Asymmetric Data Rate Transmission in Two-Way Relaying Systems With Network Coding

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***Abstract*—We propose a novel transmission scheme for the broadcast phase of two-way relaying systems. The proposed scheme employs network coding at the relay and is able to transmit with asymmetric data rates to the receivers according to their individual link qualities. The idea is that the weaker link receiver exploits *a priori* bit information in each transmit symbol, so that it only needs to decode on a subset of the transmit symbol constellation. Subject to the same bit error rate constraint, the weaker link receiver can decode at lower signal-to-noise ratio compared to the stronger link. The signal labeling used for mapping bits to symbols at the relay is shown to be crucial for the performance at the receivers, and we provide the criterion and method for finding the optimized labeling schemes. Simulations show that the proposed transmission scheme can be applied to practical scenarios with asymmetric channel qualities, and the optimized labeling greatly outperforms conventional ones at both receivers.**

I. Introduction

The *two-way relaying* protocol [1] is a novel technique proposed to recover the spectral efficiency loss in half-duplex relaying systems, where two wireless stations exchange data via a half-duplex wireless relay. We consider two-way decode-and-forward (DF) relaying systems in this paper, where the data from the two stations are exchanged in two phases: the multiple-access (MAC) phase and the broadcast (BRC) phase. In the MAC phase, the two stations transmit their data to the relay and the relay decodes the received signal; the decoded data are combined at the relay and are retransmitted back to the two stations in the BRC phase.

We employ *network coding* [2] at the relay for the BRC phase, where the relay combines the data on the bit level using the XOR operation before modulation. Compared to other data-combining schemes, network coding does not split the power for transmitting the two sets of data, and has advantages in many scenarios [3]. Network coding requires that both receiving stations decode the combined data bits from the same transmit symbols in the BRC phase. It was shown in [4] that network coding is optimal for transmitting the same amount of data to both stations when the channel qualities from the relay to the two stations are equal. When the channel qualities to the two stations are asymmetric, how to transmit data, so that the data rates from the relay are not limited by the weaker link of the two stations, is an important problem for practical systems. Information theory shows that by using random coding approaches [4], [5], it is possible for the relay to transmit information rates equal to the individual link capacities simultaneously to the two receiving stations. The authors of [6] and [7] respectively proposed schemes of combining channel coding for binary transmission and lattice coding with network coding in two-way relaying systems and derived the achievable rate regions for each case. However, real-world applications call for practical and low-complexity transmission schemes, especially for multi-antenna systems.

In this paper, we propose a novel transmission scheme for the BRC phase of two-way DF relaying systems when network coding is applied. In the proposed scheme, the data rates transmitted by the relay to the two receiving stations can be adjusted according to their individual link qualities subject to certain bit error rate (BER) constraints, and we call it *asymmetric data rate transmission*. The proposed scheme has low complexity and can be applied to systems with single or multiple antennas. The idea is that the relay combines the data in such a way that some bits in each transmit symbol are *a priori* known to the weaker link receiver. That receiver can hence exploit the known bits and only needs to decode on a subset of the transmit signal constellation. Therefore, the *a priori* bit information is translated into the coding gain, and enables the weaker link receiver to achieve comparable decoding performance as the stronger link. We show that the *signal labeling* at the relay, i.e., the assignment of bit patterns to each symbol in the signal constellation, plays an important role in the system performance. The criterion and method for finding the optimized labeling schemes are also proposed. Furthermore, we show an example of systems with 8PSK constellation on each transmit antenna, and provide the optimized labeling schemes with their performance results. To the best of our knowledge, this is the first scheme that exploits the *a priori* bit information on the symbol level for network coding schemes in two-way DF relaying systems.

The rest of the paper is organized as follows: the two-way DF relaying protocol with network coding is recapitulated in Section II. The details of the proposed transmission scheme are discussed in Section III, where we provide the transceiver structures and show how they work. The criterion and method for designing optimized labeling schemes are discussed in Section IV. Simulation results that compare the decoding per-

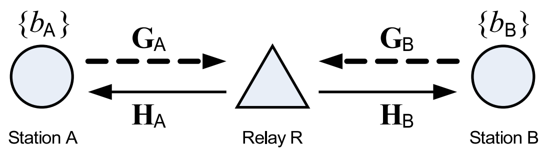


Fig. 1. Two-way DF relaying system. The dashed arrows represent the transmission in the MAC phase, and the solid arrows represent the transmission in the BRC phase.

formance of the optimized labeling with that of conventional ones are presented in Section V. Conclusions and outlooks are drawn in Section VI.

Notation We use bold uppercase letters to denote matrices and bold lowercase letters to denote vectors. $\mathbf{I}_N$ is an $N \times N$ identity matrix. $\mathcal{CN}(\mathbf{0}, \mathbf{K})$ denotes a circularly symmetric complex normal zero mean random vector with covariance matrix $\mathbf{K}$. Furthermore, $\mathbb{E}[\cdot]$, $|\cdot|$, $\|\cdot\|$, $(\cdot)^T$ and $(\cdot)^H$ denote the expectation, the cardinality of a set, the Euclidean norm of a vector, the transpose and the conjugate transpose, respectively. $\{a\}$ denotes a sequence composed of elements a.

II. System Model

We consider a relaying system where two wireless stations A and B exchange data via a half-duplex relay as shown in Fig. 1. We assume that there is no direct connection between stations A and B (e.g., due to shadowing). The number of antennas at Station A, Relay R and Station B are N_{A}, N_{R} and N_{B}, respectively. $\mathbf{G}_{\mathsf{A}} \in \mathbb{C}^{N_{\mathsf{R}} \times N_{\mathsf{A}}}$ and $\mathbf{G}_{\mathsf{B}} \in \mathbb{C}^{N_{\mathsf{R}} \times N_{\mathsf{B}}}$ respectively denote the channel matrices from stations A and B to the relay in the MAC phase. $\mathbf{H}_{\mathsf{A}} \in \mathbb{C}^{N_{\mathsf{A}} \times N_{\mathsf{R}}}$ and $\mathbf{H}_{\mathsf{B}} \in \mathbb{C}^{N_{\mathsf{B}} \times N_{\mathsf{R}}}$ denote the channel matrices from the relay to stations A and B in the BRC phase, respectively. Station A wants to send the bit sequence $\{b_{\mathsf{A}}\}$ to Station B, and Station B wants to send the bit sequence $\{b_{\mathsf{B}}\}$ to Station A.

