# When Do Non-Regenerative Two-Hop Relaying Networks Require a Global Phase Reference?

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***Abstract*— We consider a distributed wireless multiuser network with non-regenerative relays, where the gain factors are computed from channel estimates. They are to be chosen such that multiple signal paths add up coherently at the destinations ('distributed coherent beamforming'). In the presence of channel estimation errors, coherency is generally destroyed because the gain factors cannot be computed correctly. In this work, the channel estimates are assumed to be noiseless, but exhibit phase errors due to random and unknown local oscillator (LO) phases at the terminals. Without global phase reference at the nodes, the phase errors depend on the direction in which the individual point-to-point channels are measured. In some cases, coherency is completely destroyed while in others the system performance is not affected. We will still call the latter cases 'coherent', even if perfect channel state information (CSI) is not available. Based on this observation we derive a framework to determine which nodes in a two-hop network require a global phase reference in order to allow for coherent distributed beamforming. We consider four traffic patterns that differ in the utilization of the direct link and discuss all combinations of directions in which the single-hop channel matrices can be estimated.**

## I. Introduction

In recent years, the predicted benefits of user cooperation in wireless networks have triggered a lot of research in the field of wireless cooperative communication. The basic idea is that multiple autonomous nodes cooperate in order to improve the overall network performance. A distributed array gain increases the receive signal-to-noise ratio (SNR) at the destinations (e.g. [1]). The spatial diversity, that is inherently available in distributed networks, can be exploited to increase the outage rate, thus making the communication more robust against deep fades [2]–[5]. Finally, a distributed spatial multiplexing gain can be realized by allowing multiple source/destination pairs to communicate concurrently on the same physical channel. In [6]–[8] it was shown that the source/destination links can be orthogonalized in a completely distributed manner.

We consider distributed wireless networks with dedicated amplify-and-forward (AF) (non-regenerative) relays. These terminals simply multiply their received signals with a complex-valued gain factor prior to retransmission. In order to achieve a distributed array or spatial multiplexing gain, multiple signal paths from the relays have to add up coherently at the destinations ('distributed coherent beamforming'). Gain allocation schemes achieving these gains are for example discussed in [9]–[11]. It is often believed that perfect instantaneous CSI is required to compute the corresponding gain factors. However, in the context of distributed AF relaying, this assumption has to be adapted. In some cases, the presence of an unknown and random phase offset on the channel estimates has no impact on the system performance. We call those cases 'coherent', even if perfect CSI is not available.

This work was triggered by the fact that in the presence of unknown and random LO phases, the direction in which wireless point-to-point (single-hop) channels are measured has an impact on the estimates (in the form of a phase rotation). Although the propagation channels are reciprocal, the equivalent baseband channels are generally not if the nodes are not phase synchronous (e.g. [12]). Since the gain factors are computed from channel estimates, an unknown and random phase shift is in some cases introduced to the signals at the relays. This phase shift depends on the direction in which the point-to-point channels have been estimated. If no global phase reference is present, it may destroy coherency. However, it has been shown on a real-world demonstrator that in some cases coherent distributed beamforming is possible without global phase reference [13].

In this paper, we answer the following question: *Which nodes in a two-hop relaying network require a global phase reference so that coherent distributed beamforming is possible?* To this end, we derive a framework to determine the phase synchronization requirements. They will be shown to depend on the direction (relative to the data transmission) in which the point-to-point channels are estimated. We therefore investigate all possible combinations for the direction in which the channel matrices can be measured. In multi-hop relaying networks there are different reasons to estimate the single-hop channels in one direction or the other. For example, the number of required channel uses depends on the direction the channels are measured if the number of transmitters and receivers is different. Furthermore, consider a network where the assignment of the nodes (sources, relays, and destinations) is dynamic. Reorganizing the network, e.g. sources becoming destinations and vice versa, automatically changes the direction of the current channel estimates relative to the data transmission direction. A specification of the phase synchronization requirements is a valuable tool to organize such networks or to compare the effort required for phase synchronization with the benefits of different directions of channel estimation.

**Organization of the paper**: The system model is introduced in Section II. We explain the impact of

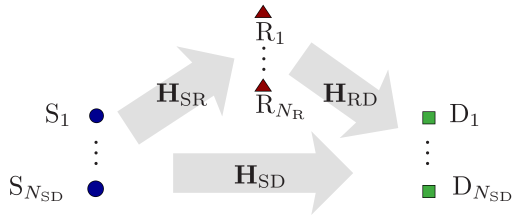


Fig. 1. Two-hop relaying system configuration

unknown and random LO phase offsets on the equivalent baseband channels. Furthermore, we identify four traffic patterns that differ in the utilization of the direct link. Section III then treats the estimation of the point-to-point channel coefficients based on which the gain factors have to be computed. In Section IV, we discuss the notion of distributed coherent beamforming and derive a requirement that can formally be used as an indicator for required global phase synchronization. Finally, the phase synchronization requirements are derived exemplarily for one single channel estimation protocol. A table summarizes the remaining results that are not treated in details because of space limitation.

**Notation**: Bold upper and lower case letters denote matrices and vectors, respectively. The entry in row $i$ and column $j$ of a matrix $\mathbf{X}$ is written as $\mathbf{X}[i,j]$. The operators $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ are the matrix transpose and Hermitian transpose, respectively. $\mathrm{E}_x[\cdot]$ denotes the expectation with respect to $x$ and $\mathrm{diag}(\mathbf{x})$ writes the elements of $\mathbf{x}$ into a diagonal matrix. Finally, vectors with entries that are taken from a zero-mean complex normal distribution with variance $\sigma^2$ are denoted by $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$, where $\mathbf{I}$ is the identity matrix.

