Time Domain Precoding for MIMO-OFDM Systems

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***Abstract*— Broadband multiple-input multiple-output (MIMO) communication systems using spatial multiplexing are facing multi-stream interference (MSI) from the transmit antennas and intersymbol interference (ISI) due to the time-dispersive nature of the channel. Appropriate detection requires the minimization of both sources of interference, usually at the receiver. Orthogonal frequency division multiplexing (OFDM) is a well-known technique implemented in IEEE 802.11a chipsets that can eliminate ISI, and therefore MSI remains the only source of interference in a MIMO-OFDM system. Removal can be realized at the transmitter utilizing channel state information (CSI) in the frequency domain [1]. In some practical cases no signal processing at the sub-carrier (frequency domain) level is possible and MSI removal (equalization) has to be done in time domain.**

In this paper we consider a MIMO-OFDM system built as a set of signal generating off-the-shelf IEEE 802.11a chipsets connected to a spatial multiplexing convergence layer. We present a time domain based full rate precoding scheme for removing MSI in frequency selective channels under assumption of perfect CSI at the transmitter that is still benefiting from the OFDM ability to remove ISI. The proposed system uses low complexity receivers and can be applied to a multiple-antenna transmitter communicating with several single antenna receivers (downlink scenario).

***Index Terms*— MIMO, spatial multiplexing, time-domain precoding,spatial multiplexing convergence layer**

I. Introduction

A. Framework

Multiple-input multiple-output (MIMO) systems for single user communication enable high data rate transmissions and a lower symbol error rate (SER) without additional bandwidth cost when compared to a single-input single-output (SISO) system. Multi-antenna systems are at the center of the upcoming IEEE 802.11n standard which promises up to 630Mbps for a 4x4 MIMO system. The penalty for this perfomance is that MIMO systems require a specific signal processing: in general the received signal must be free from both multi-stream interference (MSI) and inter-symbol interference (ISI). MSI originates from the superposition of all the transmitted signals at the receiver. ISI refers to the frequency selectivity of the channel: when the channel impulse response consists in a sequence of pulses, the received symbol at a given time instant is not directly proportional to the transmitted symbol but is equal to the convolution of the input sequence with the channel impulse response. In a multi-user scheme, data streams have to be separated to provide the corresponding data stream to each user. The processing can be realized at the receiver and/or transmitter. In this article we concentrate on a simple low cost and low complexity receiver downlink structure and place the complexity on the transmitter side. A solution for suppressing ISI is the orthogonal frequency division multiplexing (OFDM) technique which transforms a frequency selective channel into a set of parallel orthogonal flat fading channels. For practical realizations this operation can be performed by IEEE 802.11a chipsets and we consider in this article that our MIMO multi-user system is made from 802.11a chips operating in parallel at the transmitter and the receiver side (Fig. 1). The chips feed their data into a spatial multiplexing convergence layer (SMCL).
We propose in this article a SMCL that performs a linear time domain MSI removal precoding. The proposed precoder removes MSI and devolves ISI mildering/suppression to the OFDM processing within the 802.11a chipset, thus allowing the use of low complexity receivers.

B. State of the Art

MSI and ISI removal is a key topic for MIMO systems and it has been treated under different aspects. Precoding in time domain for a non dispersive channel is investigated in [2] whereas the study of

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dispersive channels is an ongoing area of research. Frequency equalization [1] using OFDM seems to be the simplest solution but doesn't fit into our framework. In [3], [4], [5] ISI/MSI are suppressed using optimized receivers and transmitters in time domain. [6] is investigating equalizers at the receiver side that jointly remove ISI and MSI. But so far no literature has be found in the multi-user case using low complexity receivers. Increasing the number of receive antennas is presented in [7] as a solution to remove ISI using an appropriate precoding. But in a more general way we look for a linear precoding which would work with any $N_{\text{RX}} \times N_{\text{TX}}$ MIMO system.

