Linear Scalable Dispersion Codes for Frequency Selective Channels

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***Abstract*—We investigate a combination of linear scalable space-time block codes with OFDM and present a systematic code design. This approach leads to space-frequency coded MIMO-OFDM with a variable tradeoff between improving link reliability (by using diversity techniques) and increasing data rate (using spatial multiplexing). The proposed coding scheme achieves spatial, temporal and frequency diversity and is able to utilize the potential of rich arrays (antenna arrays with a large number of antennas) with reasonable (and scalable) decoding complexity.**

I. Introduction

Future wireless communication systems will have to support high data rates at a high link reliability. In a rich scattering environment this can be done using Multiple Input Multiple Output (MIMO) signaling techniques. There are two basic coding methods, namely space-time coding to improve link reliability and spatial multiplexing to increase spectral efficiency.

Space-time codes [1] combat the fading effects of wireless multipath channels by utilizing the diversity of the communication channel given for example by the use of an antenna array at the transmitter (Tx) and/or at the receiver (Rx). Such methods aim to achieve transmit diversity without channel state information (CSI) at the Tx.

Due to the use of multiple antennas at the Tx and the Rx there is an increase in channel capacity. In an uncorrelated Rayleigh fading environment the capacity grows linearly as the number of Tx and Rx antennas grow simultaneously [2]. This increase in capacity is possible without an increase of the bandwidth. Spatial multiplexing exploits this offered capacity by breaking up the data stream into parallel substreams which are then transmitted simultaneously on individual antennas [3]. It exists a tradeoff between spatial multiplexing and transmit diversity [4], [5].

Because future wireless communication systems (e.g. next generation wireless LANs) will have to support high data rates the required system bandwidth will increase. Wideband channels provide an extra degree of diversity (frequency diversity), which can be exploited through the use of Orthogonal Frequency Division Multiplexing (OFDM) with an appropriate coding across the OFDM tones.

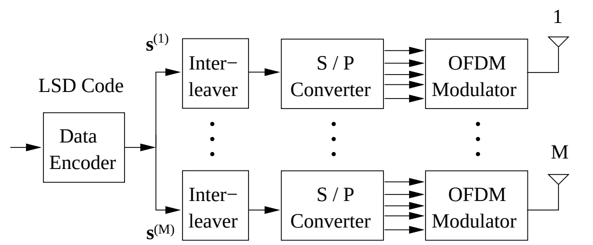

Fig. 1. System block diagram of a space-frequency coded OFDM transmitter

In this paper we present a space-frequency signaling scheme for frequency selective channels. The scheme is based on the class of high rate space-time codes presented in [6], [5]. The resulting space-frequency coded MIMO-OFDM exploits frequency diversity, is highly adaptive and scalable and able to trade spatial multiplexing gain for diversity gain.

II. Channel Model

We will consider a multi-antenna system with M Tx antennas and N Rx antennas. The following channel model for a time-varying wideband MIMO channel is used:

$$\mathbf{H}(\tau) = \sum_{i=1}^{L} \mathbf{H}_i \cdot \delta(\tau - \tau_i) \qquad (1)$$

The MIMO channel is modelled as the sum of several delayed random fading components (tapped delay line model). $\mathbf{H}(\tau)$ is a complex $(N \times M)$ matrix. In this paper we assume that $\mathbf{H}_i$ with $i \in [1, 2, .., L]$ are $(N \times M)$ Rayleigh fading matrices; i.e. the elements of these matrices are uncorrelated circularly symmetric complex Gaussian random variables with zero mean and variance σ_i^2 (path gains σ_i^2 are given by the power delay profile of the channel). Different taps are assumed to be uncorrelated.

III. Coding Scheme

A coded OFDM communication system with N_S OFDM subcarriers (OFDM tones), M Tx and N Rx antennas is considered. Fig. 1 shows the transmitter of such a system (complex baseband representation). The data encoding is based on the linear scalable dispersion (LSD) codes presented in [6], [5].

