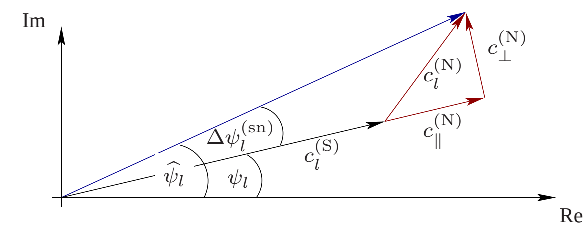

Fig. 2. Noisy phase estimation

system setup is shown in **Fig. 1**. All nodes in the network employ a single antenna. They are furthermore assumed to be equipped with individual voltage controlled oscillators (VCOs) that oscillate with equal frequency but exhibit *independent and unknown phases* $\varphi_{\mathrm{X}_i} \in (-\pi, \pi]$, where $\mathrm{X} \in \{\mathrm{S}, \mathrm{R}, \mathrm{D}\}$. Since all nodes are perfectly frequency synchronous we can restrict our discussion to baseband signals. We define

$$\mathbf{\Phi}_{\mathrm{X}} = \operatorname{diag}\left(\left[e^{j\varphi_{\mathrm{X}_1}}, \ldots, e^{j\varphi_{\mathrm{X}_{N_{\mathrm{X}}}}}\right]\right) \in \mathbb{C}^{N_{\mathrm{X}} \times N_{\mathrm{X}}}, \tag{1}$$

where again $\mathrm{X} \in \{\mathrm{S}, \mathrm{R}, \mathrm{D}\}$. The matrices $\mathbf{\Phi}_{\mathrm{S}}$, $\mathbf{\Phi}_{\mathrm{R}}$, and $\mathbf{\Phi}_{\mathrm{D}}$ thus comprise the individual LO phase offsets of the sources, relays, and destinations, respectively.

III. Global Carrier Phase Synchronization

The relays jointly assist the communication between all source/destination pairs. In order to realize coherent cooperative relaying schemes, e.g. distributed beamforming, we require a global phase reference at all relays. This is normally not provided in a network of independent nodes with free-running oscillators. In [8] we present a scheme where each relay measures the phase difference between its own LO and the LO of a global master. All LO phases are assumed as constant during the procedure. In the following, we shortly recapitulate the scheme and characterize the synchronization error.

A. Synchronization Scheme

Relay l transmits a defined sequence $p[\tau]$ of length T_{p} to a *master node* M, whose LO phase offset is denoted by φ_{M}.

Remark 1: Any source or destination node in the network can act as master. The choice and assignment of M is out of the scope of this work.

The channel impulse response from relay to master is modeled as a tapped delay line with T_{h} taps. It is assumed to be reciprocal and remains constant during the synchronization procedure. The master receives the relay signal and retransmits a conjugate complex and time-inverted version of it. Subsequently, the relay calculates the convolution $c_l[\tau]$ of its received signal with $p[\tau]$ at time $T_{\mathrm{ph}} = T_{\mathrm{p}} + T_{\mathrm{h}} - 1$. We can split

$$c_l[T_{\mathrm{ph}}] := c_l^{(\mathrm{S})} + c_l^{(\mathrm{N})} \tag{2}$$

into a signal part $c_l^{(\mathrm{S})}$ with

$$\angle\left\{c_l^{(\mathrm{S})}\right\} = -2\varphi_{\mathrm{R}_l} + 2\varphi_{\mathrm{M}} := \psi_l \tag{3}$$

and a noise part $c_l^{(\mathrm{N})} \sim \mathcal{CN}\left(0, \sigma_{\mathrm{N}_l}^2\right)$. Consequently, the relay can locally generate an estimate $\widehat{\psi}_l = \angle\{c_l[T_{\mathrm{ph}}]\}$ of ψ_l, which is two times the difference between the LO phases. We will see later that the knowledge of ψ_l instead of φ_{R_l} is enough to allow for coherent distributed relay cooperation.

B. Phase Estimation Error

In this section we characterize the phase estimation error at time t due to a noisy or outdated estimate. Let $t = 0$ be the time when the estimate $\widehat{\psi}_l$ is calculated. We define

$$\Delta\psi_l[t] = \widehat{\psi}_l - \psi_l[t] = \Delta\psi_l^{(\mathrm{sn})} - \Delta\psi_l^{(\mathrm{pn})}[t], \tag{4}$$

which is the difference between the estimate $\widehat{\psi}_l$ at time $t = 0$ and the actual value ψ_l at time $t \geq 0$. The phase estimation error $\Delta\psi_l[t]$ can be separated into two independent sources of error: $\Delta\psi_l^{(\mathrm{sn})}$ is due to measurement inaccuracy because of additive *signal noise* and $\Delta\psi_l^{(\mathrm{pn})}[t]$ is due to outdated phase estimation because of *phase noise*. Omitting the explicit dependency on time t we define the matrices

$$\mathbf{\Psi} = \operatorname{diag}\left(\left[e^{j\psi_1}, \ldots, e^{j\psi_{N_{\mathrm{R}}}}\right]\right), \tag{5}$$

$$\widehat{\mathbf{\Psi}} = \operatorname{diag}\left(\left[e^{j\widehat{\psi}_1}, \ldots, e^{j\widehat{\psi}_{N_{\mathrm{R}}}}\right]\right), \tag{6}$$

$$\text{and} \quad \mathbf{\Psi}_{\Delta} = \widehat{\mathbf{\Psi}}\mathbf{\Psi}^* = \operatorname{diag}\left(\left[e^{j\Delta\psi_1}, \ldots, e^{j\Delta\psi_{N_{\mathrm{R}}}}\right]\right). \tag{7}$$

We will now characterize both sources of error for a given channel realization and show that for $|c_l^{(\mathrm{S})}| \gg |c_l^{(\mathrm{N})}|$ we have

$$\Delta\psi_l \sim \mathcal{N}\left(0, \sigma_{\Delta\psi_l}^2[t]\right), \tag{8}$$

where

$$\sigma_{\Delta\psi_l}^2[t] = \frac{1}{2}\frac{\sigma_{\mathrm{N}_l}^2}{|c_l^{(\mathrm{S})}|^2} + 4\left(\alpha_{\mathrm{R}_l} + \alpha_{\mathrm{M}}\right)t. \tag{9}$$

$\alpha_{\mathrm{R}_l}, \alpha_{\mathrm{M}} \in \mathbb{R}$ are constant hardware parameters.