C. Notations

A block bold letter $\mathbf{H}$ represents a 2-dimensional array with entries $[\mathbf{H}]_{i,j}$ in the i^{th} row and the j^{th} column. $\mathbf{H}_{i,j}$ is the matrix obtained by deleting the i^{th} row and the j^{th} column from $\mathbf{H}$. If $\mathbf{H}$ is a square matrix, $\det(\mathbf{H})$ is the determinant of $\mathbf{H}$ and $\mathrm{adj}(\mathbf{H})$ is its adjoint (also termed adjugate). A small bold letter $\mathbf{v}$ is a vector of norm $|\mathbf{v}|$ with transposed form $\mathbf{v}^T$. $*$ denotes convolution. The number of ways of picking p unordered outcomes from n possibilities is written $\binom{n}{p}$.

II. Channel and System Model

A. Channel Model

We consider in this article a scenario where the L channel taps in time domain are all zero-mean complex normal independent identically distributed (uniform power delay profile). Channel coefficients are spatially and temporally uncorrelated, and the channel energy of each subchannel is normalized to unity. A subchannel represents the propagation path between a transmitter-emitter pair. It is assumed that the channel remains constant during the transmission of one OFDM symbol. $\mathbf{H}$ is a $N_{\text{RX}} \times N_{\text{TX}}$ matrix representing the channel in the time domain. Each $[\mathbf{H}]_{i,j}$ is a complex-valued function defined over the finite discrete domain $[0, \ldots, L-1]$ and models the impulse response of the channel between transmitter j and receiver i.

B. System Model

Fig. 1 shows a MIMO system with N_{TX} transmit antennas and N_{RX} receive antennas. The data streams are processed using OFDM and precoded by the matrix $\frac{1}{k_{\text{TD}}}\mathbf{F}$ before transmission. The OFDM part consists in applying an inverse fast Fourier transform (IFFT) to the input signal and then adding

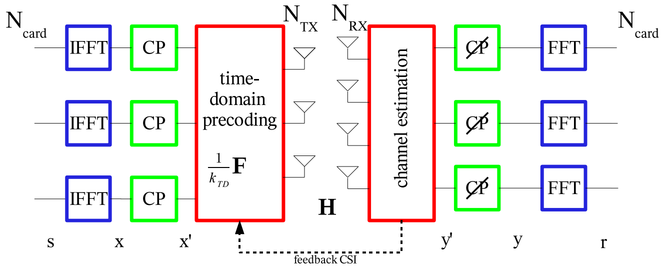

Fig. 1. System model. The noise source has been omitted

a cyclic prefix (CP). The precoder is designed to remove the MSI but not the ISI. A power constraint k_{TD} at the transmit antenna keeps the signal energy constant at the output of the precoder. The precoded signal is transmitted through a frequency-selective matrix channel where each subchannel impulse response has L equidistant channel taps. At the receiver the channel is estimated (using for example [8]) and sent back to the transmitter using an ideal channel state information (CSI) feedback link. For each received stream a zero mean circular symmetric complex Gaussian (ZMCSCG) noise term of covariance matrix $N_0\mathbf{I}$ is added, the CP is removed, a fast Fourier transform (FFT) is performed and data are decoded. The FFT/IFFT transform applies to Q points. $\mathbf{r} = [\mathbf{r_1} \ldots \mathbf{r}_{\mathbf{N}_{\text{card}}}]^T$ where $\forall k = 1 \ldots N_{\text{card}}$, $\mathbf{r_k} = [r_k[0] \ldots r_k[Q-1]]$ contains the Q taps corresponding to the frequency domain signals received on stream k. It is possible to consider the entries of $\mathbf{r_k}$ as the output of a complex function of a discrete variable. $\mathbf{y}' = [\mathbf{y}'_1 \ldots \mathbf{y}'_{\mathbf{N}_{\text{card}}}]^T$ where $\forall k = 1 \ldots N_{\text{card}}$, $\mathbf{y}'_\mathbf{k} = [y'_k[0] \ldots y'_k[P-1]]$ contains the P time domain signals received on stream k during one OFDM symbol duration (in [9], P=80). Here as well it is possible to consider the entries of $\mathbf{y}'_\mathbf{k}$ as the output of a complex function of a discrete variable. $\mathbf{s}$ and $\mathbf{x}' = [\mathbf{x}'_1 \ldots \mathbf{x}'_{\mathbf{N}_{\text{card}}}]^T$ are defined similarly to $\mathbf{r}$ and $\mathbf{y}'$ at the transmitter side.