When the two-way DF relaying protocol is applied, the bit sequences $\{b_{\mathsf{A}}\}$ and $\{b_{\mathsf{B}}\}$ are respectively modulated and transmitted to the relay by stations A and B in the MAC phase. The receiver structure at the relay can be found in, e.g., [8].[1] In the following, we focus on the BRC phase and assume the MAC phase has been completed, i.e., the bit sequences $\{b_{\mathsf{A}}\}$ and $\{b_{\mathsf{B}}\}$ have already been sent to the relay.

In the BRC phase, we apply network coding [2] at the relay to retransmit the data. The basic idea is that the relay combines the decoded bit sequences on the bit level using the XOR operation, and remodulates the combined bit sequence into transmit symbols, i.e.,

$$\{b_{\mathsf{A}} \oplus b_{\mathsf{B}}\} = \{b_{\mathsf{R}}\} \longmapsto \{\mathbf{s}_{\mathsf{R}}\}. \tag{1}$$

The received signals at stations A and B are

$$\mathbf{y}_{\mathsf{A}} = \mathbf{H}_{\mathsf{A}}\mathbf{s}_{\mathsf{R}} + \mathbf{n}_{\mathsf{A}} \tag{2}$$

$$\mathbf{y}_{\mathsf{B}} = \mathbf{H}_{\mathsf{B}}\mathbf{s}_{\mathsf{R}} + \mathbf{n}_{\mathsf{B}} \tag{3}$$

where $\mathbf{n}_{\mathsf{A}} \sim \mathcal{CN}(\mathbf{0}, \sigma_{\mathsf{A}}^2\mathbf{I}_{N_{\mathsf{A}}})$ and $\mathbf{n}_{\mathsf{B}} \sim \mathcal{CN}(\mathbf{0}, \sigma_{\mathsf{B}}^2\mathbf{I}_{N_{\mathsf{B}}})$ are the additive noise vectors at stations A and B, respectively. The two stations demodulate the received signals and reveal the unknown data bits by XOR-ing the decoded data $\{\hat{b}_{\mathsf{R}}\}$ with their own transmitted data on the bit level. That is,

$$\{\hat{b}_{\mathsf{B}}\} = \{\hat{b}_{\mathsf{R}} \oplus b_{\mathsf{A}}\}, \quad \text{at Station A;}$$
$$\{\hat{b}_{\mathsf{A}}\} = \{\hat{b}_{\mathsf{R}} \oplus b_{\mathsf{B}}\}, \quad \text{at Station B.}$$

[1] A simpler, albeit suboptimal, approach is to transmit $\{b_{\mathsf{A}}\}$ and $\{b_{\mathsf{B}}\}$ separately to the relay as in [2].

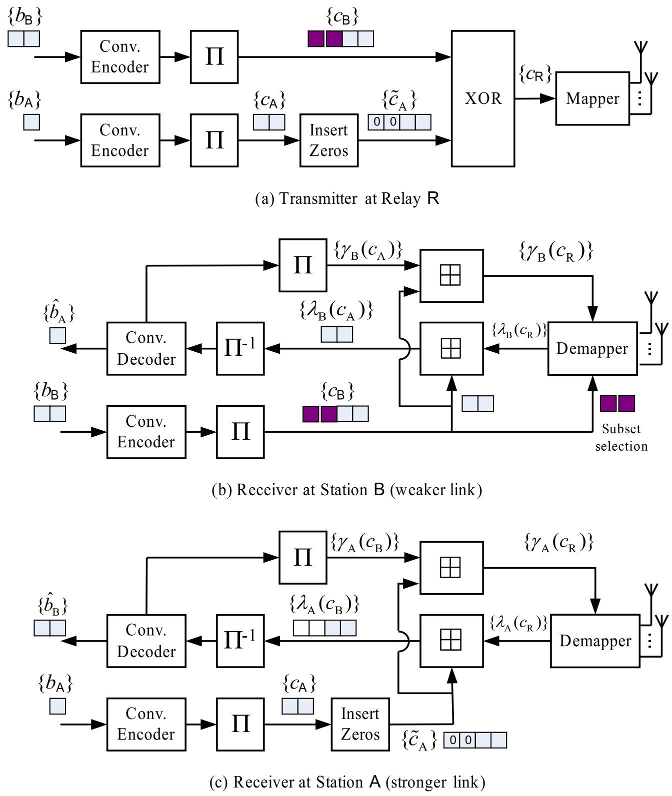


Fig. 2. Transmitter and receiver diagrams for asymmetric data rate transmission in the BRC phase. The box-plus "⊞" module at the receiver side is defined in (13).

Since both receiving stations have to decode the data contained in the symbol $\mathbf{s}_{\mathsf{R}}$, it was conventionally thought that the relay must transmit at a data rate that can be supported by both links. This sacrifices the stronger link, and is not desirable in practice. In the following, we propose a practical scheme that can transmit with asymmetric data rates simultaneously from the relay to the two stations according to their individual link qualities in the BRC phase.

III. Transceiver Structures for Asymmetric Data Rate Transmission

The transmitter and receiver diagrams of the proposed scheme are shown in Fig. 2. We assume the link from the relay to Station A has better channel quality, e.g., higher signal-to-noise ratio (SNR), than the link to Station B in the BRC phase. The aim of the proposed scheme is to utilize the stronger link to transmit more data bits per channel use to Station A, while at the same time transmitting to Station B at a data rate that can be supported by its link.[2] We assume

[2] This requires more data bits $\{d_{\mathsf{B}}\}$ to be available at the relay for the BRC phase transmission. Hence, it may require Station B to use more temporal or spectral resources, e.g., more subcarriers in OFDMA systems, to transmit those data bits to the relay in the MAC phase.

the channel matrices $\mathbf{H}_{\mathsf{A}}$ and $\mathbf{H}_{\mathsf{B}}$ are respectively known to stations A and B. The system diagram in Fig. 2 applies bit-interleaved coded modulation with iterative decoding (BICM-ID). However, the proposed idea can actually be applied to both coded and uncoded systems.