A. *Short review of LSD space-time coding*

Originally the high rate LSD space-time coding scheme is designed for flat fading channels [6]; it consists of two concatenated but decoupled linear block codes: the time variant inner code and the time invariant outer code. The outer code is optimized for diversity performance in fast fading channels. The time variant inner code is optimized with respect to the outage channel capacity available for the outer code; due to the time variance the inner code transforms a block fading channel in a fast fading channel to enable high diversity gains of the outer code. In [6] efficient code matrices are presented. No a priori channel knowledge is required at the Tx.

The possible tradeoff between diversity and spatial multiplexing is demonstrated in [5] for Rayleigh and Ricean flat fading; the more spatial subchannels are used (for a given number of Tx and Rx antennas) the higher the rate, but the symbol error probability increases too, because the diversity gain decreases. So, using LSD codes it is possible to trade diversity gain (link reliability) for data rate in a very flexible way.

B. *Space-frequency coded MIMO-OFDM*

In this work we derive a MIMO signaling scheme for frequency selective channels. Because the LSD codes are originally designed as space-time codes for flat fading channels, OFDM is used to convert the frequency selective channel into a set of N_S independent frequency flat subchannels. OFDM is realized by an IFFT; the OFDM modulators shown in Fig. 1 include a cyclic extension of the OFDM symbol in a guard interval. The channel is is considered constant over one OFDM symbol.

The symbols transmitted on the ν-th OFDM subcarrier are given by the column vector $\mathbf{s}_\nu = (\mathbf{s}_\nu[1], \mathbf{s}_\nu[2], ..., \mathbf{s}_\nu[M])^T$; the complex scalar $\mathbf{s}_\nu[i]$ denotes the symbol transmitted by the i-th antenna. In one timestep the antenna i transmits one OFDM symbol that is derived by frequency interleaving, serial-parallel conversion and IFFT of the vector $\mathbf{s}^{(i)} = (\mathbf{s}_{\nu=1}[i], \mathbf{s}_2[i], ..., \mathbf{s}_{N_S}[i])^T$ (Fig. 1).

The $(N \times 1)$ receive vector $\mathbf{r}_\nu$ of subcarrier ν follows to

$$\mathbf{r}_\nu = \sqrt{\text{SNR}} \cdot \mathcal{H}_\nu \cdot \mathbf{s}_\nu + \mathbf{w}_\nu \qquad (2)$$

where the $(N \times M)$ matrix $\mathcal{H}_\nu$ is the discrete Fourier transform of the MIMO channel $\mathbf{H}$ at the frequency ν, SNR is the average signal-to-noise-ratio at each Rx antenna and $\mathbf{w}_\nu$ is complex-valued AWGN at the ν-th subcarrier.

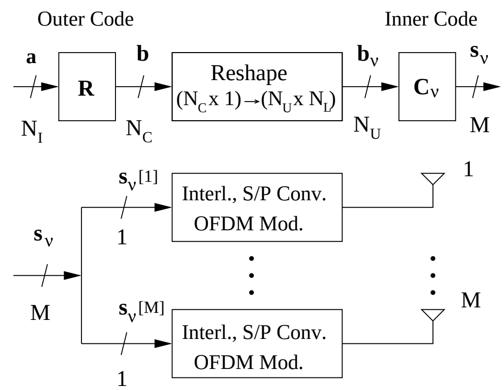

Fig. 2. Symbol discrete model of the LSD encoding scheme

Fig. 2 shows the combination of the LSD codes with OFDM and multiple Tx antennas. As in the case of the space-time codes the LSD coding scheme consists of two concatenated but decoupled linear block codes: the outer code $\mathbf{R}$ and the inner code $\mathbf{C}_\nu$; the inner code is *frequency variant* (a different Matrix $\mathbf{C}_\nu$ for each OFDM subcarrier ν) instead of time variant in the space-time coding case. In this paper we assume that the Tx has no a priori CSI.

