Abstract—We consider two-way communication between multiple MIMO transceiver pairs that is assisted by non-regenerative half-duplex two-way relays. A generalized system model is formulated in which an arbitrary number of multi-antenna relays can make use of symbol extensions in frequency, e.g. OFDM subcarriers. In contrast to conventional approaches, the relays can forward arbitrary linear combinations of their receive symbols in the spatial as well as frequency domain. We apply a closed form zero-forcing approach to orthogonalize the different transceiver pairs and compare simple heuristic power allocation with numerically optimized relay gain allocation schemes. Based on simulation results, we study the advantages of cooperation between subcarriers to improve the sum rate of two-way relaying. Furthermore, we show how such a two-way relaying scheme can beneficially be implemented in a cellular network.

I. INTRODUCTION

Future generations of cellular networks such as LTE-Advanced include the possibility to enhance the performance by coordinated multipoint (CoMP) transmission and/or by relays that assist the communication between base stations and mobiles [1]. While CoMP is a promising approach to overcome the impairments of interference that limits the exploitation of the degrees of freedom in cellular networks, relaying is in the (upcoming) standards primarily intended for range extension. However, many research results indicate that relaying can also be used as an interesting alternative to CoMP. E.g. amplify-and-forward (AF) relays can be able to change the effective channel between the terminals, such that different users are orthogonalized or achievable data rates are maximized (see e.g. [2], [3]). Since the communication in cellular networks is usually bidirectional, i.e. they consist of a down- as well as uplink, two-way relaying has been identified as an efficient concept to combine both directions of communication without a loss in spectral efficiency, even with half-duplex relays [4]. Also in this case, relays can perform distributed optimization of the network performance or can help to achieve the degrees of freedom [5].

Most research in this area has focused on narrow-band systems in which the channels can be described by single matrices, exceptions are e.g. filter-and-forward relays [6], [7]. Since current (and probably also future) systems are orthogonal frequency division multiplexing (OFDM) based, the assumption of frequency flat narrow-band channels is certainly valid, as such systems can be described by parallel channels that operate on orthogonal subcarriers. In the literature, most models therefore treat only the single-carrier case, in which the system is optimized for a single subcarrier. An extension to the wide-band case can then be done by treating each subcarrier independently and combining them with a power allocation across the subcarriers. While this treatment is certainly comfortable, recent research has shown that this is suboptimal in certain cases. Reference [8] proves for the broadcast channel (BC), that separate treatment can be strictly suboptimal for certain channel realizations, when linear precoding is used. In [9], the same authors show that in a BC with linear precoding and cooperation among different subcarriers, i.e. when arbitrary linear combinations of all symbols across different channels are allowed, a performance gain to the carrier-non-cooperative case can indeed be achieved.

Because a two-way relay network inherently contains a multiple access (MAC) as well as a BC phase and since AF relays are restricted to perform linear operations on their signals, we can expect a similar behavior as in the BC. To this end, we consider a two-way AF relay network with multiple orthogonal subcarriers and allow the multi-antenna relays to forward arbitrary linear combinations of their receive signals in the spatial as well as frequency domain. Since finding optimal relay gain matrices is generally a very difficult problem, we apply a zero-forcing approach that allows us to orthogonalize different user pairs in closed-form. The zero-forcing approach is similar to [10]. There, however, only a single relay and single antenna terminals are considered, and the study is based on a single subcarrier. In the following, we apply two-way relaying to a more general network and study the gains that can be achieved by subcarrier cooperation.

II. SUBCARRIER-COOPERATIVE RELAYING

The network under consideration consists of $K$ transceiver pairs that wish to communicate in a bidirectional way. The pair of terminals $T_k$ and $T_j$ that exchange information with each other is represented by $(k, j)$ such that the $p$th pair is denoted by $(2p-1, 2p)$. The communication is assisted by $L$ relays while it is assumed that there is no direct link between the

![Fig. 1. Two-way relay network with multiple AF relays.](image-url)
terminals. The terminal and relay nodes are equipped with an arbitrary number of antennas. The number of antennas at $T_k$ is denoted by $M_k$ and $N_l$ is the number of antennas at relay station $R_{S_l}$, $l \in \{1, \ldots, L\}$. A sketch of the system model can be seen in Fig. 1.