The input/output relationship of the time domain part of our system is

$$\mathbf{y}' = \mathbf{H} * \frac{1}{k_{\text{TD}}}\mathbf{F} * \mathbf{x}' + \mathbf{n}' \tag{1}$$

with k_{TD} such that

$$\left| \frac{1}{k_{\text{TD}}}\mathbf{F} * \mathbf{x}' \right|^2 = \left| \mathbf{x}' \right|^2 \tag{2}$$

and $\mathbf{n}' = [\mathbf{n}'_1 \ldots \mathbf{n}'_{\mathbf{N}_{\text{card}}}]^T$ where $\forall k = 1 \ldots N_{\text{card}}$, $\mathbf{n}'_\mathbf{k} = [n'_k[0] \ldots n'_k[P-1]]$

C. System Assumptions

Binary data are fed into the system by N_{card} streams. Each of these streams is individually processed and then precoded. In this article we assume perfect CSI at the transmitter There are $N_{\text{TX}} \geq N_{\text{card}}$ available transmit antennas and $N_{\text{RX}} \geq N_{\text{card}}$ available receive antennas. First the case where N_{card}=N_{TX}=N_{RX} is considered, then the precoding scheme is extended to asymmetric systems where N_{TX}=N_{card} and $N_{\text{RX}} > N_{\text{card}}$ ($N_{\text{TX}} > N_{\text{card}}$ and $N_{\text{RX}} = N_{\text{card}}$)

III. Time-Domain Precoder

A. Precoding Matrix for N_{TX}=N_{RX}=N_{card} Systems

$\mathcal{R}$ is defined as the set of the square matrices with entries being complex-valued functions of a discrete variable. Let $\mathcal{M}$ be the set of the square matrices with scalar complex entries. $\mathcal{S}$ is the set of the complex functions of a discrete variable.

The basic idea of our precoding scheme is to consider $\mathbf{H}$ as a matrix with complex-valued functions as entries and to generalize the precoding techniques formerly applied to frequency flat channels to frequency selective channels.

1) Frequency Flat Fading Channel: For a matrix $\mathbf{A} \in \mathcal{M}$ the determinant $\det(\mathbf{A})$ is recursively defined by its Laplacian expansion: $\forall i \in [1 \ldots N_{\text{card}}]$, $\det(\mathbf{A}) = \sum_{j=1}^{N_{\text{card}}} (-1)^{i+j} \left[\mathbf{A}\right]_{i,j} \cdot \det(\mathbf{A}_{i,j})$ and $\det(\mathbf{A}) = \mathbf{A}$ when $\mathbf{A}$ is a 1×1 degenerated matrix. The adjoint is defined for matrices from $\mathcal{M}$. $\text{adj}(\mathbf{H})$ is a $N_{\text{card}} \times N_{\text{card}}$ matrix with entries at position (i, j):

$$\left[\text{adj}(\mathbf{H})\right]_{i,j} = (-1)^{i+j} \det(\mathbf{H}_{j,i}). \tag{3}$$

Let us now suppose that $\mathbf{H}$ is full rank. $\mathbf{H}$, $\det(\mathbf{H})$ and $\text{adj}(\mathbf{H})$ are linked by equation (4) [10].

$$\mathbf{H} \cdot \text{adj}(\mathbf{H}) = \det(\mathbf{H}) \cdot \mathbf{I} \tag{4}$$

with $\mathbf{I}$ the square unity matrix for $(\mathcal{M}, \cdot)$.
Thus when using $\mathbf{F}$=$\text{adj}(\mathbf{H})$ as the precoding matrix, the equivalent channel becomes diagonal and the input/output relationship (1) is

$$\mathbf{y}^{'} = \det(\mathbf{H}) \cdot \mathbf{I} \cdot \frac{1}{k_{TD}} \mathbf{x}^{'} + \mathbf{n}^{'} \tag{5}$$

2) Frequency Selective Channel: For a matrix $\mathbf{H} \in \mathcal{R}$ the adjoint and determinant are defined for matrices from $\mathcal{R}$ in a similar way as in III-A.1 since $(\mathcal{S}, +, *)$ is a commutative ring (c.f. appendix). We have $\det(\mathbf{H}) = \sum_{j=1}^{N_{\text{card}}} (-1)^{i+j} \left[\mathbf{H}\right]_{i,j} * \det(\mathbf{H}_{i,j})$.