The input symbol vector $\mathbf{a}$, consisting of N_I information symbols, is multiplied with the $(N_C \times N_I)$ outer code matrix $\mathbf{R}$ to form the transmit symbol vector $\mathbf{b}$. The dimensions of the code matrix determine the code rate to $r_C = \frac{N_I}{N_C}$. Note that a simple modification of the code rate is possible by adding/deleting columns of the outer code matrix. Because of the interdependence of code rate and required decoder complexity this feature is highly useful in networks with heterogeneous node capabilities [7]. Throughout this paper code rate $r_C = 1$ is used. Thereafter the transmit symbol vector $\mathbf{b}$ of dimension $(N_C \times 1)$ is reshaped into a $(N_U \times N_L)$ matrix. The columns of this matrix are the consecutive $N_L = \frac{N_C}{N_U}$ input vectors $\mathbf{b}_\nu$ of the linear inner code $\mathbf{C}_\nu$; a different $\mathbf{C}_\nu$ is used for each OFDM subcarrier. Coding across the tones is required to exploit the inherent frequency diversity of the frequency selective channel. We first consider the situation that the outer coding is done across all OFDM subcarriers, i.e. $N_L = N_S$.

The M elements of the vector $\mathbf{s}_\nu$ are transmitted by the M OFDM modulators in Fig. 2. The system transmits $N_U \leq \text{rank}(\mathcal{H})$ symbols over one OFDM subcarrier in one timestep. We refer to N_U as the number of *spatial subchannels* to be used for spatial multiplexing. The remaining spatial dimensions can be used by the code achieving an additional diversity gain.

Due to the code concatenation we have the possibility to optimize two decoupled linear block codes.

1) Coding for pure transmit diversity (MISO case): In the MISO case the receiver uses only one Rx antenna, $N = 1$. The channel matrix $\mathcal{H}_\nu$ becomes a row vector $\mathbf{h}_\nu^H$ with M elements. For only one Rx antenna no spatial multiplexing is possible, i.e. $N_U = 1$ and $N_L = N_C$. The matrix $\mathbf{C}_\nu$ and the vector $\mathbf{b}_\nu$ become a column vector $\mathbf{c}_\nu$ and a scalar b_ν, respectively. The vectors $\mathbf{c}_\nu$ ($\nu \in [1, 2, ..., N_L]$) are of dimension $(M \times 1)$ and are used to drive the Tx antenna array (normalized to maintain the total transmit power).

After linear signal processing of the received signal vector of subcarrier ν - given by (2) - follows the estimation of b_ν to (superscript ^{H}: conjugate transpose):

$$\hat{b}_\nu = \sqrt{SNR} \cdot \mathbf{c}_\nu^H \cdot \mathbf{h}_\nu \cdot \mathbf{h}_\nu^H \cdot \mathbf{c}_\nu \cdot b_\nu + w_\nu \tag{3}$$

The scalar equivalent fading coefficients are

$$z_\nu = \mathbf{c}_\nu^H \cdot \mathbf{h}_\nu \cdot \mathbf{h}_\nu^H \cdot \mathbf{c}_\nu \tag{4}$$

Without CSI at the Tx the inner code vector is independent of the current realization of the channel vector. If the elements of $\mathbf{h}_\nu$ are random variables, z_ν is a random variable for any ν, when viewed over all channel realizations. If the channel is not frequency selective ($\mathbf{h}_\nu = \mathbf{h}$), a frequency invariant inner code $\mathbf{c}_\nu = \mathbf{c}$ does not allow to exploit a Tx diversity gain; since in this case follows from (4) that all N_C transmit symbols b_ν of one OFDM symbol are affected by the same fading coefficient z. Therefore no Tx diversity gain can be achieved. But if $\mathbf{c}_\nu$ is frequency variant (e.g. two different subcarrier use two different Tx antennas), different transmit symbols b_ν are affected by different fading variables. A diversity gain due to the Tx antenna array is possible (even if the channel is not frequency selective); but it requires an appropriate outer code that has to be optimized for diversity gain in fast fading.