Remark 2: The expectation of $e^{j\Delta\psi_l}$ with respect to the phase estimation error $\Delta\psi_l$ is

$$\mathrm{E}_{\Delta\psi_l}\left[e^{j\Delta\psi_l}\right] = e^{-\frac{1}{2}\sigma_{\Delta\psi_l}^2[t]}. \tag{10}$$

This follows directly from the definition of the characteristic function of a Gaussian random variable.

1) Estimation error due to additive signal noise: The noise part $c_l^{(\mathrm{N})}$ of $c_l[T_{\mathrm{ph}}]$ (2) can be split into a part parallel and one perpendicular to the desired part $c_l^{(\mathrm{S})}$ (see **Fig. 2**). As $c_l^{(\mathrm{N})}$ is a circularly symmetric, complex Gaussian random variable with variance $\sigma_{\mathrm{N}_l}^2$, $|c_\parallel^{(\mathrm{N})}|$ and $|c_\perp^{(\mathrm{N})}|$ are independent, real-valued Gaussians with variance $\frac{1}{2}\sigma_{\mathrm{N}_l}^2$ each. For $|c_l^{(\mathrm{S})}| \gg |c_l^{(\mathrm{N})}|$ we approximate

$$\Delta\psi_l^{(\mathrm{sn})} \approx \tan^{-1}\left(\frac{|c_\perp^{(\mathrm{N})}|}{|c_l^{(\mathrm{S})}|}\right). \tag{11}$$

It can then be shown that the probability density function (pdf) of $\Delta\psi_l^{(\mathrm{sn})}$ is

$$f\left(\Delta\psi_l^{(\mathrm{sn})}\right) \approx \frac{1}{|c_l^{(\mathrm{S})}|\left(1+\tan^2\left(\Delta\psi_l^{(\mathrm{sn})}\right)\right)} \frac{1}{\sqrt{\pi\sigma_{\mathrm{N}_l}^2}} \cdot \\ \cdot \exp\left(-\frac{|c_l^{(\mathrm{S})}|^2 \tan^2\left(\Delta\psi_l^{(\mathrm{sn})}\right)}{\sigma_{\mathrm{N}_l}^2}\right). \quad (12)$$

The angle $\Delta\psi_l^{(\mathrm{sn})}$ will be small for $|c_l^{(\mathrm{S})}| \gg |c_l^{(\mathrm{N})}|$. This means that we can further approximate

$$\tan\left(\Delta\psi_l^{(\mathrm{sn})}\right) \approx \Delta\psi_l^{(\mathrm{sn})} \quad (13)$$

$$\text{and} \quad 1+\tan^2\left(\Delta\psi_l^{(\mathrm{sn})}\right) \approx 1. \quad (14)$$

With (13) and (14) the pdf (12) simplifies to

$$f\left(\Delta\psi_l^{(\mathrm{sn})}\right) \approx \frac{1}{\sqrt{\pi|c_l^{(\mathrm{S})}|^2\sigma_{\mathrm{N}_l}^2}} \exp\left(-\frac{\left(|c_l^{(\mathrm{S})}|^2\Delta\psi_l^{(\mathrm{sn})}\right)^2}{|c_l^{(\mathrm{S})}|^2\sigma_{\mathrm{N}_l}^2}\right), \quad (15)$$

according to which $|c_l^{(\mathrm{S})}|^2\Delta\psi_l^{(\mathrm{sn})} \sim \mathcal{N}\left(0, \frac{1}{2}|c_l^{(\mathrm{S})}|^2\sigma_{\mathrm{N}_l}^2\right)$. This finally leads to

$$\Delta\psi_l^{(\mathrm{sn})} \sim \mathcal{N}\left(0, \frac{1}{2}\frac{\sigma_{\mathrm{N}_l}^2}{|c_l^{(\mathrm{S})}|^2}\right) \quad (16)$$

which is the first part of (9).

2) Estimation error due to phase noise: Wiener phase noise is a widely used model for the phase noise of VCOs (e.g. [9]). The VCO phases are modeled as Wiener processes. Thus, $\varphi_{\mathrm{R}_l}[t]$ and $\varphi_{\mathrm{M}}[t]$ are zero-mean, Gaussian random variables with variance $\alpha_{\mathrm{R}_l}t$ and $\alpha_{\mathrm{M}}t$, respectively. We have

$$\psi_l[t] = -2\left(\varphi_{\mathrm{R}_l}[0] + \Delta\varphi_{\mathrm{R}_l}[t]\right) + 2\left(\varphi_{\mathrm{M}}[0] + \Delta\varphi_{\mathrm{M}}[t]\right) = \\ = \psi_l[0] + \Delta\psi_l^{(\mathrm{pn})}[t], \quad (17)$$

where $\Delta\psi_l^{(\mathrm{pn})}[t] = -2\Delta\varphi_{\mathrm{R}_l}[t] + 2\Delta\varphi_{\mathrm{M}}[t]$. It follows that

$$\Delta\psi_l^{(\mathrm{pn})}[t] \sim \mathcal{N}\left(0, 4\left(\alpha_{\mathrm{R}_l} + \alpha_{\mathrm{M}}\right)t\right). \quad (18)$$

With (16) this finally leads to (8), (9).