## II. System Model

Consider a distributed wireless network where $N_{\mathrm{SD}}$ source/destination pairs communicate with the help of $N_{\mathrm{R}}$ linear AF relays. For the sake of simplicity it is assumed that all nodes in the network employ a single antenna only. The extension to multi-antenna terminals is straightforward. It is assumed that the relays are not able to transmit and receive at the same time (*half-duplex constraint*). Consequently, a transmission cycle consists of two timeslots: one for the *first-hop* transmission from the sources to all relays and one for the *second-hop* transmission from the relays to the destinations. Additionally, the direct link constitutes the transmission from the sources to the destinations and can be used in both timeslots (see Fig. 1). All channels are assumed to be independent and frequency flat. The probability density function (pdf) of the coefficients is not important in this work. The matrices $\mathbf{H}_{\mathrm{SR}} \in \mathbb{C}^{N_{\mathrm{R}} \times N_{\mathrm{SD}}}$ and $\mathbf{H}_{\mathrm{RD}} \in \mathbb{C}^{N_{\mathrm{SD}} \times N_{\mathrm{R}}}$ are the first-hop and second-hop channel matrices, respectively. $\mathbf{H}_{\mathrm{SD}} \in \mathbb{C}^{N_{\mathrm{SD}} \times N_{\mathrm{SD}}}$ denotes the direct link channel. The propagation environment is assumed to be quasi-static, i.e. the channels are constant during at least one transmission cycle.

### A. LO Phase Offsets and Equivalent Channel Matrices

Consider two single-antenna nodes A and B with independent LOs. Let $h := h_{\mathrm{AB}} = h_{\mathrm{BA}}$ denote the equivalent baseband coefficient of the reciprocal, frequency-flat propagation channel between them. The LO phase offsets of nodes A and B are denoted by $\varphi_{\mathrm{A}}$ and $\varphi_{\mathrm{B}}$, respectively. They introduce phase rotations to the signals during the mixing operations. With positive sign when mixing from baseband to passband and with negative sign when mixing from passband to baseband (e.g. [12]). Consequently, the respective equivalent complex baseband channels from A to B and from B to A are

$$\widetilde{h}_{\mathrm{AB}} = h e^{j(\varphi_{\mathrm{A}} - \varphi_{\mathrm{B}})} \quad (1)$$

$$\text{and} \quad \widetilde{h}_{\mathrm{BA}} = h e^{j(\varphi_{\mathrm{B}} - \varphi_{\mathrm{A}})} = \widetilde{h}_{\mathrm{AB}} e^{2j(\varphi_{\mathrm{B}} - \varphi_{\mathrm{A}})}. \quad (2)$$

They become reciprocal, i.e., $\widetilde{h}_{\mathrm{AB}} = \widetilde{h}_{\mathrm{BA}}$, if A and B are phase synchronous, i.e., $\varphi_{\mathrm{A}} = \varphi_{\mathrm{B}}$.

All terminals of the system shown in Fig. 1 are independent, stand-alone nodes. It is thus sensible to assume that each of them employs its own LO. Let $\varphi_{\mathrm{S}_k}$, $\varphi_{\mathrm{R}_l}$, and $\varphi_{\mathrm{D}_m}$ denote the LO phase offset of source $k$, relay $l$, and destination $m$, respectively. We define

$$\mathbf{\Phi}_{\mathrm{S}} := \mathrm{diag}\left(e^{j\varphi_{\mathrm{S}_1}}, \ldots, e^{j\varphi_{\mathrm{S}_{N_{\mathrm{SD}}}}}\right), \quad (3)$$

$$\mathbf{\Phi}_{\mathrm{R}} := \mathrm{diag}\left(e^{j\varphi_{\mathrm{R}_1}}, \ldots, e^{j\varphi_{\mathrm{S}_{N_{\mathrm{R}}}}}\right), \quad (4)$$

$$\text{and} \quad \mathbf{\Phi}_{\mathrm{D}} := \mathrm{diag}\left(e^{j\varphi_{\mathrm{D}_1}}, \ldots, e^{j\varphi_{\mathrm{D}_{N_{\mathrm{SD}}}}}\right). \quad (5)$$

The matrices

$$\widetilde{\mathbf{H}}_{\mathrm{SD}} := \mathbf{\Phi}_{\mathrm{D}}^{\mathrm{H}} \mathbf{H}_{\mathrm{SD}} \mathbf{\Phi}_{\mathrm{S}}, \quad (6)$$

$$\widetilde{\mathbf{H}}_{\mathrm{SR}} := \mathbf{\Phi}_{\mathrm{R}}^{\mathrm{H}} \mathbf{H}_{\mathrm{SR}} \mathbf{\Phi}_{\mathrm{S}}, \quad (7)$$

$$\text{and} \quad \widetilde{\mathbf{H}}_{\mathrm{RD}} := \mathbf{\Phi}_{\mathrm{D}}^{\mathrm{H}} \mathbf{H}_{\mathrm{RD}} \mathbf{\Phi}_{\mathrm{R}} \quad (8)$$

are then called 'equivalent channel matrices'. They comprise the propagation channel coefficients as well as the phase rotations due to the LO phases of the respective transmitter and receiver.

### B. Input/Output Relation

In the following, four two-hop traffic patterns are described within the framework of the system model. They differ in the utilization of the direct link. Traffic patterns I – III correspond to protocols I – III in [14].