A. Transmission Strategy at the Relay

Fig. 2(a) shows the proposed transmitter structure at Relay R. The information bit sequences $\{b_{\mathsf{A}}\}$ and $\{b_{\mathsf{B}}\}$ decoded in the MAC phase are encoded individually by a convolutional encoder with coding rate r. Here we assume the two sequences are encoded by the same encoder for ease of implementation at the relay. Without loss of generality, we consider a spatial-multiplexing structure with N_{R} independent data streams. Similar discussions apply to less data streams, e.g., transmitting $\min(N_{\mathsf{R}}, N_{\mathsf{A}}, N_{\mathsf{B}})$ streams to enable efficient decoding. In each transmission, the relay sends $r \cdot mN_{\mathsf{R}}$ information bits to Station A and $r \cdot nN_{\mathsf{R}}$ information bits to Station B simultaneously, where $n < m$. The relay determines m and n according to the knowledge of the average receive SNRs at A and B. The transmit symbol on each relay antenna belongs to the M-ary QAM or PSK symbol alphabets, where $M = 2^m$.

The transmitter works as follows: the output bits of the convolutional encoders are bitwise interleaved to form the code sequences $\{c_{\mathsf{A}}\}$ and $\{c_{\mathsf{B}}\}$, where $c_{\mathsf{A}}, c_{\mathsf{B}} \in \{0, 1\}$. Then the bit-interleaved codewords are respectively partitioned into groups of nN_{R} and mN_{R} bits. Each pair of the two corresponding bit groups is denoted as $\mathbf{c}_{\mathsf{A}} = [c_{\mathsf{A}}^{nN_{\mathsf{R}}}, \cdots, c_{\mathsf{A}}^{1}]^T$ and $\mathbf{c}_{\mathsf{B}} = [c_{\mathsf{B}}^{mN_{\mathsf{R}}}, \cdots, c_{\mathsf{B}}^{1}]^T$, respectively. For each $\mathbf{c}_{\mathsf{A}}$, we insert $(m - n)N_{\mathsf{R}}$ *dummy* zeros in it and obtain

$$\tilde{\mathbf{c}}_{\mathsf{A}} = [\underbrace{0, \cdots, 0}_{(m-n)N_{\mathsf{R}}}, c_{\mathsf{A}}^{nN_{\mathsf{R}}}, \cdots, c_{\mathsf{A}}^{1}]^T.$$

Those dummy zeros contain no information. Their positions are fixed and known to both stations A and B. After inserting zeros, the corresponding bits of $\tilde{\mathbf{c}}_{\mathsf{A}}$ and $\mathbf{c}_{\mathsf{B}}$ are combined into $\mathbf{c}_{\mathsf{R}} = [c_{\mathsf{R}}^{mN_{\mathsf{R}}}, \cdots, c_{\mathsf{R}}^{1}]^T$ using the XOR operation, i.e.,

$$\mathbf{c}_{\mathsf{R}} = \mathbf{c}_{\mathsf{B}} \oplus \tilde{\mathbf{c}}_{\mathsf{A}} \quad (4)$$

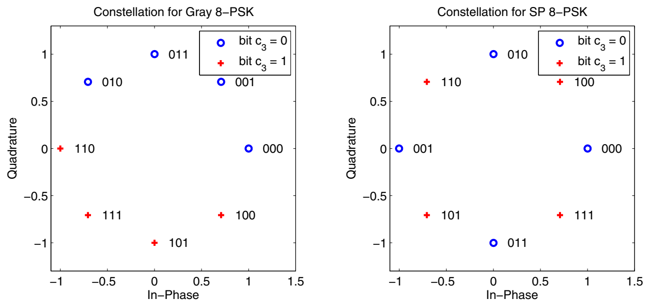
$$= [\underbrace{c_{\mathsf{B}}^{mN_{\mathsf{R}}}, \cdots, c_{\mathsf{B}}^{nN_{\mathsf{R}}+1}}_{(m-n)N_{\mathsf{R}}}, c_{\mathsf{B}}^{nN_{\mathsf{R}}} \oplus c_{\mathsf{A}}^{nN_{\mathsf{R}}}, \cdots, c_{\mathsf{B}}^{1} \oplus c_{\mathsf{A}}^{1}]^T. \quad (5)$$

Due to the dummy zeros in $\tilde{\mathbf{c}}_{\mathsf{A}}$, $[c_{\mathsf{B}}^{mN_{\mathsf{R}}}, \cdots, c_{\mathsf{B}}^{nN_{\mathsf{R}}+1}]$ are kept unchanged after the XOR operation when $\mathbf{c}_{\mathsf{R}}$ is generated, and those bits are known to Station B.

Each of the combined bit group $\mathbf{c}_{\mathsf{R}}$ is mapped to an N_{R} dimensional complex symbol vector $\mathbf{s}_{\mathsf{R}} = [s_{\mathsf{R},1}, \cdots, s_{\mathsf{R},N_{\mathsf{R}}}]^T = \mu(\mathbf{c}_{\mathsf{R}})$ on the relay antennas by the mapper, where $\mu(\cdot)$ denotes the mapping function. Each element $s_{\mathsf{R},i}$, $i \in \{1, \cdots, N_{\mathsf{R}}\}$, belongs to the M-ary QAM or PSK symbol alphabets $\mathcal{A} = \{a_1, \cdots, a_M\}$, where $M = 2^m$. The N_{R} dimensional signal constellation is denoted as $\mathcal{X}$, i.e.,

$$\mathcal{X} = \{\mathbf{s} \mid \mathbf{s} = \mu(\mathbf{c}), \forall \mathbf{c} \in \{0, 1\}^{mN_{\mathsf{R}}}\} = \mathcal{A}^{N_{\mathsf{R}}}$$

and $|\mathcal{X}| = 2^{mN_{\mathsf{R}}}$. Furthermore, $\mathbb{E}(\mathbf{s}_{\mathsf{R}}\mathbf{s}_{\mathsf{R}}^H) = P_{\mathsf{R}}/N_{\mathsf{R}}\mathbf{I}_{N_{\mathsf{R}}}$, where P_{R} is the average transmit power constraint at the relay in the BRC phase. Both the encoding scheme and the mapping scheme $\mu(\cdot)$ at the relay are known to stations A and B.



(a) 8PSK with Gray labeling (b) 8PSK with SP labeling

Fig. 3. 8PSK labeling schemes, single transmit antenna case.