An equivalent situation was found for the optimization of the space-time LSD codes for flat fading in [6]. As shown there we can optimize the inner code with respect to the outage capacity of the channel available for the outer code. An appropriate objective function for the outer code is the fading averaged pairwise error probability (PEP).

a) Design of the inner code: According to [6] for Rayleigh fading channels $\mathbf{h}$ a good set of inner coding vectors is given by the first M columns of the $(N_L \times N_L)$ discrete Fourier matrix $\mathbf{F}$:

$$\mathbf{c}_\nu = \mathbf{F}[\nu, 1 : M]^T \tag{5}$$

with

$$\mathbf{F}[k, l] = \frac{1}{\sqrt{N_L}} \cdot e^{-j \cdot 2\pi (l-1)(k-1)/N_L}. \tag{6}$$

At the frequency ν the M elements of the vector $\mathbf{s}_\nu$ are transmitted over M antennas; due to this inner code $\mathbf{c}_\nu$ every element is a phase-shifted variation of the scalar b_ν.

b) Design of the outer code: A suitable cost function for the outer code is the fading averaged pairwise probability (PEP) of message error. The PEP of two different input symbol vectors $\mathbf{a}^{(1)}$ and $\mathbf{a}^{(2)}$ is the probability that $\mathbf{a}^{(1)}$ is transmitted and the Rx decides erroneously in favor of $\mathbf{a}^{(2)}$:

$$P[\mathbf{a}^{(1)} \rightarrow \mathbf{a}^{(2)}] = P[\hat{\mathbf{a}} = \hat{\mathbf{a}}^{(2)} | \mathbf{a}^{(1)}] \tag{7}$$

The inner code and the MISO channel (i.e. the reshaped equivalent SISO channel z_ν) are modelled as a Rayleigh fast fading channel $\mathbf{x}$ with variance σ_x^2; the matrix $\mathbf{D}_x = \text{diag}(\mathbf{x})$ is a diagonal matrix of dimension $(N_C \times N_C)$ with the elements of vector $\mathbf{x}$ on the main diagonal:

$$\mathbf{r} = \mathbf{D}_x \mathbf{R} \cdot \mathbf{a} + \mathbf{w} = \mathbf{D}_x \cdot \mathbf{b} + \mathbf{w} \tag{8}$$

with $\mathbf{b} = \mathbf{R} \cdot \mathbf{a}$ (Fig. 2).

For a given channel realization $\mathbf{D}_x$ the PEP is given by [8]

$$P[\hat{\mathbf{a}} = \hat{\mathbf{a}}^{(1)} | \mathbf{a}^{(2)}, \mathbf{D}_x] = P[\mathbf{b}_x^{(1)} \rightarrow \mathbf{b}_x^{(2)} | \mathbf{D}_x] = Q\left(\sqrt{\frac{d_{ED}^2}{2\sigma_w^2}}\right) \tag{9}$$

with σ_w^2 is the variance of AWGN $\mathbf{w}$, $d_{ED} = \|\Delta \mathbf{b}_x\|$ the Euclidian distance of $\mathbf{b}_x^{(1)} = \mathbf{D}_x \mathbf{R} \cdot \mathbf{a}^{(1)}$ and $\mathbf{b}_x^{(2)} = \mathbf{D}_x \mathbf{R} \cdot \mathbf{a}^{(2)}$. All pairs (i, j) of input symbol vectors leading to the same $\Delta \mathbf{b}_x$ show the same PEP. Using the relation $Q(x) \leq \frac{1}{2} \cdot e^{-x^2/2}$ we get an upper bound

$$P[\Delta \mathbf{b}, \mathbf{D}_x] \leq \frac{1}{2} \cdot e^{-\frac{d_{ED}^2}{4\sigma_w^2}} \tag{10}$$