IV. Linear Multiuser Relaying

Having characterized the phase estimation error $\Delta\psi_l$ at relay R_l, we now investigate the influence of individual LO phase offsets on linear multiuser relaying. Based on this we derive the multiuser zero-forcing gain allocation [1] taking the individual LO phase offsets into account and analyze the impact of phase synchronization errors on this gain allocation scheme. To make notation easier, we will in the following omit the explicit dependency of the LO phases on time t.

Consider again the system model depicted in **Fig. 1**. For data transmission the nodes employ a channel-flattening technique like OFDM. We assume a quasi-static or slow fading environment. The channels are uncorrelated, exhibiting frequency-flat Rayleigh fading. Consequently, the channel coefficients are i.i.d. complex normal random variables with variance σ_{h}^2. They remain constant for at least one transmission cycle and change independently afterwards (blockfading). The communication follows a two-hop relay traffic pattern, i.e. each transmission cycle includes two channel uses: one for the *first hop* transmission from the sources to all relays and one for the *second hop* transmission from the relays to the destinations. The direct link is not taken into account. Every source k operates in spatial multiplexing mode generating an independent data stream $s_k[t]$. The scalar transmit symbols of all sources are stacked in the vector $\mathbf{s}[t] \in \mathbb{C}^{N_{\mathrm{SD}}}$. No channel state information (CSI) is present at the sources. Consequently, no power or bit loading is performed and each source is assumed to use the same transmit power $\mathrm{E}_t\left[|s_k[t]|^2\right] = \sigma_s^2$. The source signal vector $\mathbf{s}[t]$ is first transmitted over the first hop matrix channel $\mathbf{H}_{\mathrm{SR}} \in \mathbb{C}^{N_{\mathrm{R}} \times N_{\mathrm{SD}}}$ to the relays. Their received equivalent baseband signals are stacked in the vector

$$\mathbf{r}^{(\mathrm{rx})}[t] = \mathbf{\Phi}_{\mathrm{R}}^* \mathbf{H}_{\mathrm{SR}} \mathbf{\Phi}_{\mathrm{S}} \cdot \mathbf{s}[t] + \mathbf{n}_{\mathrm{R}}[t], \quad (19)$$

where $\mathbf{n}_{\mathrm{R}}[t] \sim \mathcal{CN}\left(\mathbf{0}, \sigma_{n_{\mathrm{R}}}^2 \mathbf{I}_{N_{\mathrm{R}}}\right)$ comprises the additive, white, Gaussian noise (AWGN) contributions at the relay nodes. After multiplication with the gain matrix $\mathbf{G} \in \mathbb{C}^{N_{\mathrm{R}} \times N_{\mathrm{R}}}$, the signal

$$\mathbf{r}^{(\mathrm{tx})}[t] = \mathbf{G}\mathbf{r}^{(\mathrm{rx})}[t], \quad (20)$$

is passed through the second hop matrix channel $\mathbf{H}_{\mathrm{RD}} \in \mathbb{C}^{N_{\mathrm{SD}} \times N_{\mathrm{R}}}$ to the destination nodes. Assuming $\mathbf{G}$ is fixed, the average sum transmit power of all relays is given by

$$\mathrm{E}_{\mathbf{s},\mathbf{n},\mathbf{H}_{\mathrm{SR}}}\left[\left\|\mathbf{r}^{(\mathrm{tx})}[t]\right\|_2^2\right] = N_{\mathrm{SD}}\left(\sigma_{\mathrm{s}}^2\sigma_{\mathrm{h}}^2 + \sigma_{n_{\mathrm{R}}}^2\right) \operatorname{tr}\left(\mathbf{G}\mathbf{G}^{\mathrm{H}}\right). \quad (21)$$

The received signals at the destinations are stacked in the vector

$$\mathbf{d}[t] = \mathbf{\Phi}_{\mathrm{D}}^* \mathbf{H}_{\mathrm{RD}} \mathbf{\Phi}_{\mathrm{R}} \cdot \mathbf{r}^{(\mathrm{tx})}[t] + \mathbf{n}_{\mathrm{D}}[t], \quad (22)$$

where $\mathbf{n}_{\mathrm{D}} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_{n_{\mathrm{D}}}^2 \mathbf{I}_{N_{\mathrm{SD}}}\right)$ comprises the AWGN contributions at the destinations. For the sake of simplicity we let $\sigma_{n_{\mathrm{R}}}^2 = \sigma_{n_{\mathrm{D}}}^2 := \sigma_n^2$. The vector of received signals can be written as

$$\mathbf{d}[t] = \mathbf{\Phi}_{\mathrm{D}}^* \mathbf{H}_{\mathrm{RD}} \mathbf{\Phi}_{\mathrm{R}} \mathbf{G} \mathbf{\Phi}_{\mathrm{R}}^* \mathbf{H}_{\mathrm{SR}} \mathbf{\Phi}_{\mathrm{S}} \cdot \mathbf{s}[t] + \\ + \mathbf{\Phi}_{\mathrm{D}}^* \mathbf{H}_{\mathrm{RD}} \mathbf{\Phi}_{\mathrm{R}} \mathbf{G} \cdot \mathbf{n}_{\mathrm{R}}[t] + \mathbf{n}_{\mathrm{D}}[t]. \quad (23)$$

Note that the relays do not share their received signals. Thus the gain matrix is diagonal as a relay can only forward its own received signal. Using the diagonality constraint $\mathbf{G} = \operatorname{diag}(\mathbf{g})$ in (23) the *equivalent channel matrix* is

$$\mathbf{H}_{\mathrm{SD}} = \mathbf{\Phi}_{\mathrm{D}}^* \mathbf{H}_{\mathrm{RD}} \mathbf{G} \mathbf{H}_{\mathrm{SR}} \mathbf{\Phi}_{\mathrm{S}} \quad (24)$$

since the matrices $\mathbf{\Phi}_{\mathrm{R}}$ and $\mathbf{\Phi}_{\mathrm{R}}^*$ cancel. The element $[m,k]$ of $\mathbf{H}_{\mathrm{SD}}$ corresponds to the equivalent channel from source k over all relays to destination m. In order to decode the signal from source k, destination m has to know $\mathbf{H}_{\mathrm{SD}}[m,k]$.