$\text{adj}(\mathbf{H})$ is a $N_{\text{card}} \times N_{\text{card}}$ matrix with entries at position (i, j):

$$\left[\text{adj}(\mathbf{H})\right]_{i,j} = (-1)^{i+j} \det(\mathbf{H}_{j,i}). \tag{6}$$

and

$$\mathbf{H} * \text{adj}(\mathbf{H}) = \det(\mathbf{H}) * \mathbf{I} \tag{7}$$

with $\mathbf{I}$ the square unity matrix for $(\mathcal{R}, *)$. If $\det(\mathbf{H}) \neq \mathbf{0}$ we choose the precoding matrix $\mathbf{F} = \text{adj}(\mathbf{H})$: the equivalent matrix channel becomes diagonal and thus free from MSI. In this case the input-output relationship (1) is:

$$\mathbf{y}^{'} = \det(\mathbf{H}) * \mathbf{I} * \frac{1}{k_{\text{TD}}} \cdot \mathbf{x}^{'} + \mathbf{n}^{'} \tag{8}$$

N_{card} identical virtual SISO channels result with the same impulse response given by $\frac{1}{k_{\text{TD}}}\det(\mathbf{H})$. The remaining ISI is removed by the OFDM, assuming that the length M of the resulting impulse response ($M = N_{\text{card}} L - (N_{\text{card}} - 1)$) does not exceed the length of the cyclic prefix.

In the frequency domain,

$$\mathbf{r} = \frac{1}{k_{\text{TD}}} \begin{pmatrix} \mathbf{\Omega} & \ldots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \ldots & \mathbf{\Omega} \end{pmatrix} \cdot \mathbf{s} + \mathbf{n} \tag{9}$$

where

$$\mathbf{\Omega} = \text{diag}\left(\omega\left[0\right], \ldots, \omega\left[Q-1\right]\right)$$

with

$$\forall k = 0 \ldots Q-1,\ \omega\left[k\right] = \sum_{l=0}^{M} \det(\mathbf{H})\left[l\right] e^{-\frac{j2\pi kl}{Q-1}}$$

and $\mathbf{n}$ a ZMCSCG noise term with same variance as $\mathbf{n}^{'}$ [2].

B. Precoding Matrix for $N_{TX} > N_{card}$ or $N_{RX} > N_{card}$

1) $N_{TX} > N_{card}$ and $N_{RX} = N_{card}$: In the case there are more transmit antennas available than input data streams, additional antennas can be integrated into a selection combining scheme to improve the system performance. The SMCL is selecting the transmit antennas such that the equivalent subchannels (they are all identical) provide the highest signal to noise ratio (SNR) at the receiver. The best equivalent subchannel to be selected is the one which conveys the maximum energy, i.e. the one which has the largest Frobenius norm.
Let $\{\mathbf{H}_k\}, k = 1 \ldots \binom{N_{\text{TX}}}{N_{\text{card}}}$ be the set of the square $N_{\text{card}} \times N_{\text{card}}$ channel matrices built by extracting N_{card} from N_{TX} columns in the channel matrix $\mathbf{H}$. The precoder selects the antennas corresponding to the channel matrix $\mathbf{H}_k$ such that $k = \arg\max_{k=1\ldots\binom{N_{\text{TX}}}{N_{\text{card}}}} (|\det(\mathbf{H}_k)|)$. The scheme is transparent for the receiver.

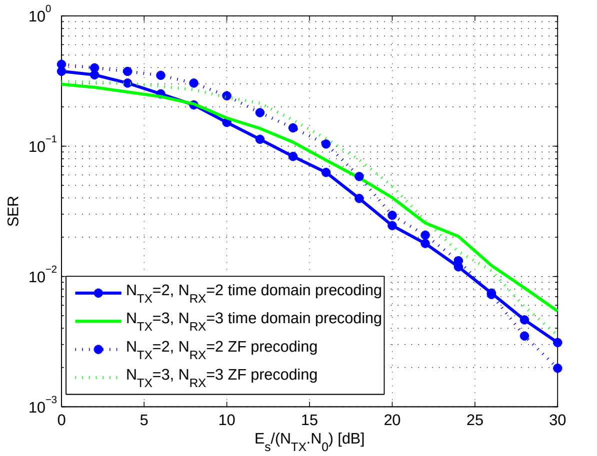

Fig. 2. Symbol error rate for time domain precoding and frequency domain zero forcing precoder for a 3-taps channel with uniform power delay profile