One element $\Delta \mathbf{b}_x[i]$ of the vector $\Delta \mathbf{b}_x$ has the variance $\lambda_i = \sigma_x^2 \cdot |\Delta \mathbf{b}[i]|^2$, the corresponding eigenvalue of the matrix $\Lambda_{b_x b_x}$. Averaging over the Gaussian distributed random realizations of $\Delta \mathbf{b}_x$ we get an upper bound for the *fading averaged PEP* for a given $\Delta \mathbf{b}$:

$$P[\Delta \mathbf{b}] \leq \frac{1}{2} \cdot \prod_{i=1}^{N_C} \left[\frac{1}{\frac{\lambda_i}{4 \cdot \sigma_w^2} + 1} \right] \tag{11}$$

Clearly a repetition code of rate $\frac{1}{N_C}$ minimizes the objective function, the fading averaged PEP, it achieves full diversity in fast fading; but the spectral efficiency is low. Therefore spectral efficiency is a reasonable constraint for the optimization: a high diversity gain should be reached at code rate $r_C = 1$. To require $\mathbf{R}$ to be unitary is another sensible constraint. So the Euclidean distance and thus the error performance on an AWGN channel is maintained; this allows a low complexity decoder under AWGN conditions (and in Ricean fading environments with large Ricean K-factor).

In [6] the choice to meet these constraints is a complex and unitary coding matrix based on the impulse response of a cyclic chirp filter. The columns are found by cyclic shifting of the first column $\mathbf{R}[:,1]$, that is given by the Fourier transform of the vector $\mathbf{m}$

$$\mathbf{m}[i] = e^{j\cdot 2\pi c\cdot(i-1)^2/N_C^2}. \tag{12}$$

The optimal value of the parameter c is found as follows: For each of the $V = v^{N_C}$ (v is size of the input alphabet) transmit symbol vectors $\mathbf{b}^{(i)}$ that vector $\mathbf{b}^{(j)}$ ($j \neq i$) is determined that leads to the highest fading averaged PEP. Then we determine that value of c that minimizes the maximum of all V possible PEPs.

2) Coding for spatial multiplexing in combination with transmit diversity (MIMO case): In the MIMO case it is possible to multiplex independent data streams and transmit them over spatial subchannels. If there are more Tx antennas than used spatial subchannels, inner and outer code have to be designed to use spatial multiplexing in combination with Tx diversity. The number of used spatial subchannels should be adaptive in a very flexible way, because this allows a flexible tradeoff between diversity to increase link reliability and spatial multiplexing to increase the data rate. This tradeoff is controlled by the inner code. We use the same outer code as in the MISO case.

Again the inner code is optimized with respect to the outage capacity of the channel available for the outer code. Using spatial multiplexing an efficient choice of $\mathbf{C}_\nu$ is given by [6]

$$\mathbf{C}_\nu = \mathrm{diag}(\mathbf{c}_\nu)\cdot\mathbf{M} = \mathbf{D}_{\mathbf{c}_\nu}\cdot\mathbf{M} \tag{13}$$

The N_L vectors $\mathbf{c}_\nu$ are defined in (5) and (6). The matrix $\mathbf{M}$ is a $(M \times N_U)$ unitary chirp matrix. The elements of $\mathbf{M}$ are determined as described in section III-B.1.b for the outer code. The adaption to a different number N_U of spatial subchannels can simply be done by adding/deleting columns of $\mathbf{M}$.

Outer coding across all OFDM subcarriers may lead to large block lengths N_I and therefore to high decoder complexities. But usually the distance between two neighboring OFDM subcarriers is small and therefore their fading may be highly correlated. Hence, the frequency diversity actually available in the channel is limited. Design criteria for space-frequency codes are derived in [9]. It is more effective to code across subcarriers that are widely separated.