B. Decoding Strategies at the Receivers

Upon receiving $\mathbf{y}_{\mathsf{A}}$ and $\mathbf{y}_{\mathsf{B}}$ as in (2) and (3), stations A and B demodulate the received signals, and reveal the unknown data based on the bits contained in $\mathbf{s}_{\mathsf{R}}$ and their own data bits. The receiver structures are shown in Fig. 2(b) and Fig. 2(c). There are two important issues in the design of the receivers: firstly, in order to make it possible for the weaker link receiver B to decode at lower SNR, we must *exploit* its *a priori* known bits contained in $\mathbf{c}_{\mathsf{R}}$ in the demapping process; secondly, we use the box-plus "$\boxplus$" module defined in Section III-B2 in the iterative decoding process to keep the reliability information for the decoded bits intact .

1) Exploiting a priori bit information at Station B: The transmit symbol vector $\mathbf{s}_{\mathsf{R}}$ contains different *useful* bits for stations A and B: in order to obtain $\mathbf{c}_{\mathsf{B}}$, every bit $c_{\mathsf{R}}^{i}, \forall i \in \{1, \cdots, mN_{\mathsf{R}}\}$, is useful for Station A according to (5) and needs to be decoded, whereas the useful bits for Station B are c_{R}^{i}, $i \in \{1, \cdots, nN_{\mathsf{R}}\}$, because the $(m - n)N_{\mathsf{R}}$ bits $[c_{\mathsf{B}}^{mN_{\mathsf{R}}}, \cdots, c_{\mathsf{B}}^{nN_{\mathsf{R}}+1}]$ in (5) are *a priori* known at its receiver. Instead of decoding every bit in $\mathbf{c}_{\mathsf{R}}$ and discarding the known bits, we propose to exploit this *a priori* bit information, so that the receiver B only needs to demap on the subset of the transmit signal constellation whose labels contain $[c_{\mathsf{B}}^{mN_{\mathsf{R}}}, \cdots, c_{\mathsf{B}}^{nN_{\mathsf{R}}+1}]$ at the corresponding positions.

An example with 8PSK ($m = 3$) constellation transmission is shown in Fig. 3, where $N_{\mathsf{R}} = 1$ and two labeling schemes for the symbol s_{R} are shown: the Gray labeling in Fig. 3(a) and the set partitioning (SP) labeling [9] in Fig. 3(b). We assume $n = 2$ and $c_{\mathsf{B}}^{3} = 0$. Since Station B knows $c_{\mathsf{B}}^{3} = 0$, it only needs to consider the symbols whose 3rd bit is 0 (indicated by circles in Fig. 3) for the demapping process. Given the known bits $[c_{\mathsf{B}}^{mN_{\mathsf{R}}}, \cdots, c_{\mathsf{B}}^{nN_{\mathsf{R}}+1}]$, we define the subset of symbol constellation, whose labels contain those known bits at the corresponding positions, as $\mathcal{S}(c_{\mathsf{B}}^{mN_{\mathsf{R}}}, \cdots, c_{\mathsf{B}}^{nN_{\mathsf{R}}+1}) \subset \mathcal{X}$, i.e.,

$$\mathcal{S}(c_{\mathsf{B}}^{mN_{\mathsf{R}}}, \cdots, c_{\mathsf{B}}^{nN_{\mathsf{R}}+1}) = \left\{\mathbf{s} \mid c_{mN_{\mathsf{R}}}(\mathbf{s}) = c_{\mathsf{B}}^{mN_{\mathsf{R}}}, \cdots, c_{nN_{\mathsf{R}}+1}(\mathbf{s}) = c_{\mathsf{B}}^{nN_{\mathsf{R}}+1}, \mathbf{s} \in \mathcal{X}\right\} \quad (6)$$

where $c_j(\mathbf{s})$ denotes the jth bit associated with the label of symbol $\mathbf{s}$. Given $c_{\mathsf{B}}^{3} = 0$ in Fig. 3, the subset to be demapped at Station B can be denoted as $\mathcal{S}(0)$. Fig. 3 shows that different labeling schemes lead to different subsets for the given *a priori* bits and influence the decoding performance at the receivers. For SP labeling, the components in $\mathcal{S}(0)$ are same as those of QPSK, and the minimum Euclidean distance (MED) between symbols in $\mathcal{S}(0)$ is increased compared to that of the original 8PSK. This leads to better decoding performance at Station B when it demaps only on $\mathcal{S}(0)$ instead of on $\mathcal{X}$. However, the MED of $\mathcal{S}(0)$ for Gray labeling is not increased, and simulations show that demapping only on $\mathcal{S}(0)$ in Gray labeling does not improve the decoding performance. How to find the optimized labeling schemes will be discussed in Section IV.

The demappers at stations A and B work as follows. In each iteration, the soft-output demapper of Station A calculates the *a posteriori* LLR values $\Lambda_{\mathsf{A}}(c_{\mathsf{R}}^i)$ for each of the coded bit c_{R}^i, $i \in \{1, \cdots, mN_{\mathsf{R}}\}$ associated with $\mathbf{y}_{\mathsf{A}}$ [8]:

$$\begin{aligned}\Lambda_{\mathsf{A}}(c_{\mathsf{R}}^i) &= \ln \frac{p(c_{\mathsf{R}}^i = 1|\mathbf{y}_{\mathsf{A}})}{p(c_{\mathsf{R}}^i = 0|\mathbf{y}_{\mathsf{A}})} \qquad (7)\\ &\approx \min_{\mathbf{s}_{\mathsf{R}} \in \mathcal{X}_i^0} \frac{\|\mathbf{y}_{\mathsf{A}} - \mathbf{H}_{\mathsf{A}}\mathbf{s}_{\mathsf{R}}\|^2}{\sigma_{\mathsf{A}}^2} - \min_{\mathbf{s}_{\mathsf{R}} \in \mathcal{X}_i^1} \frac{\|\mathbf{y}_{\mathsf{A}} - \mathbf{H}_{\mathsf{A}}\mathbf{s}_{\mathsf{R}}\|^2}{\sigma_{\mathsf{A}}^2} \qquad (8)\end{aligned}$$

where $\mathcal{X}_i^1$ and $\mathcal{X}_i^0$ represent the sets of transmit symbol vectors whose ith bit labeling is 1 and 0, respectively. Similarly, the *a posteriori* LLR values $\Lambda_{\mathsf{B}}(c_{\mathsf{R}}^i)$ for the coded bits c_{R}^i, $i \in \{1, \cdots, nN_{\mathsf{R}}\}$ are calculated at the demapper B as