In this work, we consider AF relays that operate in a half-duplex mode and allow for two-way relaying. Furthermore, the communication can use orthogonal subcarriers $c \in \{1, \ldots, C\}$ that can e.g. correspond to OFDM subcarriers. On each subcarrier $c$, the channel is assumed to be frequency flat. We describe the channel between $T_k$ and $R_{S_l}$ on subcarrier $c$ by $H_{l,k}^{(c)}$ and assume reciprocity, i.e. the channel from $R_{S_l}$ to $T_k$ is $H_{k,l}^{(c)\text{T}}$. For the two-way relaying protocol, we apply two time slots for each transmission cycle. In the first time slot, all terminals transmit simultaneously to the relays. The receive signal of $R_{S_l}$ on subcarrier $c$ thus can be written as

$$r_l^{(c)} = \sum_{k=1}^{2K} H_{l,k}^{(c)} \cdot x_k^{(c)} + n_l^{(c)}, \quad c = 1, \ldots, C,$$

(1)

with $x_k^{(c)} \in \mathbb{C}^{M_k}$ the transmit signal of $T_k$ and $n_l^{(c)} \in \mathbb{C}^{N_l}$ the noise induced in $R_{S_l}$.

In order to get a more compact notation for all subcarriers, we rewrite the channels into the equivalent block-diagonal form

$$H_{l,k} = \text{blkdiag} \left\{ H_{l,k}^{(1)}, \ldots, H_{l,k}^{(C)} \right\}.$$

The corresponding form of (1) over all subcarriers is thus

$$r_l = \sum_{k=1}^{2K} H_{l,k} \cdot x_k + n_l,$$

(2)

where $x_k \in \mathbb{C}^{CM_k}$ and $n_l \in \mathbb{C}^{CN_l}$ are the transmit and noise signals of all subcarriers stacked into a single vector.

The AF relays multiply their received signal with a gain matrix $G_l$ and broadcast the resulting signal $G_l \cdot r_l$ back to the terminals. In conventional approaches, the relays process their input separately for all subcarriers, i.e. a separate gain matrix $G_l^{(c)} \in \mathbb{C}^{N_l \times N_l}$ is applied to each subcarrier $c$. The resulting gain over all subcarriers is thus a block-diagonal matrix $G_l = \text{blkdiag} \left\{ G_l^{(1)}, \ldots, G_l^{(C)} \right\}$. This block-diagonal structure ensures that the transmission on the different subcarriers remain orthogonal. We will refer to this case as to the conventional, subcarrier non-cooperative case.

Subcarrier cooperation, on the other hand, allows the relays to form arbitrary linear combinations across all subcarriers, i.e., the gain matrices are no longer restricted to be block-diagonal but can have an arbitrary form $G_l \in \mathbb{C}^{N_l \times CN_l}$.

The receive signal over all subcarriers at terminal $T_k$ is in either case (subcarrier cooperative or non-cooperative)

$$y_k = \sum_{l=1}^{L} H_{l,k}^{\text{T}} \cdot G_l \cdot \left( \sum_{n=1}^{2K} H_{l,n} \cdot x_n + n_l \right) + w_k,$$

(3)

where $w_k \in \mathbb{C}^{CM_k}$ is the noise induced in terminal $T_k$.