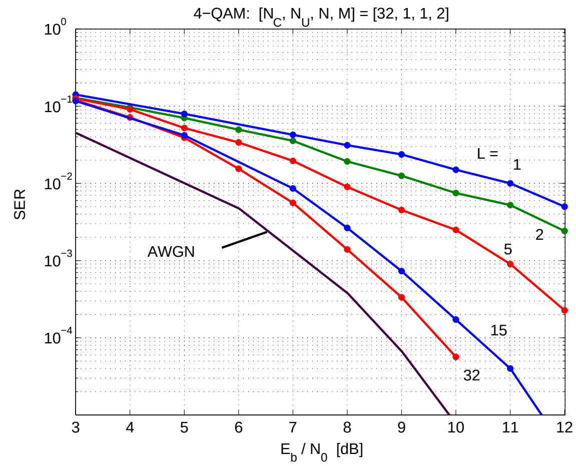

Fig. 3. Combination of Tx diversity and frequency diversity

This is possible by using k LSD encoders (instead of one as in Fig. 1). Then each encoder uses only $\frac{N_S}{k}$ OFDM subcarriers. In this case $\mathbf{s}^{(i)}$ ($i \in [1, 2, ..., M]$) in Fig. 1 consists of k stacked vectors, one from each encoder.

Space-time-frequency coding is useful if in addition to the spatial diversity of the MIMO system also time and frequency diversity are available. In this case the coding is done across m OFDM subcarriers and across n symbols on each subcarrier, i.e. $N_L = m \cdot n$. For $m = N_S$ - only one LSD encoder is used - this means that e.g. $\mathbf{s}_{(\nu=1)}$ and $\mathbf{s}_{(\nu=m+1)}$ are transmitted over the same subcarrier; time diversity is exploited by the outer code. Altogether the system is able to use frequency, spatial and temporal diversity.

IV. Decoder

As appropriate decoder for the LSD codes in [10] a suboptimal reduced complexity ISI decoder (MAP-DFE) is presented. The ISI results from interfering spatial subchannels and from the optimized diversity performance of the outer code in fading. This decoder is feasible for large block length N_C showing a reasonable and scalable complexity. For the LSD codes it performs better and shows a lower complexity as the V-BLAST decoder (MMSE-DFE) according to [11]; alternatively other interference compensation algorithms (e.g. Zero-Forcing, MMSE equalization etc.) can be used.

V. Performance Results

In the following we will analyze the performance of the combination of LSD codes and OFDM based on simulations for different scenarios. In all simulations we use the MAP-DFE [10] as decoder and consider a

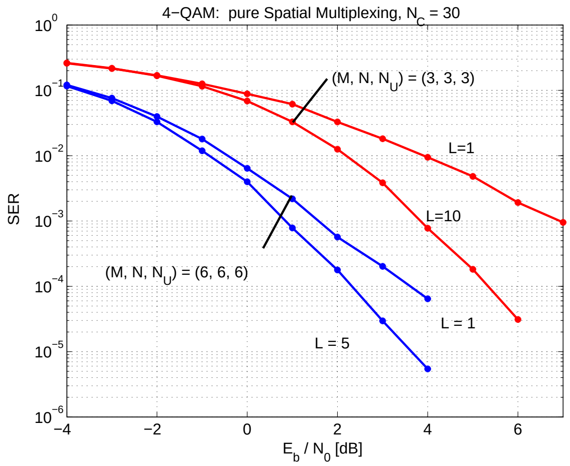

Fig. 4. Combination of spatial multiplexing and frequency diversity

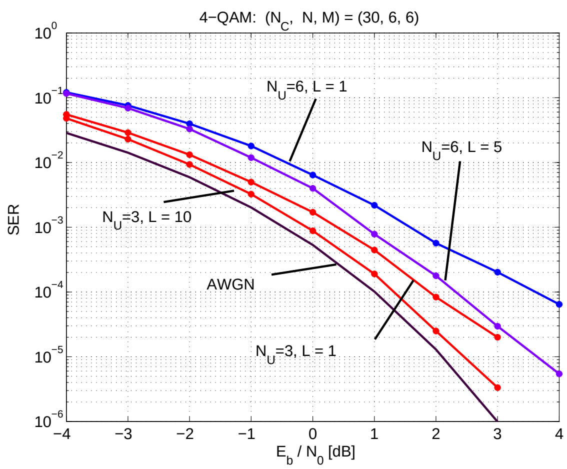

Fig. 5. Combination of spatial multiplexing, Tx/Rx diversity and frequency diversity

uniform power delay profile (path gains $\sigma_i^2 = 1$ for $i = 1, 2, ...L$).