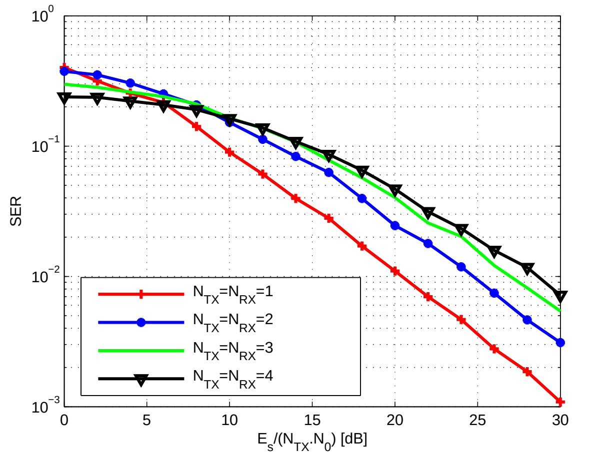

Fig. 3. Symbol error rate for MIMO systems using time domain precoding with power constraint at the transmitter for a 3-taps channel with uniform power delay profile

2) $N_{TX} < N_{RX}$ and $N_{TX} = N_{card}$: The scenario is the same as in paragraph III-B.1. The precoder must choose N_{card} receive antennas among the N_{RX} receive antennas available to perform the transmission [2].

IV. Numerical Results

A. $N_{card} \times N_{card}$ System

1) Comparison with a frequency domain zero forcing precoder: We simulate the symbol error rate (SER) performance of a MIMO system with the proposed time-domain precoding and compare it with a frequency domain zero forcing (ZF) precoder using the same transmit power constraint (Fig. 2). For low SNR, the system with time domain precoding achieves a lower SER than the system based on frequency domain precoding. But the slope of the SER curve of the frequency domain precoder is steeper than the time domain precoder, and for higher SNR the frequency domain precoder overperforms the proposed scheme.

When using a frequency domain ZF precoder, the equivalent channel is constant for each SISO link and each OFDM frequency ($\Omega = \mathbf{I}$ in equation (9)), whereas the equivalent channel for time domain precoding is fading ($\det(\mathbf{H})$ is a random variable). Let k_{ZF} be the scaling factor for the ZF precoder and E_s/N_{card} is the input energy per input stream. For the subcarrier k and a given SISO link, the subchannel coefficient in the ZF case is $1/k_{\text{ZF}}$ whereas for the time-domain precoder the equivalent channel is $1/(k_{\text{TD}} \cdot \omega[k])$. The system with time domain precoder undergoes the fading effect from both k_{TD} and $\det(\mathbf{H})$.

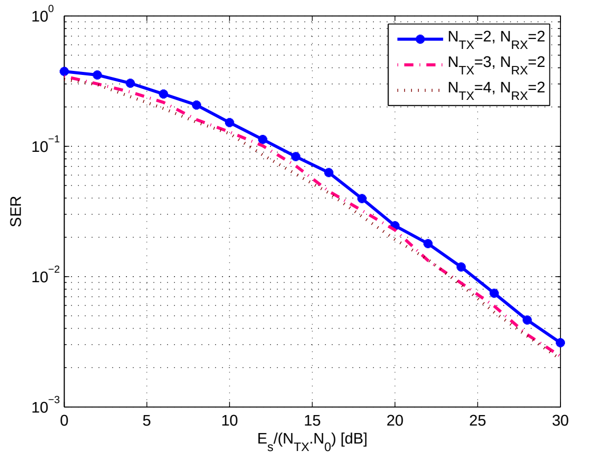

Fig. 4. Symbol error rate for MIMO systems using antenna selection combining

2) Influence of N_{card}: When increasing the number of input streams with a constant transmitted power E_s we obtain Fig. 3. SISO links are resulting from precoding and no diversity is observed. The performance degradation when increasing N_{card} originates from the power constraint: the same amount of energy is distributed over a larger number of SISO links, leading to a lower SNR at the decoder.