**Traffic Pattern I:** In timeslot one, the sources transmit to all relays and destinations. In timeslot two, the sources and relays transmit to the destinations.

**Traffic Pattern II:** The sources transmit to relays and destinations in the first timeslot. While the relays retransmit in the second timeslot, the sources are silent. This traffic pattern might be used in a scenario where the sources receive data from other terminals in the second timeslot and are thus not able to transmit.

**Traffic Pattern III:** In the first timeslot, the sources transmit only to the relays. In the second timeslot, both the sources and relays transmit to the destinations. This

TABLE I

SUMMARY OF TRAFFIC PATTERNS I – IV; A $\rightarrow$ B DENOTES TERMINAL A TRANSMITTING TO B.

| Traffic Pattern | Timeslot 1 | Timeslot 2 |
|---|---|---|
| I | S $\rightarrow$ {R, D} | {S, R} $\rightarrow$ D |
| II | S $\rightarrow$ {R, D} | R $\rightarrow$ D |
| III | S $\rightarrow$ R | {S, R} $\rightarrow$ D |
| IV | S $\rightarrow$ R | R $\rightarrow$ D |

traffic pattern might be used in a scenario where the destinations transmit data to other terminals in the first timeslot thus not being able to receive.

**Traffic Pattern IV:** The direct link is not utilized at all. This corresponds to pure two-hop relaying which may be relevant for situations where the source/destination link is blocked.

Table I summarizes the four traffic patterns. We will subsequently derive the input/output relation for traffic pattern I. The ones for traffic patterns II – IV can be readily derived from the result.

Let the vector $\mathbf{s}^{(1)}$ comprise the mutually independent transmit symbols of all sources in the first timeslot. They are transmitted over the equivalent first-hop channel and the equivalent direct link channel to the relays and destinations, respectively. The vectors of received symbols at the relays and destinations are

$$\mathbf{r} = \widetilde{\mathbf{H}}_{\mathrm{SR}}\mathbf{s}^{(1)} + \mathbf{n}_{\mathrm{R}} \quad (9)$$

$$\text{and} \quad \mathbf{d}^{(1)} = \widetilde{\mathbf{H}}_{\mathrm{SD}}\mathbf{s}^{(1)} + \mathbf{n}_{\mathrm{D}}^{(1)}, \quad (10)$$

where $\mathbf{n}_{\mathrm{R}}$ and $\mathbf{n}_{\mathrm{D}}^{(1)}$ comprise additive white Gaussian noise (AWGN) samples. Prior to retransmission, $\mathbf{r}$ is multiplied with the matrix $\mathbf{G}$ comprising the complex-valued gain factors of the relays. Since all relays employ a single antenna only, $\mathbf{G}$ is diagonal. In the second timeslot, the relay transmit signal is passed through the equivalent second-hop channel to the destinations while the sources transmit $\mathbf{s}^{(2)}$. The received signals at the destinations in timeslot two are stacked in the vector

$$\begin{aligned}\mathbf{d}^{(2)} &= \widetilde{\mathbf{H}}_{\mathrm{RD}}\mathbf{G}\mathbf{r} + \widetilde{\mathbf{H}}_{\mathrm{SD}}\mathbf{s}^{(2)} + \mathbf{n}_{\mathrm{D}}^{(2)} = \\ &= \widetilde{\mathbf{H}}_{\mathrm{RD}}\mathbf{G}\left(\widetilde{\mathbf{H}}_{\mathrm{SR}}\mathbf{s}^{(1)} + \mathbf{n}_{\mathrm{R}}\right) + \widetilde{\mathbf{H}}_{\mathrm{SD}}\mathbf{s}^{(2)} + \mathbf{n}_{\mathrm{D}}^{(2)},\end{aligned}$$

where $\mathbf{n}_{\mathrm{D}}^{(2)}$ comprises AWGN at the destinations.

The most demanding requirements for phase synchronization are imposed on the network if the signals from all possible paths (direct link in the first timeslot as well as direct and two-hop link in the second timeslot) have to combine coherently at the destinations. This is the case for traffic pattern I if both of the following assumptions hold:

- The sources transmit a scaled version of $\mathbf{s}^{(1)}$ in the second timeslot, i.e., $\mathbf{s}^{(2)} = \mathbf{\Gamma}_{\mathrm{S}}\mathbf{s}^{(1)}$, where the matrix $\mathbf{\Gamma}_{\mathrm{S}}$ is diagonal with complex-valued entries $\gamma_{\mathrm{S}_k} \in \mathbb{C}$, $k \in \{1, \ldots, N_{\mathrm{SD}}\}$.
- The destinations compute
$$\mathbf{d}_{\mathrm{I}} = \mathbf{\Gamma}_{\mathrm{D}}\mathbf{d}^{(1)} + \mathbf{d}^{(2)}, \quad (11)$$
prior to decoding, where $\mathbf{\Gamma}_{\mathrm{D}}$ is a diagonal matrix of scaling factors $\gamma_{\mathrm{D}_m}$, $m \in \{1, \ldots, N_{\mathrm{SD}}\}$ at the destinations.

Relaxing any of the two assumptions relaxes the phase synchronization requirements for traffic pattern I:

- If the symbols in $\mathbf{s}^{(1)}$ and $\mathbf{s}^{(2)}$ are mutually independent, they need not combine coherently at the destination. In this case, the phase synchronization requirements are the same as for traffic pattern II, where the source is silent in the second timeslot.
- If the vectors $\mathbf{d}^{(1)}$ and $\mathbf{d}^{(2)}$ are not required to combine coherently, the phase synchronization requirements are the same as for traffic pattern III, where the destinations do not listen in the first timeslot.
- If both assumptions are dropped, the phase synchronization requirements are the same as for traffic pattern IV.