Remark 3: Once $\mathbf{G}$ is calculated, relay phase noise has no impact on the linear relaying scheme as the relay phase offsets φ_{R_l} do not appear in (24). This also holds for a blockdiagonal gain matrix $\mathbf{G}$ as encountered when the relays have multiple colocated antennas.

A. Zero-forcing gain allocation

In a perfect world, $\mathrm{N}_{\mathrm{SD}}^2 - \mathrm{N}_{\mathrm{SD}} + 1$ relays can completely orthogonalize all N_{SD} source/destination links by choosing their gain factors accordingly [1]. As a result, interstream interference is cancelled (zero-forcing). However, each relay needs to have *global* channel state information, i.e. it actually needs to know the matrices $\mathbf{H}_{\mathrm{SR}}\mathbf{A}$ and $\mathbf{B}\mathbf{H}_{\mathrm{RD}}$, where $\mathbf{A}$ and $\mathbf{B}$ can be arbitrary diagonal matrices. Due to the unknown phase offsets at the relays this is however not feasible. When each relay l has an estimate $\widehat{\psi}_l$ of

$$\psi_l = -2\varphi_{\mathrm{R}_l} + 2\varphi_{\mathrm{M}}, \tag{25}$$

as described in Section III-A, the following procedure nevertheless allows to calculate a gain matrix that orthogonalizes all source/destination links:

The $2N_{\mathrm{SD}}$ source and destination nodes broadcast an estimation sequence, e.g. an m-sequence, to the relays. Every relay can thus estimate its local first hop and second hop channel coefficients with respect to their individual LO phases. Assuming a noiseless and interference-free estimate, each relay l obtains

$$\widehat{h}_{\mathrm{S}_k\mathrm{R}_l} = h_{\mathrm{S}_k\mathrm{R}_l} e^{j\left(\varphi_{\mathrm{S}_k} - \varphi_{\mathrm{R}_l}\right)},\ k \in \{1, \ldots, N_{\mathrm{SD}}\} \tag{26}$$

$$\widehat{h}_{\mathrm{R}_l\mathrm{D}_m} = h_{\mathrm{R}_l\mathrm{D}_m} e^{j\left(\varphi_{\mathrm{D}_m} - \varphi_{\mathrm{R}_l}\right)},\ m \in \{1, \ldots, N_{\mathrm{SD}}\}\,. \tag{27}$$

Using its phase estimate $\widehat{\psi}_l$, every relay then computes

$$\widetilde{h}_{\mathrm{S}_k\mathrm{R}_l} = \widehat{h}_{\mathrm{S}_k\mathrm{R}_l} \cdot e^{-j\frac{1}{2}\widehat{\psi}_l},\ k \in \{1, \ldots, N_{\mathrm{SD}}\} \tag{28}$$

$$\widetilde{h}_{\mathrm{R}_l\mathrm{D}_m} = \widehat{h}_{\mathrm{R}_l\mathrm{D}_m} \cdot e^{-j\frac{1}{2}\widehat{\psi}_l},\ m \in \{1, \ldots, N_{\mathrm{SD}}\} \tag{29}$$

and disseminates this information to all other relays. We will see later that the relays need to know $\widetilde{h}_{\mathrm{S}_k\mathrm{R}_l}$ and $\widetilde{h}_{\mathrm{R}_l\mathrm{D}_m}$ instead of $\widehat{h}_{\mathrm{S}_k\mathrm{R}_l}$ and $\widehat{h}_{\mathrm{R}_l\mathrm{D}_m}$ in order to compute the zero-forcing gain matrix.

Remark 4: There exists a π-ambiguity when calculating $\frac{1}{2}\widehat{\psi}_l$ as required in (28) and (29). Note that we have

$$\frac{1}{2}\psi_l = \begin{cases} -\varphi_{\mathrm{R}_l} + \varphi_{\mathrm{M}} & \text{for} \quad -\varphi_{\mathrm{R}_l} + \varphi_{\mathrm{M}} < \pi \\ -\varphi_{\mathrm{R}_l} + \varphi_{\mathrm{M}} - \pi & \text{for} \quad -\varphi_{\mathrm{R}_l} + \varphi_{\mathrm{M}} \geq \pi \end{cases} \tag{30}$$

This will, however, be of no consequence to our scheme as we will see later.

Assume without loss of generality that source k and destination k, $k \in \{1, \ldots, N_{\mathrm{SD}}\}$ form a source/destination pair. With (24) the equivalent channel coefficient from source k to destination m can be written as

$$\mathbf{H}_{\mathrm{SD}}[m,k] = \mathbf{g}^{\mathrm{T}} \left(\mathbf{h}_{\mathrm{RD}_m} \odot \mathbf{h}_{\mathrm{S}_k\mathrm{R}}\right) \cdot e^{j\left(\varphi_{\mathrm{S}_k} - \varphi_{\mathrm{D}_m}\right)}, \tag{31}$$

where $\mathbf{h}_{\mathrm{S}_k\mathrm{R}}$ and $\mathbf{h}_{\mathrm{RD}_m}$ are column vectors comprising the channel coefficients from source k to all relays and from all relays to destination m, respectively. The operator $\odot$ denotes the elementwise (Hadamard) product.

