A New Cascade Decoder for Linear Block Codes OFDM-System

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***Abstract*—Space-Time Codes and OFDM represent a key technology for future broadband wireless communication systems. In this paper a class of space-time block codes according to [3], in combination with OFDM, is used which can lead to intersymbol interference (ISI) due to an optimized diversity performance. Therefore ISI compensation is an important task of the decoding process of these codes. Several known ISI compensation methods can be applied for example MMSE equalization or parallel or serial ISI cancellation.**

In this publication a new scalable decoder that works according to the parallel ISI cancellation method is presented. This decoder bases on a cascade structure with two branches for each cascade stage. The sub decoders in each branch were calculated coupledly according to the MMSE criterion. The inputs of the cascade are the receiver symbols and an estimation of the transmitter symbols realized with a MMSE estimator. The prioritization of the branches is made through the a priori symbol error probability. This probability bases on the transmitter symbols estimation. By varying the number of the cascade stages or the block length the performance of the decoder can be scaled.

I. INTRODUCTION

If we consider the impact of strongly frequency-selective channels on the transmission by OFDM it can be said that this fading is the reason why the transmission performance (bit error rate BER) decreases. Classical methods to avoid the degradation of the bit error rate use channel coding techniques like convolution or trellis codes. These methods have the disadvantage that, when adding redundancy, the data rate decreases. In contrast to this method the new linear blockcode presented in [3] works with code rate $r_c = 1$. However, this code allows us to use every code rate desired so that an immense increase in performance is obtained. The codes used in the paper have been developed for high rated wireless communication as a new class of *Space–Time–Codes* [3][2]. The structure of this codes is shown in Figure 1. On the transmitter side, the space-time-codes consist of two components, the *outer*– ($\mathbf{C}_\mathrm{A}$) and the *inner* ($\mathbf{C}_\mathrm{I}$) code. Both of them are *linear block codes* which are determined by matrices. The *MIMO*-channel results from an antenna array at the transmitter and at the receiver and is described as matrix $\mathbf{X}$. The receiver consists of a matched channel adaption ($\mathbf{X}^H$) as well as an inner ($\mathbf{C}_\mathrm{I}^H$) and outer ($\mathbf{C}_\mathrm{A}^H$) code matched matrix. Finally, the decoder makes an estimation of the transmit symbol vector $\vec{\hat{s}}$.

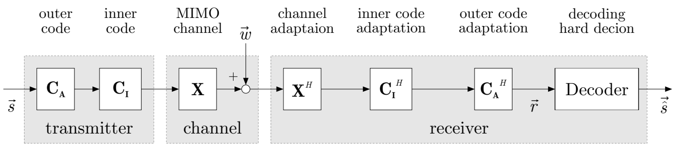


Fig. 1. System model for the linear space–time code

Now we will take only the outer code into our consideration because we focus on SISO channels and therefore we do not need the inner code. The subscript for differentiation between outer and inner codes is omitted and we identify the code matrix by $\mathbf{C}$. With these declarations we get the simplified system model shown in Figure 2.

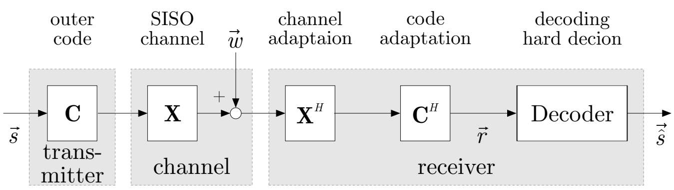


Fig. 2. System model for the SISO channel

II. THE SYSTEM MODEL

The OFDM system used in this paper should be dimensioned in such a way that the channel impact is only a scalar multiplication of the transmitted symbols $\hat{s}'_i$ with the complex transfer factors $H_{c,i}$ of the channel. So, the received symbol vector $\vec{r}'$ is given by

$$\vec{r}' = \mathbf{D}_{Hc} \cdot \vec{s}' + \vec{n}' \quad (1)$$

where $\mathbf{D}_{Hc}$ describes the diagonal channel matrix with the complex transfer factor on the diagonal and $\vec{n}'$ is the additive coloured noise.