Fig. 3 shows the symbol error rate (SER) over $\frac{E_b}{N_0}$ per Rx antenna for the MISO case ($N_C = N_L = 32$), with and without additional frequency diversity; the coding is done across 32 OFDM subcarriers. For $L = 1$ only spatial Tx diversity ($M = 2$) is achieved; with an increasing number L of (symbol discrete) channel taps the SER decreases. The system clearly exploits the frequency diversity of the frequency selective MIMO channel. In the pure spatial multiplexing MIMO case (without spatial Tx diversity) the codes are able to achieve an additional frequency diversity gain. Fig. 4 demonstrates this for two different scenarios: (i) $N_C = 30, M = N = N_U = 3, N_L = 10$ with $L = 1$ and $L = 10$ (coding across 10 subcarriers). (ii) $N_C = 30, M = N = N_U = 6, N_L = 5$ with $L = 1$ and $L = 5$ (coding across 5 subcarriers).

An example for the possible tradeoff between spatial multiplexing and diversity is shown in Fig. 5. The simulated system ($N_C = 30$) uses 6 Tx and 6 Rx antennas and two different numbers of spatial subchannels; for $N_U = 6$ follows $N_L = 5$ (coding across 5 tones); $N_U = 3$ leads to $N_L = 10$ (coding across 10 tones). The higher the number N_U of used spatial subchannels, the higher the data rate, but the SER increases too. In addition to the spatial Rx and Tx diversity gain a frequency diversity gain is achieved (shown in Fig. 5 for $L = N_L$).

VI. Conclusions

The considered combination of LSD codes and OFDM leads to space-frequency - or space-time-frequency - coded MIMO OFDM. The code construction allows to exploit spatial, temporal and frequency diversity. A rich tradeoff between spatial multiplexing to increase the data rate and diversity to improve the link reliability is possible while using a large number of antennas with reasonable complexity.

References

[1] V. Tarokh, N. Seshadri, and A.R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, no. 2, pp. 744–765, Mar. 1998.

[2] I. Emre Telatar, "Capacity of multi-antenna Gaussian channels," Tech. Rep., AT&T Bell Laboratories, 1995.

[3] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell System Techn. J.*, vol. 1, no. 2, pp. 41–59, Autumn 1996.

[4] Lizhong Zheng and David N. C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Inform. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.

[5] Marc Kuhn, Ingmar Hammerstroem, and Armin Wittneben, "Linear scalable space-time codes: Tradeoff between spatial multiplexing and transmit diversity," in *Proc. SPAWC*, 2003.

[6] Armin Wittneben and Marc Kuhn, "A new concatenated linear high rate space-time block code," in *Proc. 55th IEEE Veh. Tech. Conf.*, May 6–9, 2002, vol. 1, pp. 289–293.

[7] Marc Kuhn and Armin Wittneben, "Low complexity decoding of a class of linear space-time block codes by subspace partitioning," in *Proc. 56th IEEE Veh. Tech. Conf.*, September 24–28, 2002, vol. 4, pp. 2182–2186.

[8] J. G. Proakis, *Digital Communications*, John Wiley & Sons, first edition, 2002.

[9] H. Bölcskei and A. J. Paulraj, "Space-frequency coded broadband OFDM systems," in *Proc. IEEE Wirel. Comm. and Netw. Conf. (WCNC)*, Chicago, IL, September 23–28, 2000, pp. 1–6.

[10] Marc Kuhn and Armin Wittneben, "A new scalable decoder for linear block codes with intersymbol interference," in *Proc. 55th IEEE Veh. Tech. Conf.*, May 6–9, 2002, vol. 4, pp. 1795–1799.

[11] G.D. Golden, G.J. Foschini, R.A. Valenzuela, and R.W. Wolniansky, "Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture," *Electron. Lett.*, vol. 35, no. 1, pp. 14–15, Jan. 1999.