Remark 5: There is no interstream interference when all off-diagonal elements of the equivalent channel matrix $\mathbf{H}_{\mathrm{SD}}$ vanish, i.e. $\mathbf{H}_{\mathrm{SD}}[m,k] = 0$ for $k \neq m$.

We define a matrix $\mathbf{H}$ such that the multiplication $\mathbf{g}^{\mathrm{T}}\mathbf{H}$ results in a vector containing all off-diagonal elements of $\mathbf{H}_{\mathrm{SD}}$. It comprises the column vectors $\mathbf{h}_{\mathrm{mk}}$, where

$$\mathbf{h}_{\mathrm{mk}} = \left(\mathbf{h}_{\mathrm{RD}_m} \odot \mathbf{h}_{\mathrm{S}_k\mathrm{R}}\right) \cdot e^{j\left(\varphi_{\mathrm{S}_k} - \varphi_{\mathrm{D}_m}\right)} \quad \forall \quad k \neq m. \tag{32}$$

Zero-forcing condition: In order to cancel all interstream interference, we require

$$\mathbf{g}^{\mathrm{T}}\mathbf{H} \overset{!}{=} \mathbf{0}^{\mathrm{T}} \quad \text{and thus} \quad \mathbf{H}^{\mathrm{H}}\mathbf{g}^* \overset{!}{=} \mathbf{0}. \tag{33}$$

Let $\mathbf{Z}_{\mathrm{H}} = \mathcal{N}\left(\mathbf{H}^{\mathrm{H}}\right)$ be the complex-valued, orthonormal basis spanning the nullspace of $\mathbf{H}^{\mathrm{H}}$. Requiring $N_{\mathrm{R}} \geq N_{\mathrm{SD}}^2 - N_{\mathrm{SD}} + 1$ for the nullspace to be nonempty, we speak of a *minimum relay configuration* when $N_{\mathrm{R}} = N_{\mathrm{SD}}^2 - N_{\mathrm{SD}} + 1$. In this case the basis $\mathbf{Z}_{\mathrm{H}}$ is, apart from a complex-valued scaling factor, a uniquely defined vector. For the rest of the paper we will deal with this case.

Remark 6: Projecting an arbitrary vector $\mathbf{g}_{\mathrm{init}} \in \mathbb{C}^{N_{\mathrm{R}}}$ onto the vectorspace spanned by $\mathbf{Z}_{\mathrm{H}}$ delivers $\mathbf{g}^*$ fulfilling (33).

However, the relays do not know $\mathbf{H}$. They can only calculate $\widetilde{\mathbf{H}}$ comprising the columns

$$\widetilde{\mathbf{h}}_{mk} = \left(\widetilde{\mathbf{h}}_{\mathrm{RD}_m} \odot \widetilde{\mathbf{h}}_{\mathrm{S}_k\mathrm{R}}\right) \ \forall\ k, m \in \{1, \ldots, N_{\mathrm{SD}}\}\,, \tag{34}$$

where

$$\widetilde{\mathbf{h}}_{\mathrm{S}_k\mathrm{R}} = \left[\widetilde{h}_{\mathrm{S}_k\mathrm{R}_1}, \ldots, \widetilde{h}_{\mathrm{S}_k\mathrm{R}_{N_{\mathrm{R}}}}\right]^{\mathrm{T}} \tag{35}$$

$$\widetilde{\mathbf{h}}_{\mathrm{RD}_m} = \left[\widetilde{h}_{\mathrm{R}_1\mathrm{D}_m}, \ldots, \widetilde{h}_{\mathrm{R}_{N_{\mathrm{R}}}\mathrm{D}_m}\right]^{\mathrm{T}}. \tag{36}$$

Using the definition of $\widetilde{h}_{\mathrm{S}_k\mathrm{R}_l}$ in (28) and of $\widetilde{h}_{\mathrm{R}_l\mathrm{D}_m}$ in (29), we can write

$$\widetilde{\mathbf{h}}_{mk} = \widehat{\mathbf{\Psi}}^* \mathbf{\Phi}_{\mathrm{R}}^* \mathbf{\Phi}_{\mathrm{R}}^* \cdot \mathbf{h}_{mk} \cdot e^{j2\varphi_{\mathrm{D}_m}}. \tag{37}$$

Let $\mathbf{Z}_{\widetilde{\mathrm{H}}} = \mathcal{N}\left(\widetilde{\mathbf{H}}^{\mathrm{H}}\right)$ be the complex-valued, orthonormal basis spanning the nullspace of $\widetilde{\mathbf{H}}^{\mathrm{H}}$. In the case of perfect phase estimation, i.e. $\widehat{\mathbf{\Psi}} = \mathbf{\Psi}$, we have

$$\widetilde{\mathbf{h}}_{mk} = e^{-j2\varphi_{\mathrm{M}}} \cdot \mathbf{h}_{mk} \cdot e^{j2\varphi_{\mathrm{D}_m}}. \tag{38}$$

By definition, $\mathbf{Z}_{\widetilde{\mathrm{H}}}$ fulfills $\widetilde{\mathbf{H}}^{\mathrm{H}}\mathbf{Z}_{\widetilde{\mathrm{H}}} = \mathbf{0}^{\mathrm{T}}$ implying

$$e^{j(2\varphi_{\mathrm{M}} - 2\varphi_{\mathrm{D}_m})} \cdot \mathbf{h}_{mk}^{\mathrm{H}} \cdot \mathbf{Z}_{\widetilde{\mathrm{H}}} = 0,$$

$$\text{and thus} \quad \mathbf{h}_{mk}^{\mathrm{H}} \cdot \mathbf{Z}_{\widetilde{\mathrm{H}}} = 0 \quad \forall \quad k \neq m. \tag{39}$$