One component thereof is self-interference, i.e. the signal of the terminal itself that has been retransmitted back to it. This self-interference can be cancelled at the terminal when the effective channel via the relays is known. Hence, the receive signal (3) becomes

$$y_k = \sum_{l=1}^{L} H_{l,k}^{\text{T}} \cdot G_l \cdot \left( \sum_{n=1}^{2K} H_{l,n} \cdot x_n + \sum_{n \notin \{k,l\}}^{2K} H_{l,n} \cdot x_n \right) + \sum_{l=1}^{L} H_{l,k}^{\text{T}} \cdot G_l \cdot n_l + w_k.$$

(4)

In the following, we focus on the design of the relay gain matrices $G_l$ and assume that the terminals do not apply any beamforming or precoding. To this end, the transmit signals $x_k$ of the terminals are assumed to be i.i.d $\mathcal{CN}(\mathbf{0}, P_s \cdot \mathbf{I})$. In this case, the achievable rate of user $k$ can be stated as

$$R_k = \frac{1}{2} \log_2 \det \left\{ \mathbf{I} + \left( K_k^{(i)} + K_k^{(n)} \right)^{-1} \cdot K_k^{(s)} \right\},$$

(5)

with the covariance matrices of the desired signal, interference, and noise given by

$$K_k^{(i)} = P_s \sum_{l=1}^{L} \sum_{i=1}^{L} H_{l,k}^{\text{T}} \cdot G_l \cdot H_{l,i} \cdot G_l^\ast \cdot H_{i,k},$$

$$K_k^{(s)} = P_s \sum_{l=1}^{L} \sum_{n \notin \{n,l\}}^{2K} H_{l,k}^{\text{T}} \cdot G_l \cdot H_{l,n} \cdot G_l^\ast \cdot H_{n,k},$$

$$K_k^{(n)} = \sigma_n^2 \sum_{l=1}^{L} H_{l,k}^{\text{T}} \cdot G_l \cdot H_{l,k} + \sigma_w^2 \mathbf{I}.$$

Therein, $\sigma_n^2$ and $\sigma_w^2$ are the variances of the noise induced in the relays and terminals, respectively.

III. DESIGN OF RELAY GAIN MATRIX

Relay gain matrices that optimize the performance of the network, e.g. sum rate, are generally hard to find. Possible approaches optimize the objective function by iterative algorithms as e.g. in [3]. In this paper, the relay gain matrices are generalized to form arbitrary linear combinations of the relay input signals in the spatial as well as frequency domain. This additionally increases the complexity of the optimization problem, especially when many subcarriers are involved. To this end, we apply a block zero-forcing approach in which the relay gains cancel the interference between the terminal pairs. Note that zero-forcing is not optimal with respect to achievable rate. However, it allows us to reduce the optimization problem to only finding combination weights of null space vectors.

A. Block Zero-Forcing

The zero-forcing conditions can be stated as follows

$$\sum_{l=1}^{L} H_{l,k}^{\text{T}} \cdot G_l \cdot H_{l,n} = \mathbf{0}, \quad \forall k, n, \quad n \notin \{k,j\}$$

(6)

$$\operatorname{rank} \left( \sum_{l=1}^{L} H_{l,k}^{\text{T}} \cdot G_l \cdot H_{l,j} \right) = d,$$

(7)

where $d \leq \min \{M_k, M_j\}$ is the number of desired spatial streams user pair $(k, j)$ wishes to exchange. Note that self-interference can be cancelled at the terminals and is therefore not part of these conditions.
In order to solve the zero-forcing conditions in closed form, we rewrite the left hand side of (6) into the product $H_k^T G H_n$, in which the matrices are with respect to all relays, i.e. $H_k^T = [H_{1,k}^T, \ldots, H_{L,k}^T]$ and $G = \text{blkdiag} \{G_1, \ldots, G_L\}$. The equation system $H_k^T G H_n = 0$ is linear in the entries of $G$. It can be solved by rewriting it into the equivalent form

$$A_{k,n} \cdot b = O, \quad \forall k, n, \quad n \not= \{k, j\},$$

where $b = \text{vec}(G)$ is the vector that contains all relay gain coefficients. The matrix $A_{k,n}$ is constructed by copying and permuting combinations of entries of $H_k$ and $H_n$ as follows

$$A_{k,n} = \left[ (I_{M_n \times 1} \otimes I_{M_k}) \cdot H_k^T \cdot E \right] \odot \left[ (H_n^T \cdot F) \otimes I_{M_k \times 1} \right],$$