Equation (11) can be written as

$$\mathbf{d}_{\mathrm{I}} = \widetilde{\mathbf{H}}_{\mathrm{I}}\mathbf{s} + \widetilde{\mathbf{n}}_{\mathrm{I}}, \quad (12)$$

where

$$\widetilde{\mathbf{H}}_{\mathrm{I}} = \mathbf{\Gamma}_{\mathrm{D}}\widetilde{\mathbf{H}}_{\mathrm{SD}} + \widetilde{\mathbf{H}}_{\mathrm{RD}}\mathbf{G}\widetilde{\mathbf{H}}_{\mathrm{SR}} + \widetilde{\mathbf{H}}_{\mathrm{SD}}\mathbf{\Gamma}_{\mathrm{S}} \quad (13)$$

$$\text{and} \quad \widetilde{\mathbf{n}}_{\mathrm{I}} = \mathbf{\Gamma}_{\mathrm{D}}\mathbf{n}_{\mathrm{D}}^{(1)} + \widetilde{\mathbf{H}}_{\mathrm{RD}}\mathbf{G}\mathbf{n}_{\mathrm{R}} + \mathbf{n}_{\mathrm{D}}^{(2)} \quad (14)$$

are called the 'compound channel matrix' and the 'compound noise vector', respectively. Thus, the received signal and interference power at destination $m$ are

$$P_m^{(\mathrm{s})} = \left|\widetilde{\mathbf{H}}_{\mathrm{I}}[m,m]\right|^2 \sigma_{\mathrm{s}}^2 \quad (15)$$

$$\text{and} \quad P_m^{(\mathrm{i})} = \sum_{\substack{k=1 \\ k \neq m}}^{N_{\mathrm{SD}}} \left|\widetilde{\mathbf{H}}_{\mathrm{I}}[m,k]\right|^2 \sigma_{\mathrm{s}}^2, \quad (16)$$

respectively. The input/output relation for traffic patterns II – IV are readily obtained from (12) – (14) by setting either $\mathbf{\Gamma}_{\mathrm{D}}$ or $\mathbf{\Gamma}_{\mathrm{S}}$ or both to zero. It turns out that $\widetilde{\mathbf{H}}_{\mathrm{I}}$ is independent of the LO phases of the relays because $\mathbf{\Phi}_{\mathrm{R}}\mathbf{G}\mathbf{\Phi}_{\mathrm{R}}^{\mathrm{H}} = \mathbf{G}$ if $\mathbf{G}$ is diagonal. Consequently, the gain matrix $\mathbf{G}$ as well as the scaling matrices $\mathbf{\Gamma}_{\mathrm{D}}$ and $\mathbf{\Gamma}_{\mathrm{S}}$ do not have to be updated if the relay phases change. They only have to be updated when the propagation channels change. Once $\mathbf{G}$, $\mathbf{\Gamma}_{\mathrm{S}}$, and $\mathbf{\Gamma}_{\mathrm{D}}$ are computed, the LO phases of the nodes have no impact on the receive signal-to-interference-and-noise ratio (SINR).

## III. CHANNEL ESTIMATION

Coherent distributed beamforming requires instantaneous channel knowledge in order to compute the gain factors. Otherwise, the unknown and random phase rotations due to the propagation delays will destroy coherency. Since AWGN on the channel estimates has no impact on the phase synchronization requirements of the network, all channel estimates are assumed to be noiseless. We have seen in Section II-A that the equivalent channel between any two nodes in the network is

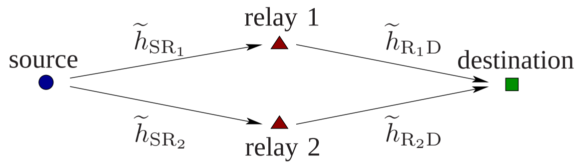


Fig. 2. Simple two-hop relaying scenario.

not reciprocal. This implies that the direction of measurement has an impact on the channel estimates. In the two-hop network in Fig. 1 each channel coefficient can be measured either in *forward direction*, i.e. from sources to relays/destinations or from relays to destinations, or in *backward direction*, i.e. from relays to sources or destinations to relays/sources. If the channels are measured in forward direction, the noiseless estimates of the channel matrices are $\widehat{\mathbf{H}}_{\mathrm{SD}} = \widetilde{\mathbf{H}}_{\mathrm{SD}}$, $\widehat{\mathbf{H}}_{\mathrm{SR}} = \widetilde{\mathbf{H}}_{\mathrm{SR}}$, and $\widehat{\mathbf{H}}_{\mathrm{RD}} = \widetilde{\mathbf{H}}_{\mathrm{RD}}$. Likewise, the estimated channel matrices are (see (2))

$$\widehat{\mathbf{H}}_{\mathrm{SD}} = \widehat{\mathbf{H}}_{\mathrm{DS}}^{\mathrm{T}} = \mathbf{\Phi}_{\mathrm{D}}\mathbf{\Phi}_{\mathrm{D}} \cdot \widetilde{\mathbf{H}}_{\mathrm{SD}} \cdot \mathbf{\Phi}_{\mathrm{S}}^{\mathrm{H}}\mathbf{\Phi}_{\mathrm{S}}^{\mathrm{H}}, \qquad (17)$$