$$\begin{aligned}\Lambda_{\mathsf{B}}(c_{\mathsf{R}}^i) &= \ln \frac{p(c_{\mathsf{R}}^i = 1|\mathbf{y}_{\mathsf{B}})}{p(c_{\mathsf{R}}^i = 0|\mathbf{y}_{\mathsf{B}})} \qquad (9)\\ &\approx \min_{\mathbf{s}_{\mathsf{R}} \in \mathcal{S}_i^0} \frac{\|\mathbf{y}_{\mathsf{B}} - \mathbf{H}_{\mathsf{B}}\mathbf{s}_{\mathsf{R}}\|^2}{\sigma_{\mathsf{B}}^2} - \min_{\mathbf{s}_{\mathsf{R}} \in \mathcal{S}_i^1} \frac{\|\mathbf{y}_{\mathsf{B}} - \mathbf{H}_{\mathsf{B}}\mathbf{s}_{\mathsf{R}}\|^2}{\sigma_{\mathsf{B}}^2} \qquad (10)\end{aligned}$$

where $\mathcal{S}_i^1$ and $\mathcal{S}_i^0$ represent the sets of transmit symbol vectors whose ith bit labeling is 1 and 0 in the constellation subset $\mathcal{S}(c_{\mathsf{B}}^{mN_{\mathsf{R}}}, \cdots, c_{\mathsf{B}}^{nN_{\mathsf{R}}+1})$, respectively.

In order to avoid error propagation, the *a priori* LLRs $\gamma_{\mathsf{A}}(c_{\mathsf{R}}^i)$ and $\gamma_{\mathsf{B}}(c_{\mathsf{R}}^i)$ from the feedback of the channel decoders are subtracted from $\Lambda_{\mathsf{A}}(c_{\mathsf{R}}^i)$ and $\Lambda_{\mathsf{B}}(c_{\mathsf{R}}^i)$ to generate the *extrinsic* LLRs $\lambda_{\mathsf{A}}(c_{\mathsf{R}}^i)$ and $\lambda_{\mathsf{B}}(c_{\mathsf{R}}^i)$ as:

$$\lambda_{\mathsf{A}}(c_{\mathsf{R}}^i) = \Lambda_{\mathsf{A}}(c_{\mathsf{R}}^i) - \gamma_{\mathsf{A}}(c_{\mathsf{R}}^i), \quad i \in \{1, \cdots, mN_{\mathsf{R}}\}, \quad (11)$$

$$\lambda_{\mathsf{B}}(c_{\mathsf{R}}^i) = \Lambda_{\mathsf{B}}(c_{\mathsf{R}}^i) - \gamma_{\mathsf{B}}(c_{\mathsf{R}}^i), \quad i \in \{1, \cdots, nN_{\mathsf{R}}\}. \quad (12)$$

2) LLR values of $\{c_{\mathsf{A}}\}$ *and* $\{c_{\mathsf{B}}\}$*:* The output of the demappers are the LLR values for $\{c_{\mathsf{R}}\}$. They must be converted to the LLR values for $\{c_{\mathsf{A}}\}$ and $\{c_{\mathsf{B}}\}$ for channel decoding. This is accomplished by the "$\boxplus$" module.

The sign of each LLR value shows the estimate that the corresponding bit is 1 or 0, and its absolute value represents the reliability of such estimation. For $i \in \{1, \cdots, nN_{\mathsf{R}}\}$, we have $c_{\mathsf{B}}^i \oplus c_{\mathsf{A}}^i = c_{\mathsf{R}}^i$. The bit c_{B}^i (resp. c_{A}^i) differs with c_{R}^i only when $c_{\mathsf{A}}^i = 1$ (resp. $c_{\mathsf{B}}^i = 1$). Given the LLR value λ of c_{R}^i and the known bit c (i.e., c_{A}^i or c_{B}^i), the LLR value of the unknown bit (i.e., c_{B}^i or c_{A}^i) can be calculated as

$$\lambda \boxplus c = \begin{cases} \lambda, & \text{if } c = 0, \\ -\lambda, & \text{if } c = 1. \end{cases} \qquad (13)$$

That is, the "$\boxplus$" module flips the sign of λ according to the corresponding input bit c. Since each bit c is perfectly known, it does not change the reliability of the decoded bits.

After the "$\boxplus$" module, the LLR values for $\{c_{\mathsf{A}}\}$ and $\{c_{\mathsf{B}}\}$ are given to the input of the convolutional decoder, where the BCJR algorithm [10] is applied. Similar to the demappers, *extrinsic* LLR values for the coded bits are generated at the output of convolutional decoders. The feedback LLR values for $\{c_{\mathsf{R}}\}$ are calculated according to the outputs of the channel decoder and the bit sequence $\{\tilde{c}_{\mathsf{A}}\}$ and $\{c_{\mathsf{B}}\}$ again using the "$\boxplus$" module. In the final iteration, the decoder outputs the hard decisions on the information bits. The overall workflow of the iterative receivers is summarized in Algorithm 1.

Algorithm 1 Workflow of receivers at stations A and B

Initialize: 1. Obtain $\{\tilde{c}_{\mathsf{A}}\}$ and $\{c_{\mathsf{B}}\}$ at A and B.
2. Set $\{\gamma_{\mathsf{A}}(c_{\mathsf{R}})\} = \{0\}$ and $\{\gamma_{\mathsf{B}}(c_{\mathsf{R}})\} = \{0\}$.
3. Set $l = 0$.

repeat

Update $l = l + 1$; In the lth iteration ($l \geq 1$):