where $\otimes$, $\odot$, and $I_{n \times m}$ denote the element wise product, Kronecker product, and the all ones matrix of size $n \times m$ and the matrices $E$ and $F$ are given by

$$E = \text{blkdiag} \{ I_{1 \times N_1} \otimes I_{N_1}, \ldots, I_{1 \times N_L} \otimes I_{N_L} \},$$

$$F = \text{blkdiag} \{ I_{1 \times N_1}, \ldots, I_{1 \times N_K}, \ldots, I_{1 \times N_L} \}.$$

Once the matrices $A_{k,n}$ are constructed, they can be stacked on top of each other into the matrix $A = [A_1^T, \ldots, A_{2K}^T, \ldots, A_{2K}^T, \ldots, A_{2K}^T]^T$ and the relay gains can be chosen to lie in the null space of $A$. To this end, the singular value decomposition $A = U \cdot \Sigma \cdot [V_1 \cdot V_0]^H$ can be applied, where the columns of $V_0$ form a basis of null $\{A\}$ [11]. The relay gain matrices that fulfill the zero-forcing conditions can then be stated as a linear combination

$$G_i = \alpha_1 \cdot V_1^{(l)} + \ldots + \alpha_D \cdot V_D^{(l)},$$

where $\alpha_i$ and $V_i^{(l)}$ are the combination weights and the null space vectors turned back into matrices according to the dimensions of $G_i$.

The dimension $D$ of the null space depends on the network topology and the number of relay antennas. In a network with a single relay, the number of relay antennas required to fulfill the zero-forcing conditions (6) and (7) is

$$N = 2 \sum_{k=1}^{K} \min \{M_{2p-1}, M_{2p} \} - \min \{M_1, \ldots, M_{2K} \},$$

irrespective of the number of subcarriers. If the network contains multiple relays, the total number of relay gain coefficients needs to be at least the same as in the single relay case.

B. Sum Rate Maximization

Solving the block zero-forcing conditions, the relay gain matrices are found to lie in the null space spanned by the basis vectors from (9). If the null space consists of $D \geq 2$ dimensions, relay gain matrices can be formed as arbitrary linear combinations of all null space vectors. Multiple null space vectors are able to improve the diversity gain [10]. However, the linear combination of the null space vectors needs to be appropriately designed to benefit from this gain. A trivial choice of combination weights might destroy the diversity. It is therefore essential to choose the different combination weights $\alpha_i$ in a beneficial way.

A plot of the sum rate for different combination weights of a sample network can be seen in Fig. 2. Therein, a typical channel realization with $C = 2$ independent subcarriers for a network with $K = 2$ user pairs, $L = 2$ relays, and $M_k = 2$ and $N_l = 4$ antennas at the different nodes is considered. The sample network has $D = 8$ null space vectors, but only the real parts of the first two are varied; the others are set to zero. It can be seen that there are multiple local optima. As a resulting optimization problem is not convex, it is difficult to find “good” linear combinations. Nevertheless, the complexity is drastically reduced as compared to a direct optimization of the relay gains without zero-forcing, as only combination weights have to be found. In order to find combination weights that achieve the expected gains of subcarrier cooperation in two-way relaying, we apply two methods that aim to maximize the sum rate of the network.

1) Null Space Selection: The first approach is to select the best null space vector and to scale it such that a power constraint at each relay is met. To this end, the null space basis is chosen according to

$$\arg\max_{i=1,\ldots,D} \sum_{k=1}^{2K} R_k(i),$$

where $R_k(i)$ is the achievable rate of user $k$ when the relays choose the $i$th null space vector, i.e. $G_i = \beta \cdot V_i^{(l)}$. The scaling factor $\beta$ is chosen such that all relays fulfill the per-node sum power constraint

$$\hat{P}_i = \text{Tr} \left\{ G_i \cdot \left( \sum_{k=1}^{2K} H_{i,k} H_{i,k}^H + \sigma_n^2 I \right) \cdot G_i^H \right\} \leq P_t.$$  \hspace{1cm} (10)

Note that all relays have to apply the same scaling factor $\beta$ otherwise the zero-forcing conditions would be violated. This is achieved by setting $\beta = P_t / \max_i \hat{P}_i$. Hence, in general, not all relays will transmit with full transmit power.