$$\widehat{\mathbf{H}}_{\mathrm{SR}} = \widehat{\mathbf{H}}_{\mathrm{RS}}^{\mathrm{T}} = \mathbf{\Phi}_{\mathrm{R}}\mathbf{\Phi}_{\mathrm{R}} \cdot \widetilde{\mathbf{H}}_{\mathrm{SR}} \cdot \mathbf{\Phi}_{\mathrm{S}}^{\mathrm{H}}\mathbf{\Phi}_{\mathrm{S}}^{\mathrm{H}}, \qquad (18)$$

$$\text{and} \quad \widehat{\mathbf{H}}_{\mathrm{RD}} = \widehat{\mathbf{H}}_{\mathrm{DR}}^{\mathrm{T}} = \mathbf{\Phi}_{\mathrm{D}}\mathbf{\Phi}_{\mathrm{D}} \cdot \widetilde{\mathbf{H}}_{\mathrm{RD}} \cdot \mathbf{\Phi}_{\mathrm{R}}^{\mathrm{H}}\mathbf{\Phi}_{\mathrm{R}}^{\mathrm{H}} \qquad (19)$$

if the channels are measured in backward direction.

## IV. Coherent Distributed Beamforming

It is often assumed that 'coherent beamforming' requires perfect CSI. In the context of distributed AF relaying networks, this definition has to be adapted. Phase errors in the channel estimates that are the result of the measurement direction, have in some cases no impact on the system performance. We want to call those cases 'coherent', even if perfect CSI is not available. In this section, a formal condition for coherent, distributed beamforming is provided that captures this issue. We start by deriving (25), which is the condition for a simple single-user scenario. Extending the result to the multiuser case finally yields (28).

Consider the simple scenario with a single source/destination pair, and two AF relays shown in Fig. 2. The received signal at the destination is

$$d = \left(\sum_{l=1}^{2} \widetilde{h}_{\mathrm{R}_l\mathrm{D}}\, g_l\, \widetilde{h}_{\mathrm{SR}_l}\right) s := \widetilde{h}_{\mathrm{SRD}} \cdot s, \qquad (20)$$

where $s \in \mathcal{CN}\left(0, \sigma_{\mathrm{s}}^2\right)$ is the source transmit symbol, $g_l$, $l \in \{1, 2\}$, are the relay gain factors, and

$$\widetilde{h}_{\mathrm{SRD}} = \sum_{l=1}^{2} \widetilde{h}_{\mathrm{R}_l\mathrm{D}}\, g_l\, \widetilde{h}_{\mathrm{SR}_l} \qquad (21)$$

is called the 'compound channel coefficient'. The power of the received signal at the destination is $P_{\mathrm{R}} = \mathrm{E}_{\mathrm{s}}[|d|^2]$. It is bounded by

$$0 \leq P_{\mathrm{R}} \leq \left(\sum_{l=1}^{2} \left|\widetilde{h}_{\mathrm{R}_l\mathrm{D}}\, g_l\, \widetilde{h}_{\mathrm{SR}_l}\right|\right)^2 \sigma_{\mathrm{s}}^2. \qquad (22)$$

If perfect CSI is available, the received signal power can be adjusted explicitly by choosing the gain factors accordingly. But what if the gain factors have to be computed from channel estimates rather than the actual coefficients? Let $\widehat{h}_{\mathrm{SR}_l}$ and $\widehat{h}_{\mathrm{R}_l\mathrm{D}}$ denote the estimates of $\widetilde{h}_{\mathrm{SR}_l}$ and $\widetilde{h}_{\mathrm{R}_l\mathrm{D}}$, respectively, based on which the relay gains have to be computed. Then, the anticipated received signal power is $\widehat{P}_{\mathrm{R}} = \mathrm{E}_{\mathrm{s}}[|\widehat{d}|^2]$, where

$$\widehat{d} = \left(\sum_{l=1}^{2} \widehat{h}_{\mathrm{R}_l\mathrm{D}}\, g_l\, \widehat{h}_{\mathrm{SR}_l}\right) s := \widehat{h}_{\mathrm{SRD}} \cdot s. \qquad (23)$$

It is bounded by

$$0 \leq \widehat{P}_{\mathrm{R}} \leq \left(\sum_{l=1}^{2} \left|\widehat{h}_{\mathrm{R}_l\mathrm{D}}\, g_l\, \widehat{h}_{\mathrm{SR}_l}\right|\right)^2 \sigma_{\mathrm{s}}^2. \qquad (24)$$

The gain factors can adjust the received signal power to any desired value within its bounds if $P_{\mathrm{R}}$ is a linear function of $\widehat{P}_{\mathrm{R}}$, i.e., $\mathrm{E}_{\mathrm{s}}[|\widehat{d}|^2] = \mathrm{E}_{\mathrm{s}}[|c\widetilde{d}|^2]$, where $c \in \mathbb{C}$. This corresponds to requiring

$$\widehat{h}_{\mathrm{SRD}} = c \cdot \widetilde{h}_{\mathrm{SRD}}. \qquad (25)$$

For the special case that $|c|^2 = 1$, we have $\widehat{P}_{\mathrm{R}} = P_{\mathrm{R}}$. Thus, the anticipated receive SNR is equal to the actual one.

*Proposition 1:* In general, we say that coherent distributed beamforming is possible if the anticipated objective function, based on which the relay gains are computed (e.g. receive SNR), is a linear function of the actual one.

If in the current example the objective function is the receive SNR, it suffices to check whether (25) is fulfilled with $|c|^2 = 1$ to find out whether coherent beamforming is possible. The following example demonstrates that this may be the case even with imperfect CSI.