1. Construct $\mathcal{S}(c_{\mathsf{B}}^{mN_{\mathsf{R}}}, \cdots, c_{\mathsf{B}}^{nN_{\mathsf{R}}+1})$ according to (6) at B;
2. Calculate $\lambda_{\mathsf{A}}(c_{\mathsf{R}}^i)$ and $\lambda_{\mathsf{B}}(c_{\mathsf{R}}^i)$ according to (7)–(12);
3. Calculate $\lambda_{\mathsf{A}}(c_{\mathsf{B}}^i) = \lambda_{\mathsf{A}}(c_{\mathsf{R}}^i) \boxplus \tilde{c}_{\mathsf{A}}^i$, $\forall\, i \in \{1, \cdots, mN_{\mathsf{R}}\}$ and $\lambda_{\mathsf{B}}(c_{\mathsf{A}}^i) = \lambda_{\mathsf{B}}(c_{\mathsf{R}}^i) \boxplus c_{\mathsf{B}}^i$, $\forall\, i \in \{1, \cdots, nN_{\mathsf{R}}\}$ according to (13);
4. Deinterleave $\{\lambda_{\mathsf{A}}(c_{\mathsf{B}})\}$ and $\{\lambda_{\mathsf{B}}(c_{\mathsf{A}})\}$; feed them to channel decoders for decoding;
5. Interleave the channel decoder outputs;
6. Calculate $\gamma_{\mathsf{A}}(c_{\mathsf{R}}^i) = \gamma_{\mathsf{A}}(c_{\mathsf{B}}^i) \boxplus \tilde{c}_{\mathsf{A}}^i$, $\forall\, i \in \{1, \cdots, mN_{\mathsf{R}}\}$ and $\gamma_{\mathsf{B}}(c_{\mathsf{R}}^i) = \gamma_{\mathsf{B}}(c_{\mathsf{B}}^i) \boxplus c_{\mathsf{B}}^i$, $\forall\, i \in \{1, \cdots, nN_{\mathsf{R}}\}$ according to (13); feed them to demappers;

until No further changes in decoded bits.

Generate the output information bits $\{\hat{b}_{\mathsf{A}}\}$ and $\{\hat{b}_{\mathsf{B}}\}$.

IV. Optimized Signal Labeling

This section discusses the criterion and method of finding the optimized signal labeling at the relay based on the decoding performance at receivers in high SNR regime. For simplicity, we assume $N_{\mathsf{A}} = N_{\mathsf{B}} \triangleq N$.

A. Optimization Criterion

For BICM-ID systems, the asymptotic decoding performance at the receivers in high SNR regime can be characterized by the *pairwise error probability* (PEP). Let $\mathbf{c}$ and $\hat{\mathbf{c}}$ denote two transmit codewords with Hamming distance d. $P(\mathbf{c} \to \hat{\mathbf{c}})$ denotes the PEP that the decoder chooses $\hat{\mathbf{c}}$ instead of the transmitted bits $\mathbf{c}$. Assuming perfect interleaving and averaged over all symbols and bit positions, $P(\mathbf{c} \to \hat{\mathbf{c}})$ at receiver A can be bounded using the union bound by [11]

$$\begin{aligned}&P(\mathbf{c} \to \hat{\mathbf{c}}) \leq E^d \\ &= \left\{ \frac{1}{mN_{\mathsf{R}} \cdot 2^{mN_{\mathsf{R}}}} \sum_{i=1}^{mN_{\mathsf{R}}} \sum_{b=0}^{1} \sum_{\mathbf{s}_{\mathsf{R}} \in \mathcal{X}_i^{(b)}} \sum_{\hat{\mathbf{s}}_{\mathsf{R}} \in \mathcal{X}_i^{(\bar{b})}} P(\mathbf{s}_{\mathsf{R}} \to \hat{\mathbf{s}}_{\mathsf{R}}) \right\}^d \qquad (14)\end{aligned}$$

where $\bar{b} = 1 - b$ for $b \in \{0, 1\}$, and $P(\mathbf{s}_{\mathsf{R}} \to \hat{\mathbf{s}}_{\mathsf{R}})$ denotes the PEP between symbol $\mathbf{s}_{\mathsf{R}}$ and $\hat{\mathbf{s}}_{\mathsf{R}}$. $\mathcal{X}_i^{(b)}$ and $\mathcal{X}_i^{(\bar{b})}$ in (14) denote the two sets of symbol vectors that only differ on their ith bit labeling. In high SNR regime and after a sufficient number of iterations, we assume the channel decoder feeds back perfect information about the other unknown bits of each symbol vector. That is why we only consider the PEP between symbol vectors s_R and $\hat{\mathsf{s}}_\mathsf{R}$ that only differ by one bit in their labeling. In i.i.d. Rayleigh fading channels, it was shown in [12] that E can be upper bounded by the Chernoff bound as $E \leq C \cdot D$, where C is a constant that is not related to the labeling, and

$$D = \frac{1}{mN_\mathsf{R} \cdot 2^{mN_\mathsf{R}}} \sum_{i=1}^{mN_\mathsf{R}} \sum_{b=0}^{1} \sum_{\mathsf{s}_\mathsf{R} \in \mathcal{X}_i^{(b)}} \sum_{\hat{\mathsf{s}}_\mathsf{R} \in \mathcal{X}_i^{(\bar{b})}} \frac{1}{\|\mathsf{s}_\mathsf{R} - \hat{\mathsf{s}}_\mathsf{R}\|^{2N}}. \tag{15}$$

In order to minimize the error bound of $P(\mathbf{c} \to \hat{\mathbf{c}})$ for receiver A, we must find the labeling scheme with the minimum D.

For given n and $[c_\mathsf{B}^{mN_\mathsf{R}}, \cdots, c_\mathsf{B}^{nN_\mathsf{R}+1}]$, demapping at Station B is performed on the subsets of the constellation $\mathcal{X}$. We define

$$\begin{aligned} &D_\mathcal{S}(c_\mathsf{B}^{mN_\mathsf{R}}, \cdots, c_\mathsf{B}^{nN_\mathsf{R}+1}) \\ &= \frac{1}{nN_\mathsf{R} \cdot 2^{nN_\mathsf{R}}} \sum_{i=1}^{nN_\mathsf{R}} \sum_{b=0}^{1} \sum_{\mathsf{s}_\mathsf{R} \in \mathcal{S}_i^{(b)}} \sum_{\hat{\mathsf{s}}_\mathsf{R} \in \mathcal{S}_i^{(\bar{b})}} \frac{1}{\|\mathsf{s}_\mathsf{R} - \hat{\mathsf{s}}_\mathsf{R}\|^{2N}} \end{aligned} \tag{16}$$

where $\mathcal{S}_i^{(b)}$ and $\mathcal{S}_i^{(\bar{b})}$ in (16) denote the two sets of symbol vectors that only differ on their ith bit labeling within the subset $\mathcal{S}(c_\mathsf{B}^{mN_\mathsf{R}}, \cdots, c_\mathsf{B}^{nN_\mathsf{R}+1})$. Since the bits $[c_\mathsf{B}^{mN_\mathsf{R}}, \cdots, c_\mathsf{B}^{nN_\mathsf{R}+1}]$ are random in the system, all of their possible realizations must be considered. For given n, the worst-case PEP bound, which determines the decoding performance in high SNR regime at Station B, can be minimized by minimizing the following

$$D_\mathcal{S}^n = \max D_\mathcal{S}(c_\mathsf{B}^{mN_\mathsf{R}}, \cdots, c_\mathsf{B}^{nN_\mathsf{R}+1}), \tag{17}$$

for all possible $[c_\mathsf{B}^{mN_\mathsf{R}}, \cdots, c_\mathsf{B}^{nN_\mathsf{R}+1}]$ realizations.