2) Numerical Optimization: The second, more complex, approach is to numerically find combination weights $\alpha_i$ that attempt to maximize the sum rate under the per-relay power constraint (10). The resulting optimization problem can be formulated as

$$\max_{\alpha_1, \ldots, \alpha_D} \sum_{k=1}^{2K} R_k \quad \text{s.t.} \quad \hat{P}_i \leq P_t, \quad \forall i.$$ \hspace{1cm} (11)

As this optimization problem is not convex, it is generally difficult to find the global optimum. There are, however,
standard optimization tools that at least converge to a local optimum. To this end, we can apply such optimization tools as e.g. gradient based optimization algorithms [12] that are designed for convex problems, but also converge to local optima in non-convex problems. We therefore apply an interior-point method with random initialization in Matlab in order to numerically find a local optimum of (11).

IV. SIMULATION RESULTS

The performance of the weight allocation schemes and the gain due to subcarrier cooperation is assessed by means of computer simulations. A comparison of the weight allocation schemes is shown in Fig. 3. There, the numerically optimized combination and the null space vector selection are compared with choosing a random null space vector and using all vectors equally weighted. For the simulation, a network with $K = L = 2$ terminal pairs and relays has been used. All terminals are equipped with $M_k = 2$ antennas, the relays with $N_f = 4$ antennas, and $C = 4$ subcarriers are considered. The elements of the channel matrices on all subcarriers are drawn i.i.d. according to $CN(0,1)$. The transmit powers are set to $P_s = P_r = C$ for all terminals and relays and the noise variances are chosen as $\sigma^2_n = \sigma^2_w = 0.01$. The numerically optimized weight allocation clearly outperforms all other schemes. Selecting the best null space vector leads to lower sum rates, but achieves the same diversity gain as indicated by the slope of the empirical cumulative distribution function (CDF) for small values. Using all null space vectors in an equally weighted way leads to further decrease in performance. Also the slope of the CDF is somewhat less steep, which indicates that not all diversity can be exploited. Using a random null space vector performs poorly and achieves a much smaller diversity gain. In this case, it can also be observed that the CDF follows a stair like behavior which is due to some null space vectors that are much worse than others.

The sum rate of the two-way relaying schemes for different numbers of subcarriers $C = 1, 2, 4, 6$ is shown in Fig. 4. The sum rates are normalized by $C$. For the simulation, the same network as before is considered. The average signal-to-noise ratio (SNR), however, is chosen to be $SNR = 30$ dB, irrespective of the number $C$ of subcarriers, i.e. $P_s = P_r = 1$ and $\sigma^2_n = \sigma^2_w = 0.001$ for any choice of $C$. The CDFs of the sum rates for the numerically optimized and null space selection method are plotted. It can be seen that the performance as well as the diversity increases with the number of subcarriers. The crossings of the null space selection CDFs in the high rate regime show a disadvantage of this scheme. When a null space vector is selected, it is scaled such that the power constraint is met at all relays. As the scaling has to be the same in all relays, only one relay exploits the power constraint with equality, the others transmit with lower power. When more subcarriers are used, the probability increases that some channels are strong which results in a lower power at the weak relays and hence in a lower sum rate.