*Example:* Let

$$\widehat{h}_{\mathrm{SR}_l} = \widetilde{h}_{\mathrm{SR}_l} e^{2j\left(\varphi_{\mathrm{R}_l} - \varphi_{\mathrm{S}}\right)} \qquad (26)$$

$$\text{and} \quad \widehat{h}_{\mathrm{R}_l\mathrm{D}} = \widetilde{h}_{\mathrm{R}_l\mathrm{D}} e^{2j\left(\varphi_{\mathrm{D}} - \varphi_{\mathrm{R}_l}\right)} \qquad (27)$$

denote the estimates of the first-hop and second-hop channels, respectively. They are obtained by estimating all channels in backward direction (see (2)). Since in this case $\widehat{P}_{\mathrm{R}} = P_{\mathrm{R}}$ it is possible to explicitly adjust the receive SNR with gain factors that are computed from imperfect CSI.

The refined definition of coherency for distributed beamforming in the single-user case also holds for multiuser networks. In order to specify the requirements for phase synchronization, we first find the requirements so that the estimated compound channel coefficient from source $k$ to destination $m$ is a linear function of the actual one, i.e.,

$$\widehat{\mathbf{H}}_{\mathrm{X}}[m,k] = c_{m,k} \cdot \widetilde{\mathbf{H}}_{\mathrm{X}}[m,k], \quad \mathrm{X} \in \{\mathrm{I}, \mathrm{II}, \mathrm{III}, \mathrm{IV}\}, \qquad (28)$$

where $c_{m,k} \in \mathbb{C}$. We identify the following propositions:

*Proposition 2:* If (28) holds with $|c_{m,k}| = |c_m|$ for all $k$, the anticipated receive signal-to-interference ratio (SIR) at destination $m$ is equal to the actual one.

*Proposition 3:* If Proposition 2 holds and furthermore $|c_{m,k}|^2 = 1$ for all $k$, the anticipated and the actual SINR at destination $m$ are equal.

Both propositions follow immediately from (15), (16).

From here, it is straightforward to find the phase synchronization requirements for any coherent gain allocation scheme by investigating its objective function: Equation (28) has to hold for $m$ and $k$ if the gain factors are a function of the signal power from source $k$ at destination $m$.

*Example:* For multiuser zero-forcing (MUZF) relaying [6], equation (28) has to hold for all $m \neq k$ and for minimum mean squared error (MMSE) relaying [9], the anticipated and the actual SINR at all destinations have to be equal.

## V. Required Phase Synchronization

In this section, the phase synchronization requirements for all four traffic patterns are discussed. Exemplarily, we derive the results for the case that the first-hop link and the direct link are estimated in forward direction and the second-hop link is estimated in backward direction. This means that the estimates

$$\widehat{\mathbf{H}}_{\mathrm{SD}} = \widetilde{\mathbf{H}}_{\mathrm{SD}}, \tag{29}$$

$$\widehat{\mathbf{H}}_{\mathrm{SR}} = \widetilde{\mathbf{H}}_{\mathrm{SR}}, \tag{30}$$

$$\text{and} \quad \widehat{\mathbf{H}}_{\mathrm{RD}} = \mathbf{\Phi}_{\mathrm{D}}\mathbf{\Phi}_{\mathrm{D}} \cdot \widetilde{\mathbf{H}}_{\mathrm{RD}} \cdot \mathbf{\Phi}_{\mathrm{R}}^{\mathrm{H}}\mathbf{\Phi}_{\mathrm{R}}^{\mathrm{H}} \tag{31}$$

are available in the network.

*Remark:* The framework presented in this work is also applicable for the case that individual channel coefficients are measured in different directions.

All results are summarized in Table II. In order to decode the data from its respective source, each destination additionally estimates its local compound channel coefficient $\widetilde{\mathbf{H}}_{\mathrm{X}}[m,m]$ with the help of a preamble during data transmission.

For traffic pattern I, the compound channel matrix $\widetilde{\mathbf{H}}_{\mathrm{I}}$ is given in (13). At the relays, its estimate is computed from the single-hop channel estimates (29) – (31):

$$\begin{aligned}\widehat{\mathbf{H}}_{\mathrm{I}} &= \widehat{\mathbf{\Gamma}}_{\mathrm{D}}\widehat{\mathbf{H}}_{\mathrm{SD}} + \widehat{\mathbf{H}}_{\mathrm{RD}}\mathbf{G}\widehat{\mathbf{H}}_{\mathrm{SR}} + \widehat{\mathbf{H}}_{\mathrm{SD}}\widehat{\mathbf{\Gamma}}_{\mathrm{S}} = \\ &= \widehat{\mathbf{\Gamma}}_{\mathrm{D}}\widetilde{\mathbf{H}}_{\mathrm{SD}} + \mathbf{\Phi}_{\mathrm{D}}\mathbf{\Phi}_{\mathrm{D}}\widetilde{\mathbf{H}}_{\mathrm{RD}}\mathbf{\Phi}_{\mathrm{R}}^{\mathrm{H}}\mathbf{\Phi}_{\mathrm{R}}^{\mathrm{H}}\mathbf{G}\widetilde{\mathbf{H}}_{\mathrm{SR}} + \widetilde{\mathbf{H}}_{\mathrm{SD}}\widehat{\mathbf{\Gamma}}_{\mathrm{S}},\end{aligned} \tag{32}$$

where $\widehat{\mathbf{\Gamma}}_{\mathrm{S}}$ and $\widehat{\mathbf{\Gamma}}_{\mathrm{D}}$ are estimates of the scaling matrices $\mathbf{\Gamma}_{\mathrm{S}}$ and $\mathbf{\Gamma}_{\mathrm{D}}$ that have to be generated locally. We have to check whether (28) is fulfilled for $\widehat{\mathbf{H}}_{\mathrm{I}}$ given in (32) and $\widetilde{\mathbf{H}}_{\mathrm{I}}$ given in (13). Since (28) has to hold for all channel realizations including the case where any of the channel coefficients is zero, we obtain two conditions:

1) For $\widetilde{\mathbf{H}}_{\mathrm{SR}} = \mathbf{0}$ or $\widetilde{\mathbf{H}}_{\mathrm{RD}} = \mathbf{0}$, we get

$$\widehat{\gamma}_{\mathrm{D}_m} + \widehat{\gamma}_{\mathrm{S}_k} = c_{m,k} \cdot \left(\gamma_{\mathrm{D}_m} + \gamma_{\mathrm{S}_k}\right). \tag{33}$$

2) And for $\widetilde{\mathbf{H}}_{\mathrm{SD}} = \mathbf{0}$, we get

$$\begin{aligned} e^{2j\varphi_{\mathrm{D}_m}} \cdot \sum_{l=1}^{N_{\mathrm{R}}} \left(e^{-2j\varphi_{\mathrm{R}_l}} \cdot \widetilde{h}_{\mathrm{R}_l\mathrm{D}_m} g_l \widetilde{h}_{\mathrm{S}_k\mathrm{R}_l}\right) &= \\ = c_{m,k} \cdot \sum_{l=1}^{N_{\mathrm{R}}} \left(\widetilde{h}_{\mathrm{R}_l\mathrm{D}_m} g_l \widetilde{h}_{\mathrm{S}_k\mathrm{R}_l}\right), \end{aligned} \tag{34}$$

where

$$\widetilde{h}_{\mathrm{S}_k\mathrm{RD}_m} = \sum_{l=1}^{N_{\mathrm{R}}} \left(\widetilde{h}_{\mathrm{R}_l\mathrm{D}_m} g_l \widetilde{h}_{\mathrm{S}_k\mathrm{R}_l}\right) \tag{35}$$

is the compound channel coefficient between source $k$ and destination $m$.

Equation (34) can only hold for all channel realizations if the relays possess a global phase reference. This means that their LO phases have to be equal:

$$\varphi_{\mathrm{R}_l} = \phi, \quad \forall\, l \in \{1, \ldots, N_{\mathrm{R}}\} \tag{36}$$

The phase offset $\phi$ may be unknown and random but has to be the same for all relays. With (36), equation (34) implies

$$e^{2j\left(\varphi_{\mathrm{D}_m} - \phi\right)} = c_{m,k}. \tag{37}$$

Inserting (37) into (33) finally yields

$$\widehat{\gamma}_{\mathrm{D}_m} + \widehat{\gamma}_{\mathrm{S}_k} = e^{2j\left(\varphi_{\mathrm{D}_m} - \phi\right)} \cdot \left(\gamma_{\mathrm{D}_m} + \gamma_{\mathrm{S}_k}\right). \tag{38}$$

We can now derive the phase synchronization requirements for all four traffic patterns from (38):

**Traffic Pattern I:** For traffic pattern I, (28) is fulfilled if (38) holds. With the CSI available in the network, there is, however, no way to compute $\widehat{\gamma}_{\mathrm{S}_k}$, $\gamma_{\mathrm{S}_k}$, $\widehat{\gamma}_{\mathrm{D}_m}$, and $\gamma_{\mathrm{D}_m}$, unequal to zero, such that (38) is fulfilled. In this case, destination $m$ is required to possess the same LO phase reference as the relays, i.e., $\varphi_{\mathrm{D}_m} = \phi$. Equation (38) can then be fulfilled for all $k$ by choosing $\widehat{\gamma}_{\mathrm{S}_k} = \gamma_{\mathrm{S}_k}$ and $\widehat{\gamma}_{\mathrm{D}_m} = \gamma_{\mathrm{D}_m}$. A particularly simple and valid choice is $\widehat{\gamma}_{\mathrm{S}_k} = \gamma_{\mathrm{S}_k} = \widehat{\gamma}_{\mathrm{D}_m} = \gamma_{\mathrm{D}_m} = 1$. Since in this case $c_{m,k} = 1$, the anticipated SINR at destination $m$ is equal to the actual one.

**Traffic Pattern II:** With $\widehat{\gamma}_{\mathrm{S}} = \gamma_{\mathrm{S}} = 0$, equation (38) becomes

$$\widehat{\gamma}_{\mathrm{D}_m} = e^{2j\left(\varphi_{\mathrm{D}_m} - \phi\right)} \cdot \gamma_{\mathrm{D}_m}. \tag{39}$$

For traffic pattern II, destination $m$ can estimate $\widetilde{h}_{\mathrm{S}_k\mathrm{RD}_m}$ (see (35)) using a preamble during data transmission. If it chooses

$$\gamma_{\mathrm{D}_m} = \widetilde{h}_{\mathrm{S}_k\mathrm{RD}_m} \tag{40}$$

and the relays use their respective estimates

$$\begin{aligned}\widehat{\gamma}_{\mathrm{D}_m} &= \widehat{h}_{\mathrm{S}_k\mathrm{RD}_m} = \\ &= e^{2j\left(\varphi_{\mathrm{D}_m} - \phi\right)} \cdot \widetilde{h}_{\mathrm{S}_k\mathrm{RD}_m},\end{aligned} \tag{41}$$

equation (39) is fulfilled for all $k$. The anticipated SINR at destination $m$ is the equal to the actual one because $|c_{m,k}|^2 = 1$ for all $k$.