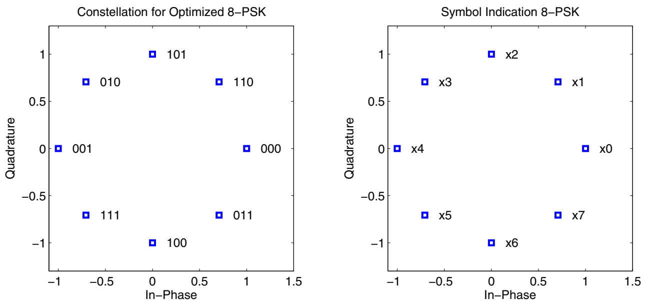
B. Optimized Labeling

In a two-way relaying system, the decoding performance at stations A and B both have to be considered. This is a *multi-objective optimization* problem. For given m and n, we propose to find labeling schemes that minimize the cost function $D + w \cdot D_\mathcal{S}^n$, where D and $D_\mathcal{S}^n$ are defined in (15) and (17), respectively. $w > 0$ is the weighting factor. When the constellation size is large, searching exhaustively for the optimum labeling that minimizes the cost function becomes impossible. So we applied the binary switching algorithm [13] to search for the optimized labeling. Here w is set to be 1.

Considering the case that the relay employs 8PSK constellation on each antenna, we select the labeling schemes that work well for both $n = 2$ and 1. When $N_\mathsf{R} = 1$, we find the optimized labeling scheme as shown in Fig. 4(a). When $N_\mathsf{R} = 2$, the 8PSK symbol on each antenna is indicated as in Fig. 4(b), and the optimized labeling assignment is shown in Table I, which shows how $\mathbf{c}_\mathsf{R}$ in decimal format is mapped to the symbol vector $[x_i, x_j]^T$ on the two antennas. For example, $\mathbf{c}_\mathsf{R} = 000000$ is mapped to the symbol vector $\mathsf{s}_\mathsf{R} = [x_3, x_4]^T$.

V. Simulation Results

In this section, we show the performance of the proposed asymmetric data rate transmission scheme. In particular, we compare the performance of the optimized labeling schemes with conventional ones (Gray and SP labeling on each antenna). At the relay transmitter, we use a convolutional encoder with coding rate $r = 1/2$ and generator $(4, 7)_8$ in octal representation. The interleaver length is 12000 bits. The data from the relay is transmitted using the OFDM technique with 1024 subcarriers. Each subcarrier corresponds to a Rayleigh fading channel and the channel of each subcarrier remains constant for two OFDM symbols.



(a) Optimized labeling, $N_\mathsf{R} = 1$ (b) Symbol indication on each antenna

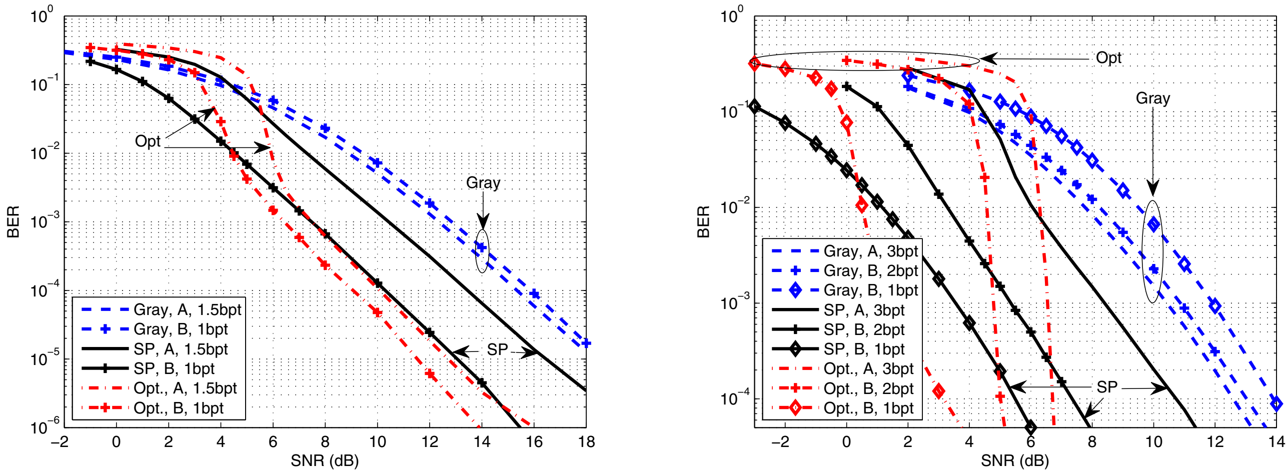
Fig. 4. 8PSK labeling scheme and symbol indication