B. Additional antennas for selection combining

Simulations when using more antennas than input data streams are in Fig. 4. Diversity remains equal to 1 but the general performance improves through a coding gain of approximatively 1dB between the case N_{TX}=N_{RX} and $N_{\text{TX}} \neq N_{\text{RX}}$.

V. Conclusion

In this article we showed that the precoder $\mathbf{F} = \text{adj}(\mathbf{H})$ actually diagonalizes the matrix channel, completely eliminating the MSI. The equivalent channel is still fading and is longer than the orginal channel but enables the parallel operation of virtual independent SISO links with all the same channel impulse response.

APPENDIX

A. Operators

+ is the term by term addition operator defined as:

$$\forall (M_1, M_2) \in (\mathcal{R} \times \mathcal{R}) \text{ such that}$$
$$M_l = \begin{pmatrix} f^l_{1,1} & \cdots & f^l_{1,N} \\ \vdots & \ddots & \vdots \\ f^l_{N,1} & \cdots & f^l_{N,N} \end{pmatrix}_{l=1,2},$$
$$M_1 + M_2 = \begin{pmatrix} f^1_{1,1} + f^2_{1,1} & \cdots & f^1_{1,N} + f^2_{1,N} \\ \vdots & \ddots & \vdots \\ f^1_{N,1} + f^2_{N,1} & \cdots & f^1_{N,N} + f^2_{N,N} \end{pmatrix}$$
$$\text{with } \forall k = 0, \ldots, L-1,$$
$$\left(f^1_{i,j} + f^2_{i,j}\right)[k] = f^1_{i,j}[k] + f^2_{i,j}[k]$$

$*$ is the term by term discrete convolution operator defined as:

$$\forall (M_1, M_2) \in (\mathcal{R} \times \mathcal{R}), \forall (i,j) \in [1, ..., N]^2,$$
$$(M_1 * M_2)_{i,j} = \left\{ \sum_{k=1}^{N} f^1_{i,k} * f^2_{k,j} \right\}$$
$$\text{with } \left(f^1_{i,k} * f^2_{k,j}\right)[k] = \sum_p f^1_{i,k}[p]\, f^2_{k,j}[k-p]$$

B. Algebraic set

As defined in III-A, $\mathcal{R}$ is the set of the $(N \times N)$ matrices of complex functions $\{f_{i,j}\}_{\substack{i=1,\ldots,N \\ j=1,\ldots,N}}$ of a discrete variable.

$\mathcal{S}$ is the set of the complex functions of a discrete variable. In this appendix we prove that $(\mathcal{S}, +, *)$ is a commutative ring by checking the axioms. For this ring the determinant of a matrix from $\mathcal{R}$ is defined as well as its adjoint (also termed adjugate).

C. Structure

1) ($\mathcal{S}$,+) abelian group:

- $\mathcal{S}$ is non empty since the null function $(\mathbf{0} : \mathbb{Z} \to \mathbb{C}, k \longmapsto 0)$ belongs to it,
- + is defined as an operator from $\mathcal{S}$ to $\mathcal{S}$ (from its definition),
- + is associative, due to the associativity of + in $\mathbb{C}$,
- the identity element is the null function,
- each function f in $\mathcal{S}$ has its inverse $-f$ such that $\forall k, (f - f)[k] = 0$
- + is commutative due to the commutativity of + in $\mathbb{C}$.

2) ($\mathcal{S}, +, *$) ring: ($\mathcal{S}$,+) is an abelian group (cf. C.1). We define a second operation $*$ which extends the discrete convolution product to elements of $\mathcal{S}$.

- $\forall (f_1, f_2) \in \mathcal{S}^2,\ f_1 * f_2 \in \mathcal{S}$ due to the discrete convolution properties,
- $\forall (f_1, f_2, f_3) \in \mathcal{S}^3,\ f_1 * (f_2 * f_3) = (f_1 * f_2) * f_3$ due to the associativity property of the discrete function convolution,
- $*$ is distributive over +,
- the identity discrete fonction $(\mathbf{1} : \mathbb{Z} \to \mathbb{C}, k \longmapsto \delta[k])$ is the identity element for $*$ and is an element of $\mathcal{S}$

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