From (39) we conclude that $\mathbf{Z}_{\widetilde{\mathrm{H}}}$ lies in the nullspace of $\mathbf{H}^{\mathrm{H}}$. Any vector $\gamma^*\mathbf{Z}_{\widetilde{\mathrm{H}}}$, $\gamma \in \mathbb{C}$ will thus satisfy

$$\mathbf{H}^{\mathrm{H}} \cdot \gamma^* \mathbf{Z}_{\widetilde{\mathrm{H}}} = \mathbf{0}. \tag{40}$$

Hence, we have to calculate the gain vector $\mathbf{g}$ according to

$$\mathbf{g} = \gamma \mathbf{Z}_{\widetilde{\mathrm{H}}}^*,\ \gamma \in \mathbb{C} \tag{41}$$

in order to satisfy the zero-forcing condition (33). Using (21) we fix γ such that the average sum transmit power of all relays is constant. Requiring $\mathrm{E}_{\mathbf{s},\mathbf{n},\mathbf{H}_{\mathrm{SR}}}\left[\left\|\mathbf{r}^{(\mathrm{tx})}[t]\right\|_2^2\right] \overset{!}{=} P_{\mathrm{R}}$ we get

$$\gamma = \sqrt{\frac{P_{\mathrm{R}}}{N_{\mathrm{SD}}\left(\sigma_{\mathrm{s}}^2\sigma_{\mathrm{h}}^2 + \sigma_{\mathrm{n}}^2\right)}}, \tag{42}$$

where we used the fact that by definition $\mathbf{Z}_{\widetilde{\mathrm{H}}}^{\mathrm{H}}\mathbf{Z}_{\widetilde{\mathrm{H}}} = 1$.

B. *Impact of phase estimation errors*

In contrast to perfect phase estimation, there will be interstream interference in the presence of phase estimation errors. In this section we will quantify the expected signal and interference power for a given channel realization when the phase estimation is noisy and the estimation error $\Delta\psi_l$ at the relays is distributed according to (8).

Remark 7: When the time t between phase estimation and channel measurement is small, the estimation error due to phase noise, i.e. $\Delta\psi_{\mathrm{pn}}[t]$, becomes negligible.
The relation between $\mathbf{Z}_{\mathrm{H}}$ and $\mathbf{Z}_{\widetilde{\mathrm{H}}}$ is given by

$$\mathbf{Z}_{\mathrm{H}} = \frac{1}{\zeta}\mathbf{\Psi}_{\Delta}\mathbf{Z}_{\widetilde{\mathrm{H}}} \quad \text{and} \quad \mathbf{Z}_{\widetilde{\mathrm{H}}} = \zeta\mathbf{\Psi}_{\Delta}^{\mathrm{H}}\mathbf{Z}_{\mathrm{H}}, \tag{43}$$

where $\zeta \in \mathbb{C}$ and $|\zeta|^2 = 1$. With (31), (32), and (41) the entries of the equivalent channel matrix $\mathbf{H}_{\mathrm{SD}}$ are

$$\mathbf{H}_{\mathrm{SD}}[m,k] = \gamma\mathbf{Z}_{\widetilde{\mathrm{H}}}^{\mathrm{H}}\mathbf{h}_{mk}. \tag{44}$$

With (43) we end up with

$$\mathbf{H}_{\mathrm{SD}}[m,k] = \sqrt{\frac{\zeta^2 P_{\mathrm{R}}}{N_{\mathrm{SD}}\left(\sigma_{\mathrm{s}}^2\sigma_{\mathrm{h}}^2 + \sigma_{n_{\mathrm{R}}}^2\right)}}\boldsymbol{\psi}_{\Delta}^{\mathrm{T}}\left(\mathbf{Z}_{\mathrm{H}}^* \odot \mathbf{h}_{mk}\right), \tag{45}$$

where we used γ as in (42) and $\boldsymbol{\psi}_{\Delta} = \mathrm{diag}\left(\mathbf{\Psi}_{\Delta}\right)$. Given a specific channel realization, we are interested in the expected signal and interference power so we have to calculate

$$\mathrm{E}_{\Delta\psi}\left[|\mathbf{H}_{\mathrm{SD}}[m,k]|^2\right] = \frac{P_{\mathrm{R}}\mathrm{E}_{\Delta\psi}\left[\boldsymbol{\psi}_{\Delta}^{\mathrm{T}}\mathbf{A}_{mk}\boldsymbol{\psi}_{\Delta}^*\right]}{N_{\mathrm{SD}}\left(\sigma_{\mathrm{s}}^2\sigma_{\mathrm{h}}^2 + \sigma_{\mathrm{n}}^2\right)}, \tag{46}$$

where we substituted $\mathbf{A}_{mk} = \left(\mathbf{Z}_{\mathrm{H}}^* \odot \mathbf{h}_{mk}\right)\left(\mathbf{Z}_{\mathrm{H}}^* \odot \mathbf{h}_{mk}\right)^{\mathrm{H}}$. As $\mathbf{A}_{mk}$ is a Hermitian matrix, i.e. $\mathbf{A}_{mk}^{\mathrm{H}} = \mathbf{A}_{mk}$, we can write

$$\mathrm{E}_{\Delta\psi}\left[\boldsymbol{\psi}_{\Delta}^{\mathrm{T}}\mathbf{A}_{mk}\boldsymbol{\psi}_{\Delta}^*\right] = \mathrm{tr}\left(\mathbf{A}_{mk}\mathbf{C}\right) + \mathbf{m}^{\mathrm{H}}\mathbf{A}_{mk}\mathbf{m}. \tag{47}$$