We used (4) and the system model from Figure 2 in order to investigate the block code. The channel adaptation

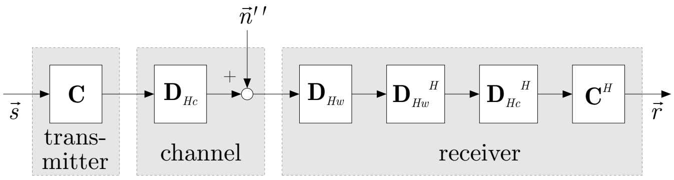


Fig. 3. System model for the linear block code and the OFDM

is dimensioned according to the matched filter principle, in this case the matched matrix. The matched matrix consists of a pre-whitening matrix $\mathbf{D}_{Hw}$ for the coloured noise, a matched matrix $\mathbf{D}_{Hw}^H$ for the pre-whitening matrix and a matched matrix $\mathbf{D}_{Hc}^H$ for the channel. The pre- whitening matrix can be realised as a diagonal matrix because the coloured noise is transformed into a white instationary noise. The outer code will be adapted to a hermitic code matrix $\mathbf{C}^H$. With these declarations the received vector $\vec{r}$ is:

$$\vec{r} = \mathbf{\Lambda}_{ISI} \cdot \vec{s} + \mathbf{C}^H \cdot \mathbf{D}_{Hc}^H \cdot \mathbf{D}_{|Hw|^2} \cdot \vec{n}'' \quad (2)$$

The notation

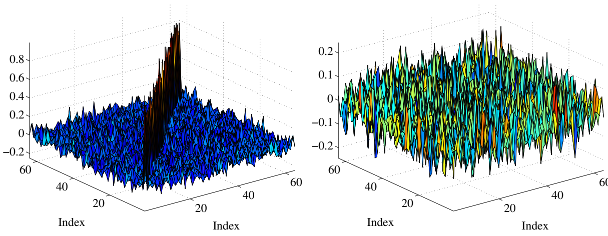
$$\mathbf{D}_{|Hw|^2} = \mathbf{D}_{Hw}^H \cdot \mathbf{D}_{Hw} \quad (3)$$

means that the elements of the diagonal matrix will be calculated as square of the magnitude and (4)

$$\mathbf{\Lambda}_{ISI} = \mathbf{C}^H \cdot \mathbf{D}_{|Hc|^2} \cdot \mathbf{D}_{|Hw|^2} \cdot \mathbf{C} \quad (4)$$

describes the system matrix, which includes the dependence of the transmitter symbols on the receiver symbols which are influenced by the channel and the coding. Thus, $\mathbf{\Lambda}_{ISI}$ describes the inter symbol interference (ISI) of the receiver symbols.

The structure of the $\mathbf{\Lambda}_{ISI}$ matrix is shown exemplarily in Figure 4. The partial image 4(a) shows the real part of the interference matrix $\mathbf{\Lambda}_{ISI}$ and the main diagonal reveals the impacts of the channel and whitening filter. The magnitude of them is not constant what is equivalent to a different attenuation of the symbols. In both partial images, the secondary diagonals show the intersymbol interference, which has in all dimensions nearly the same order of magnitude. This effect is made particularly clear in the imaginary part of Figure 4(b). The intersymbol



(a) real part (b) imaginary part

Fig. 4. Example of an $\mathbf{\Lambda}_{ISI}$ matrix (64×64)

interference, which has been constrained by coding, is the basis for the diversity gain; This is a kind of spectral spreading. If some transfer factors $H_{c,i}$ are strongly attenuated or even at zero, the intersymbol interference can be used to reconstruct the disturbed symbols. Figure 5 shows the system model from (2). Thus, we get the receiver vector $\vec{r}$

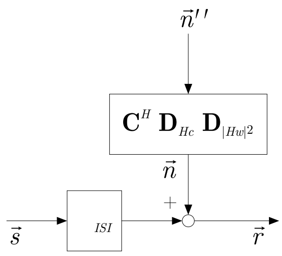


Fig. 5. Composite system model

from Figure 5

$$\vec{r} = \mathbf{\Lambda}_{ISI} \cdot \vec{s} + \vec{n} \quad (5)$$

with

$$\vec{n} = \mathbf{C}^H \cdot \mathbf{D}_{Hc}^H \cdot \mathbf{D}_{|Hw|^2} \cdot \vec{n}'' \quad (6)$$

for the noise. To compensate this ISI, a decoder using an equalization method is needed.