TABLE II

SUMMARY OF PHASE SYNCHRONIZATION REQUIREMENTS; S: SOURCES, R: RELAYS, D: DESTINATIONS; A $\rightarrow$ B INDICATES THAT THE RESPECTIVE CHANNEL MATRIX IS ESTIMATED FROM NODES A TO B.

| $\widehat{\mathbf{H}}_{\mathrm{SD}}$ / $\widehat{\mathbf{H}}_{\mathrm{SR}}$ / $\widehat{\mathbf{H}}_{\mathrm{RD}}$ | Traffic Pattern I | Traffic Pattern II | Traffic Pattern III | Traffic Pattern IV |
|---|---|---|---|---|
| S $\rightarrow$ D / S $\rightarrow$ R / R $\rightarrow$ D | - | - | - | - |
| S $\rightarrow$ D / S $\leftarrow$ R / R $\leftarrow$ D | $\mathrm{S}_k$ & $\mathrm{D}_m$ | $\mathrm{S}_k$ & $\mathrm{S}_m$ | $\mathrm{S}_k$ & $\mathrm{D}_m$ | - |
| **S $\rightarrow$ D / S $\rightarrow$ R / R $\leftarrow$ D** | **R & $\mathbf{D}_m$** | **R** | **R & $\mathbf{D}_m$** | **R** |
| S $\rightarrow$ D / S $\leftarrow$ R / R $\rightarrow$ D | R & $\mathrm{S}_k$ | R & $\mathrm{S}_k$ | R & $\mathrm{S}_k$ | R |
| S $\leftarrow$ D / S $\rightarrow$ R / R $\rightarrow$ D | $\mathrm{S}_k$ & $\mathrm{D}_m$ | $\mathrm{S}_k$ & $\mathrm{S}_m$ | $\mathrm{S}_k$ & $\mathrm{D}_m$ | - |
| S $\leftarrow$ D / S $\leftarrow$ R / R $\leftarrow$ D | - | - | - | - |
| S $\leftarrow$ D / S $\rightarrow$ R / R $\leftarrow$ D | R & $\mathrm{S}_k$ | R & $\mathrm{S}_k$ | R & $\mathrm{S}_k$ | R |
| S $\leftarrow$ D / S $\leftarrow$ R / R $\rightarrow$ D | R & $\mathrm{D}_m$ | R & $\mathrm{D}_m$ | R & $\mathrm{D}_m$ | R |

**Traffic Pattern III:** With $\widehat{\gamma}_{\mathrm{D}} = \gamma_{\mathrm{D}} = 0$, equation (38) becomes

$$\widehat{\gamma}_{\mathrm{S}_k} = e^{2j\left(\varphi_{\mathrm{D}_m} - \phi\right)} \cdot \gamma_{\mathrm{S}_k}. \tag{42}$$

From the available channel knowledge, it is not possible to compute $\widehat{\gamma}_{\mathrm{S}_k}$ and $\gamma_{\mathrm{S}_k}$, unequal to zero, so that (42) is fulfilled. Consequently, destination $m$ has to have the same LO phase as the relays, i.e., destination $m$ and the relays require a joint global phase reference. In this case, $\varphi_{\mathrm{D}_m} = \phi$ and (42) can be fulfilled by choosing $\widehat{\gamma}_{\mathrm{S}_k} = \gamma_{\mathrm{S}_k}$. Since (28) then holds for $c_{m,k} = 1$, the anticipated and the actual SINR at destination $m$ are equal.

**Traffic Pattern IV:** With $\widehat{\gamma}_{\mathrm{S}} = \gamma_{\mathrm{S}} = \widehat{\gamma}_{\mathrm{D}} = \gamma_{\mathrm{D}} = 0$, equation (38) is always fulfilled. In this case, (28) holds for $c_{m,k} = 1$. Thus, the anticipated and the actual SINR at all destinations are the same as the actual one.

Table II summarizes the phase synchronization requirements for each traffic pattern and all combinations of directions in which the first-hop, second-hop, and direct link channel matrices can be measured. Equation (28) is only fulfilled if the respective phase synchronization is available in the network. The boldface row indicates the case that has been exemplarily derived in this work. The meaning of the entries is as follows:

- '-': No global phase reference is required.
- 'R': Global phase reference is required at all relays.
- 'R & $\mathrm{S}_k$': All relays and source $k$ require a joint, global phase reference.
- 'R & $\mathrm{D}_m$': All relays and destination $m$ require a joint, global phase reference.
- '$\mathrm{S}_k$ & $\mathrm{D}_m$': Source $k$ and destination $m$ require a joint, global phase reference.
- '$\mathrm{S}_k$ & $\mathrm{S}_m$': Equation (28) holds for $k = m$ without global phase reference. However, a phase reference between source $k$ and $m$ is required for $k \neq m$.

Given that the required phase synchronization is available in the network, equation (28) can in all cases be fulfilled with $|c_{m,k}|^2 = 1$. This means that the anticipated received power from source $k$ at destination $m$ is equal to the actual one.

## VI. CONCLUSION

In this work, we investigated nonregenerative two-hop relaying networks where all nodes employ independent LOs. The channel estimates, based on which the gain factors are computed, thus suffer from unknown and random phase errors that depend on the direction of measurement. In the presence of these errors, we specified for each traffic pattern and channel estimation protocol which set of nodes requires to synchronize their LO phases so that coherency is preserved and the signals of multiple paths add up coherently at the destinations.

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