The vector $\mathbf{m}$ comprises the mean values

$$\mathbf{m}[l] = e^{-\frac{1}{2}\sigma_{\Delta\psi_l}^2} \tag{48}$$

of $\boldsymbol{\psi}_{\Delta}^*$ and $\mathbf{C}$ is the covariance matrix given by

$$\mathbf{C} = \mathrm{E}_{\Delta\psi}\left[\boldsymbol{\psi}_{\Delta}^*\boldsymbol{\psi}_{\Delta}^{\mathrm{T}}\right] - \mathbf{m}\mathbf{m}^{\mathrm{H}}. \tag{49}$$

Inserting (49) into (47) leads to

$$\mathrm{E}_{\Delta\psi}\left[\boldsymbol{\psi}_{\Delta}^{\mathrm{T}}\mathbf{A}_{mk}\boldsymbol{\psi}_{\Delta}^*\right] = \mathrm{tr}\left(\mathbf{A}_{mk}\mathrm{E}_{\Delta\psi}\left[\boldsymbol{\psi}_{\Delta}^*\boldsymbol{\psi}_{\Delta}^{\mathrm{T}}\right]\right). \tag{50}$$

Remark 8: To simplify matters, we make the following assumption: All relays perform their phase and channel estimation at *different times*. Since we use a Wiener process to model phase noise, the individual phase estimation errors at the relays become statistically independent.
Using (10) we can thus calculate

$$\mathrm{E}_{\Delta\psi}\left[e^{j(\Delta\psi_i - \Delta\psi_j)}\right] = \begin{cases} 1, & i = j \\ e^{-\frac{1}{2}\left(\sigma_{\Delta\psi_i}^2 + \sigma_{\Delta\psi_j}^2\right)}, & i \neq j \end{cases} \tag{51}$$

and get

$$\mathrm{E}_{\Delta\psi}\left[\boldsymbol{\psi}_{\Delta}^*\boldsymbol{\psi}_{\Delta}^{\mathrm{T}}\right] = \mathbf{D} + \mathbf{m}\mathbf{m}^{\mathrm{H}}, \tag{52}$$

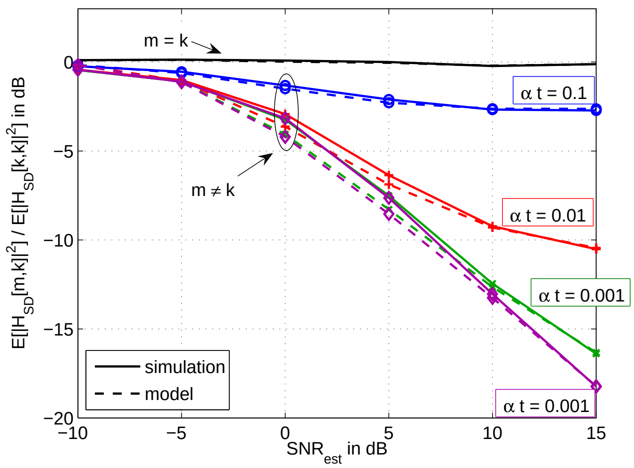

Fig. 3. Normalized expected power of the equivalent channel

where $\mathbf{D}$ is diagonal with elements $\mathbf{D}[l,l] = 1 - e^{-\sigma_{\Delta\psi_l}^2}$ for $l \in \{1,\ldots,N_{\mathrm{R}}\}$. We rewrite (50) into

$$\mathrm{E}_{\Delta\psi}\left[\boldsymbol{\psi}_{\Delta}^{\mathrm{T}}\mathbf{A}_{mk}\boldsymbol{\psi}_{\Delta}^*\right] = \mathrm{tr}\left(\mathbf{A}_{mk}\mathbf{D}\right) + \mathrm{tr}\left(\mathbf{m}^{\mathrm{H}}\mathbf{A}_{mk}\mathbf{m}\right). \tag{53}$$

Inserting (53) into (46) finally leads to

$$\mathrm{E}_{\Delta\psi}\left[|\mathbf{H}_{\mathrm{SD}}[m,k]|^2\right] = \frac{P_{\mathrm{R}}}{N_{\mathrm{SD}}\left(\sigma_{\mathrm{s}}^2\sigma_{\mathrm{h}}^2 + \sigma_{\mathrm{n}}^2\right)} \cdot \left(\mathrm{tr}\left(\mathbf{A}_{mk}\mathbf{D}\right) + \mathbf{m}^{\mathrm{H}}\mathbf{A}_{mk}\mathbf{m}\right). \tag{54}$$

Remark 9: The interference power ($k \neq m$) is bounded with respect to the phase estimation error. When $\sigma_{\Delta\psi_l}^2 \to 0$ for all relays, we get $|\mathbf{H}_{\mathrm{SD}}[m,k]|^2 = 0$ and when $\sigma_{\Delta\psi_l}^2 \to \infty$ for all relays, (54) simplifies to

$$\mathrm{E}_{\Delta\psi}\left[|\mathbf{H}_{\mathrm{SD}}[m,k]|^2\right] = \frac{P_{\mathrm{R}}\mathrm{tr}\left(\mathbf{A}_{mk}\right)}{N_{\mathrm{SD}}\left(\sigma_{\mathrm{s}}^2\sigma_{\mathrm{h}}^2 + \sigma_{\mathrm{n}}^2\right)}. \tag{55}$$

V. Simulation Results

In this section we present results of Monte-Carlo simulations. We consider a system as introduced in Section II with $N_{\mathrm{SD}} = 4$ source/destination pairs and $N_{\mathrm{R}} = 13$ relays. We use m-sequences of length T_{p} for the phase estimation because they have, apart from the small dc component, a flat spectrum. That means that they exhibit the same power in all frequency bins and we can exploit frequency diversity [8]. All channels are modelled according to the Hiperlan B channel model with total average power normalized to 1 [10]. It corresponds to a typical large open space environment with Non-Line-of-Sight (NLoS) conditions or an office environment with large delay spread. For the data transmission we use OFDM modulation with frequency-flat 128 subchannels. The relays choose their gain factors according to (41) for each subchannel.