III. The MMSE Cascade Decoder

In this paper a new ISI cancellation method with a suboptimal working decoder is presented. The decoder works according to the MMSE criterion. This decoder is based on a cascaded structure with two branches for each cascade stage. The sub decoders in each branch were calculated coupledly according to the MMSE criterion. The inputs of the cascade are the receiver symbols as well as an estimation of the transmitter symbols realized with a MMSE estimator. Every stage is followed by an threshold detector.

Figure 6 shows the decoder structure with two cascaded stages. The first part of the block diagram shows the basic system model with the transmit symbol vector $\vec{s}$ as input value and the noise receiver symbol vector $\vec{r}$ as output value as well as the noise vector $\vec{n}$. The next block

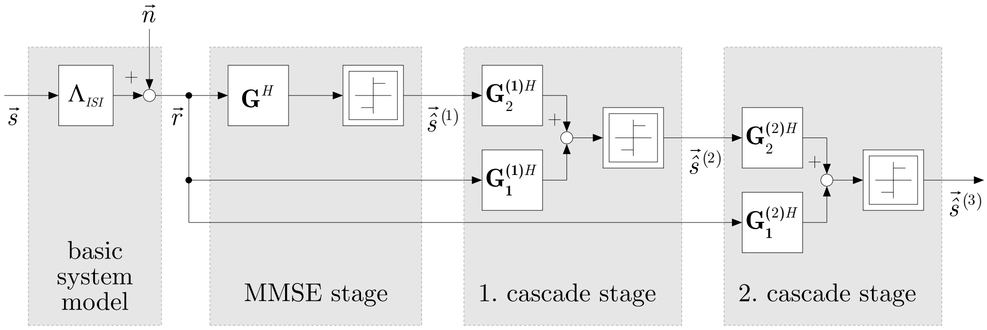


Fig. 6. System model with a two stage cascade decoder

makes a first estimation of the receiver symbol vector for the transmit symbol vector $\vec{\hat{s}}^{(1)}$ using a MMSE equalizer.

$$\boxed{\mathbf{G} = (\mathbf{\Lambda}_{ISI} \cdot \mathbf{\Lambda}_{ISI}^H + \mathbf{R}_{nn})^{-1} \cdot \mathbf{\Lambda}_{ISI}} \qquad (7)$$

The following first cascaded stage consists of two coupled linear MMSE equalizers ($\mathbf{G}_1^{(1)}$, $\mathbf{G}_2^{(1)}$) and a threshold detector. This way we get the symbol estimation $\vec{\hat{s}}^{(2)}$ that represents, together with the receiver symbols $\vec{r}$, the input values of the next cascaded stage. This stage also consists of two coupled linear MMSE equalizer and a threshold detector. It is important that the MMSE equalizer from stage k is not identical with the MMSE equalizer from stage $k+1$.

The subdecoders of each branch must be calculated coupledly according to the MMSE criterion. The MMSE criterion aims to minimize the mean squared error (MMSE). We get this error $|| \ \vec{e}^{\,(k+1)} \ ||^2$ by subtracting the transmitted symbols from the estimated symbols, see Figure 7. For the two equalizer matrices $\mathbf{G}_1^{(k)}$ and $\mathbf{G}_2^{(k)}$ we

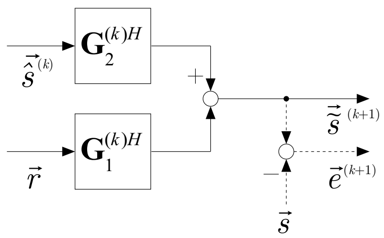


Fig. 7. MMSE decoder on the kth cascade stage

get two coupled equations with the help of the orthogonality relation.

$$\mathcal{E}\left\{\vec{r} \cdot \vec{e}^{\,(k+1)H}\right\} = \mathbf{0} \quad \text{und} \quad \mathcal{E}\left\{\vec{\hat{s}}^{\,(k)} \cdot \vec{e}^{\,(k+1)H}\right\} = \mathbf{0} \qquad (8)$$