The performance gain due to subcarrier cooperation over subcarrier non-cooperative schemes is studied in Fig. 5. There, $C = 4$ subcarriers are considered and the proposed subcarrier-cooperative scheme is compared to two conventional subcarrier non-cooperative schemes. In both conventional schemes, the relays perform block zero-forcing separately on each of the independent subcarriers. In the first one, the relay power $P_s = C$ is divided equally to all subcarriers, i.e. the transmit power on each subcarrier is $P^{(c)}_r = 1$. In the second subcarrier non-cooperative scheme, the power allocation on the different subcarriers is numerically optimized according to

$$\max_{P^{(1)},...,P^{(C)}_r} \sum_{c=1}^{C} \sum_{k=1}^{2K} R^{(c)}_k \left( P^{(c)}_r \right), \quad \text{s.t.} \quad \sum_{c=1}^{C} P^{(c)}_r = P_r,$$

where $R^{(c)}_k \left( P^{(c)}_r \right)$ is the achievable rate of terminal $T_k$ in subcarrier $c$ when a power $P^{(c)}_r$ is allocated on this subcarrier. The CDFs of the sum rates normalized to $C$ show a significant gain in the subcarrier-cooperative case. It can also be observed...
that the optimized power allocation does not lead to much improvement compared to the equal power allocation. These results show that a notable portion of the achievable rate is lost when the relays operate independently on the different subcarriers and that subcarrier cooperation leads indeed to a significant gain.

V. APPLICATION TO CELLULAR NETWORKS

An interesting application of two-way relaying with subcarrier cooperation is to use them in cellular networks for interference mitigation, e.g. between neighboring cells. Relays that are spread in the cells can thereby cancel interference between different users. In this way, the relaying scheme can act as an alternative or complement to CoMP. However, a potential drawback of the proposed scheme is the large number of relay antennas required for zero-forcing.

This number, however, can be reduced when the relaying scheme is combined with a simple form of base station (BS) cooperation as in [13]. The neighboring BSs can share their user data, which allows the BSs not only to cancel self-interference, but also interference caused by adjacent BSs. If the BS signals contain training or pilot sequences, these can be used to estimate the effective channels via the relays to allow the cancellation of the BS interference without disseminating CSI among the BSs. In this way, some zero-forcing conditions can be dropped and the relaying scheme works with fewer antennas. This number can be further reduced if the interference the different mobiles (MSs) cause to each other is also ignored by the zero-forcing. This can be justified by the assumption that MS signals are weak compared to BS signals. Ignoring them in the zero-forcing would therefore only increase the noise floor by a small amount. The required number of relay antennas for interference free communication between three BSs and MSs, each equipped with two antennas (thus two streams for each user in each direction), is shown in the following table:

<table>
<thead>
<tr>
<th>Number of relays $L$</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_l$ full antennas</td>
<td>10</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$N_l$ reduced antennas</td>
<td>7</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

The performance of such a network with three BSs serving three MSs in hexagonal cells of a diameter of 1000 m and the same channel model as in [13] is shown in Fig. 6. Therein, two-way relaying, once with the full number of antennas allowing to cancel all interference and once with a reduced number of antennas as explained above, are compared to a conventional network without relays where the three users are served on three orthogonal frequency bands (reuse 3). It can be seen that a significant gain in sum rate can be achieved by the relaying approach, especially when subcarrier cooperation is applied.

VI. CONCLUSIONS

Even though it seems counterintuitive at first glance to combine signals on different subcarriers that are orthogonal, we have shown that two-way AF relaying can be improved by subcarrier cooperation. Thereby, a part of the performance gain can be attributed to channel pairing, i.e. an advantageous mapping of the different subcarriers for the two hops, that is inherently included in the subcarrier cooperation. In this regard, subcarrier-cooperative relaying differs to the setup in [9] for the BC. Furthermore, a concept in which the proposed two-way relaying scheme can beneficially been included in cellular networks is presented. Such an approach is especially interesting when e.g. the backhaul of the BSs is not sufficient to allow for sophisticated BS cooperation.

In the considered schemes, the relays require global channel state information (CSI) to compute the zero-forcing solution and to select the combination of the null space vectors. Future work therefore includes the design of relay gain matrices in a more distributed way. Also schemes other than the suboptimal zero-forcing approach might benefit from subcarrier cooperation and should be considered in future research.

REFERENCES