In **Fig. 3** we investigate the interstream interference due to a noisy phase estimate. For a given channel realization, we define the phase estimation SNR at relay l as

$$\mathrm{SNR}_{\mathrm{est}}^{(l)} = \frac{|c_l^{(\mathrm{S})}|^2}{\sigma_{\mathrm{N}_l}^2}. \tag{56}$$

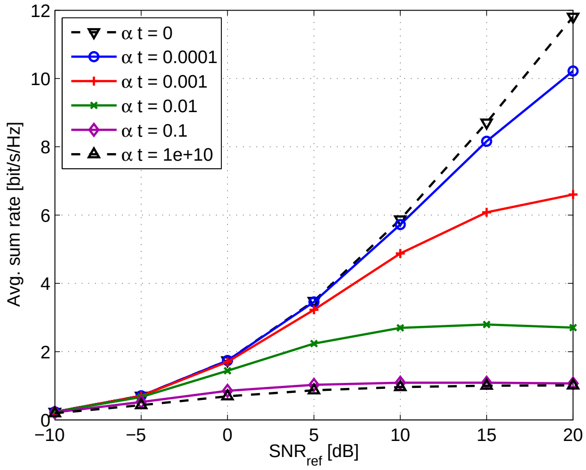

Fig. 4. Average sum rate vs. $\mathrm{SNR}_{\mathrm{ref}}$ assuming absence of additive noise

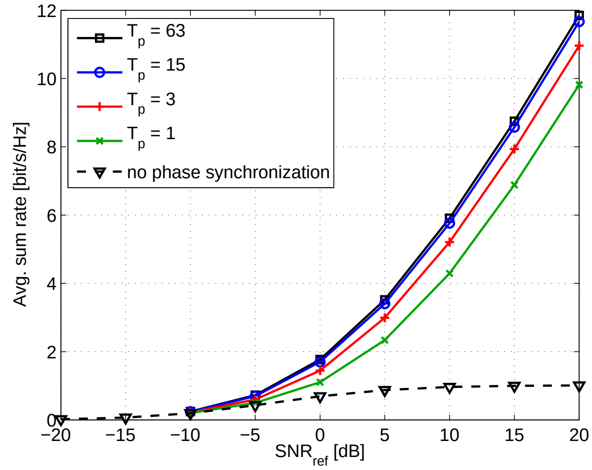

Fig. 5. Average sum rate vs. $\mathrm{SNR}_{\mathrm{ref}}$ assuming absence of phase noise

In our simulations we let $\mathrm{SNR}_{\mathrm{est}}^{(l)} = \mathrm{SNR}_{\mathrm{est}}$ for all relays and $\alpha_{\mathrm{R}_l} t = \alpha_{\mathrm{M}} t := \alpha t$. We plotted

$$10 \log_{10} \left(\frac{\mathrm{E}_{\Delta\psi} \left[|\mathbf{H}_{\mathrm{SD}}[m,k]|^2 \right]}{\mathrm{E}_{\Delta\psi} \left[|\mathbf{H}_{\mathrm{SD}}[k,k]|^2 \right]} \right) \tag{57}$$

achieved by simulation (solid lines, 'simulation') and compare it to our result (54) when the phase estimation error is modelled as in (8) (dashed lines, 'model'). The model fits quite well for small and large $\mathrm{SNR}_{\mathrm{est}}$. In between we observe a difference between model and simulation that is due to the simplifying assumptions made in Section III-B.

In order to calculate the average sum rate

$$I_{\mathrm{avg}} = \mathrm{E} \left[\sum_{m=1}^{N_{\mathrm{SD}}} \frac{1}{2} \log_2 \left(1 + \mathrm{SINR}_m \right) \right] \tag{58}$$

at a defined average SNR (denoted by $\mathrm{SNR}_{\mathrm{ref}}$), we consider a reference scenario with a single source/destination pair and only one relay in between. By simulation we determine the source transmit power that is needed to achieve $\mathrm{SNR}_{\mathrm{ref}}$ on the average at the destination. We then use this transmit power in our system that is to be evaluated at $\mathrm{SNR}_{\mathrm{ref}}$ and calculate the achieved sum rate. In (58) SINR_m denotes the instantaneous SINR at destination m while the factor $\frac{1}{2}$ is due to the half-duplex constraint. **Fig. 4** shows I_{avg} for the case that the phase estimations at the relays are performed without additive signal noise, i.e. $\Delta\psi_l^{(\mathrm{sn})} = 0$. In this way we can isolate the impact of phase noise. We see that the performance degrades quickly when αt increases. This can either happen because of phase instability of the LOs (large α) or due to large time between phase synchronization and channel estimation (large t). In **Fig. 5** we depict I_{avg} for the case that there is no LO phase noise, i.e. $\Delta\psi_l^{(\mathrm{pn})} = 0$. Additive signal noise is the only source of impairment for the phase estimation at the relays. We see that even when only 1 bit is used for phase synchronization we achieve full spatial multiplexing gain and loose 'only' about 3.5 dB SNR with respect to the perfect synchronization (matching with the curve for $T_{\mathrm{p}} = 63$).

VI. Conclusions

We investigated the impact of LO phase offsets on distributed, linear relaying. Our discussion was based on a scheme that achieves global carrier phase synchronization at all relays. Identifying additive signal noise and phase noise as sources of error, we characterized the phase estimation error. Being aware of the individual phase offsets, we adapted the multiuser zero-forcing relaying gain allocation scheme [1], that orthogonalizes all source/destination links. Finally, we quantified the impact of phase estimation errors on this scheme.

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