A. The Nulling MMSE Cascade Decoder

The special idea of this symbol estimator is based on the side condition. The two equalizer matrices $\mathbf{G}_1$ and $\mathbf{G}_2$ should be determine in a way that the mean squared error is minimized with respect to the side condition that every ith estimated symbol is set to zero.

$$\mathcal{E}\{|| \ \vec{e} \ ||^2\} \rightarrow \min \qquad (9)$$

This side condition makes it necessary to change the system model into a symbol–wise estimated model, see Figure 8. At the same time we assume that the equalizer

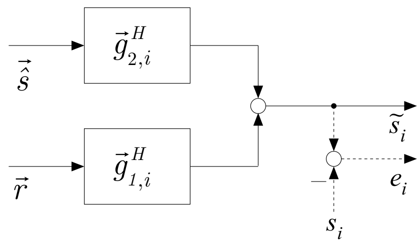


Fig. 8. The MMSE decoder for the ith symbol

matrices $\mathbf{G}_1$ and $\mathbf{G}_2$ consist of N column vectors.

$$\mathbf{G}_1 = \left[\vec{g}_{1,1} \ \ldots \ \vec{g}_{1,i} \ \ldots \ \vec{g}_{1,N}\right] \qquad (10)$$
$$\mathbf{G}_2 = \left[\vec{g}_{2,1} \ \ldots \ \vec{g}_{2,i} \ \ldots \ \vec{g}_{2,N}\right], \qquad (11)$$

$$\vec{g}_{1,i} = \left[g_{1i} \quad g_{2i} \quad \ldots \quad g_{Ni}\right]^T \qquad \text{für} \quad i \in [1;N]. \qquad (12)$$

Furthermore, we generate the input vector for the cascade from the estimated symbol vector $\vec{\hat{s}}$ from the anterior stage by setting the ith symbol of the vector to zero (nulling).

$$\vec{\hat{s}}_{0,i} = \left[\hat{s}_1 \quad \ldots \quad 0 \quad \ldots \quad \hat{s}_N\right]^T \qquad \text{für} \quad i \in [1;N], \qquad (13)$$

The error $e_i = \tilde{s}_i - s_i$ can be derived from Figure 8 with $\tilde{s}_i = \vec{g}_{1,i}^H \cdot \vec{r} + \vec{g}_{2,i}^H \cdot \vec{\hat{s}}_{0,i}$ and $\vec{r} = \mathbf{\Lambda}_{ISI} \cdot \vec{s} + \vec{n}$ to

$$\begin{aligned} e_i &= \vec{g}_{1,i}^H \cdot \vec{r} + \vec{g}_{2,i}^H \cdot \vec{\hat{s}}_{0,i} - s_i \\ &= \vec{g}_{1,i}^H \cdot (\mathbf{\Lambda}_{ISI} \cdot \vec{s} + \vec{n}) + \vec{g}_{2,i}^H \cdot \vec{\hat{s}}_{0,i} - s_i. \end{aligned} \qquad (14)$$

In order to calculate the equalizer vectors we use the orthogonal condition.

$$\mathcal{E}\{\vec{r}\cdot e_1^*\} = \vec{0} \quad (15)$$

$$\mathcal{E}\left\{\vec{\hat{s}}_{0,i}\cdot e_1^*\right\} = \vec{0} \quad (16)$$

We use the result of the error (14) to solve the equation (15).

$$\begin{aligned}\mathcal{E}\{\vec{r}\cdot e_i^*\} &= \vec{0}\\ \mathcal{E}\left\{\vec{r}\cdot(\vec{g}_{1,i}^H\cdot\vec{r}+\vec{g}_{2,i}^H\cdot\vec{\hat{s}}_{0,i}-s_i)^H\right\} &= \vec{0}\\ \mathcal{E}\left\{\vec{r}\cdot\vec{r}^H\cdot\vec{g}_{1,i}\right\}+\mathcal{E}\left\{\vec{r}\cdot\vec{\hat{s}}_{0,i}^H\cdot\vec{g}_{2,i}\right\}-\mathcal{E}\{\vec{r}\cdot s_i^*\} &= \vec{0}\\ \mathcal{E}\left\{\vec{r}\cdot\vec{r}^H\right\}\cdot\vec{g}_{1,i}+\mathcal{E}\left\{\vec{r}\cdot\vec{\hat{s}}_{0,i}^H\right\}\cdot\vec{g}_{2,i}-\mathcal{E}\{\vec{r}\cdot s_i^*\} &= \vec{0}\\ \mathbf{R}_{rr}\cdot\vec{g}_{1,i}+\mathbf{R}_{r\hat{s}_{0,i}}\cdot\vec{g}_{2,i}-\mathcal{E}\{\vec{r}\cdot s_i^*\} &= \vec{0}.\end{aligned} \quad (17)$$