TABLE I
Optimized 8PSK labeling scheme, $N_\mathsf{R} = 2$

$0 \mapsto (x_3, x_4)$	$1 \mapsto (x_6, x_0)$	$2 \mapsto (x_7, x_7)$	$3 \mapsto (x_1, x_3)$
$4 \mapsto (x_7, x_0)$	$5 \mapsto (x_2, x_4)$	$6 \mapsto (x_4, x_3)$	$7 \mapsto (x_6, x_7)$
$8 \mapsto (x_1, x_1)$	$9 \mapsto (x_4, x_4)$	$10 \mapsto (x_5, x_3)$	$11 \mapsto (x_1, x_7)$
$12 \mapsto (x_5, x_5)$	$13 \mapsto (x_0, x_0)$	$14 \mapsto (x_1, x_6)$	$15 \mapsto (x_4, x_2)$
$16 \mapsto (x_7, x_1)$	$17 \mapsto (x_2, x_5)$	$18 \mapsto (x_3, x_3)$	$19 \mapsto (x_5, x_0)$
$20 \mapsto (x_3, x_5)$	$21 \mapsto (x_6, x_1)$	$22 \mapsto (x_7, x_6)$	$23 \mapsto (x_2, x_3)$
$24 \mapsto (x_5, x_6)$	$25 \mapsto (x_0, x_1)$	$26 \mapsto (x_1, x_0)$	$27 \mapsto (x_5, x_4)$
$28 \mapsto (x_1, x_2)$	$29 \mapsto (x_4, x_5)$	$30 \mapsto (x_5, x_2)$	$31 \mapsto (x_0, x_7)$
$32 \mapsto (x_5, x_1)$	$33 \mapsto (x_0, x_4)$	$34 \mapsto (x_1, x_4)$	$35 \mapsto (x_4, x_7)$
$36 \mapsto (x_0, x_5)$	$37 \mapsto (x_4, x_0)$	$38 \mapsto (x_3, x_7)$	$39 \mapsto (x_0, x_2)$
$40 \mapsto (x_7, x_5)$	$41 \mapsto (x_2, x_0)$	$42 \mapsto (x_3, x_0)$	$43 \mapsto (x_7, x_3)$
$44 \mapsto (x_2, x_1)$	$45 \mapsto (x_6, x_4)$	$46 \mapsto (x_6, x_3)$	$47 \mapsto (x_3, x_6)$
$48 \mapsto (x_1, x_5)$	$49 \mapsto (x_4, x_1)$	$50 \mapsto (x_5, x_7)$	$51 \mapsto (x_0, x_3)$
$52 \mapsto (x_3, x_1)$	$53 \mapsto (x_0, x_6)$	$54 \mapsto (x_7, x_2)$	$55 \mapsto (x_4, x_6)$
$56 \mapsto (x_3, x_2)$	$57 \mapsto (x_6, x_5)$	$58 \mapsto (x_7, x_4)$	$59 \mapsto (x_2, x_7)$
$60 \mapsto (x_6, x_6)$	$61 \mapsto (x_2, x_2)$	$62 \mapsto (x_2, x_6)$	$63 \mapsto (x_6, x_2)$

The simulated BER performance is shown in Fig. 5. The transmission on each relay antenna employs 8PSK symbols ($m = 3$). Since the transmissions to stations A and B does not interfere with each other, we show their performance in the same figures. The "SNR" on the x-axis represents $P_\mathsf{R}/\sigma_\mathsf{A}^2$ for transmission to Station A and $P_\mathsf{R}/\sigma_\mathsf{B}^2$ for transmission to Station B. The comparison of the required SNR at BER $=$ 10^{-4} is also shown in Table II. Note the optimized labeling is found by minimizing its error bound at the high SNR regime.

Fig. 5(a) considers the case $N_\mathsf{A} = N_\mathsf{R} = N_\mathsf{B} = 1$, where the information data rate to Station A is $r \cdot mN_\mathsf{R} =$ 1.5bits/transmission (bpt), and the data rate to Station B is 1bpt ($n = 2$) in Fig. 5(a). With the SP and the optimized labeling, Fig. 5(a) shows that Station B can decode at lower SNR compared to Station A by exploiting the *a priori* bit information subject to certain BER constraints. When BER $= 10^{-4}$, the required SNR at Station B is 3.2dB lower than that of Station A for SP labeling. The optimized labeling outperforms the SP labeling by achieving lower BER at the high SNR regime (see



(a) BER performance, 15 iterations, $N_{\mathsf{A}} = N_{\mathsf{R}} = N_{\mathsf{B}} = 1$

(b) BER performance, 15 iterations, $N_{\mathsf{A}} = N_{\mathsf{R}} = N_{\mathsf{B}} = 2$

Fig. 5. BER performance at stations A and B, $N_{\mathsf{R}} = 1$ or 2. The "bpt" stands for bit/transmission.

TABLE II
REQUIRED SNR FOR DECODING AT BER $= 10^{-4}$

Labeling, $N_{\mathsf{R}} = 1$	A, 1.5bpt	B, 1bpt	B, 0.5bpt
Gray	15.3dB	15.9dB	17dB
SP	13.5dB	10.3dB	8.7dB
Opt.	10dB	9dB	8.7dB
Labeling, $N_{\mathsf{R}} = 2$	A, 3bpt	B, 2bpt	B, 1bpt
Gray	12.6dB	13dB	14dB
SP	10.7dB	7.3dB	5.5dB
Opt.	6.7dB	5dB	3.1dB

Fig. 5(a)). However, it may lead to worse BER in low SNR. With Gray labeling, the decoding performance at Station B does not improve compared to Station A . This is because the average Euclidean distance between $\mathcal{X}_i^{(0)}$ and $\mathcal{X}_i^{(1)}$ is larger when $i = 2$ and 3 than when $i = 1$ for Gray labeling.

Fig. 5(b) shows the BER performance at stations A and B when $N_{\mathsf{A}} = N_{\mathsf{R}} = N_{\mathsf{B}} = 2$. The information data rate to Station A is $r \cdot mN_{\mathsf{R}} = 3$bpt, and the data rate to Station B is 2bpt ($n = 2$) or 1bpt ($n = 1$). Similar to Fig. 5(a), Gray labeling does not provide decoding benefits at Station B compared to Station A. Compared to Fig. 5(a), the advantage of the optimized labeling to the SP labeling is more obvious. Subject to BER $= 10^{-4}$, the optimized labeling achieves coding gains of 4dB to the SP labeling for transmission to Station A. The BER bounds of 2bpt and 1bpt transmission for the optimized labeling converge at sufficiently high SNR in Fig. 5(b), which are lower than that of SP and Gray labeling.

VI. Conclusions

In this paper, we proposed a novel asymmetric data rate transmission scheme for the BRC phase of two-way DF relaying systems when network coding is applied. The idea is to exploit the *a priori* bit information in the transmit symbols at the weaker link so that it can decode at lower SNR compared to the stronger link. We also showed that the optimized labeling can significantly outperform the conventional ones in such scenario. Due to space constraints, we only showed the optimized labeling based on the error bound when the receive SNR is asymptotically high. Other criterions and performance results will be presented in our following papers.

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