For the autocorrelation matrix of the receiver symbols which can be found in the first term of (17), we get

$$\mathbf{R}_{rr} = \mathcal{E}\left\{\vec{r}\cdot\vec{r}^H\right\} = \mathbf{\Lambda}_{ISI}\cdot\mathbf{\Lambda}_{ISI}^H + \mathbf{R}_{nn}. \quad (18)$$

The cross correlation matrix between the estimated and the modified symbols, which we got in the second term of (17), can be written as

$$\mathbf{R}_{r\hat{s}_{0,i}} = \mathcal{E}\left\{\vec{r}\cdot\vec{\hat{s}}_{0,i}^H\right\} = \mathbf{\Lambda}_{ISI}\cdot\mathbf{R}_{s\hat{s}_{0,i}} + \mathbf{R}_{n\hat{s}_{0,i}}. \quad (19)$$

An exact determination of the cross correlation matrix (19) is not possible because of the nonlinear threshold detector. Therefore we assume that the estimated symbols are decorrelated from the noise.

$$\mathbf{R}_{n\hat{s}_{0,i}} = \mathbf{0} \quad (20)$$

By using this assumption, the cross correlation matrix is given for a 2 PSK to

$$\mathbf{R}_{r\hat{s}_{0,i}} = \mathcal{E}\left\{\vec{r}\cdot\vec{\hat{s}}_{0,i}^H\right\} = \mathbf{\Lambda}_{ISI}\cdot\mathbf{D}_{P_{e_{0,i}}}, \quad (21)$$

where $\mathbf{D}_{P_{e_{0,i}}}$ is a diagonal matrix with the symbol error probability P_{ei} at the diagonal.

$$\mathbf{D}_{P_{e_{0,i}}} = \begin{bmatrix} 1-2P_{e1} & 0 & \cdots & 0\\ 0 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 1-2P_{e_N}\end{bmatrix} \quad (22)$$

The ith position of the diagonal is zero. Considering the decorrelation between the estimated symbols and the noise, the third term of (17) can now be written as

$$\begin{aligned}\mathcal{E}\{\vec{r}\cdot s_i^*\} &= \mathcal{E}\{\mathbf{\Lambda}_{ISI}\cdot\vec{s}\cdot s_i^*+\vec{n}\cdot s_i^*\}\\ &= \mathbf{\Lambda}_{ISI}\cdot\mathcal{E}\{\vec{s}\cdot s_i^*\}+\mathcal{E}\{\vec{n}\cdot s_i^*\}\\ &= \mathbf{\Lambda}_{ISI}\cdot\begin{bmatrix}0 & \ldots & 1 & \ldots & 0\end{bmatrix}^T\\ &= \vec{\lambda}_{ISI,i},\end{aligned} \quad (23)$$

with

$$\mathbf{\Lambda}_{ISI} = \begin{bmatrix}\vec{\lambda}_{ISI,1} & \vec{\lambda}_{ISI,2} & \ldots & \vec{\lambda}_{ISI,N}\end{bmatrix}. \quad (24)$$

The result of (23) corresponds to the ith column of the $\mathbf{\Lambda}_{ISI}$. With these partial results the orthogonal condition from (15) can then be written as

$$\begin{aligned}\mathcal{E}\{\vec{r}\cdot e_1^*\} &= \left(\mathbf{\Lambda}_{ISI}\cdot\mathbf{\Lambda}_{ISI}^H+\mathbf{R}_{nn}\right)\cdot\vec{g}_{1,i}\\ &+ \left(\mathbf{\Lambda}_{ISI}\cdot\mathbf{D}_{P_{e_{0,i}}}\right)\cdot\vec{g}_{2,i}-\vec{\lambda}_{ISI,i}\\ &= \vec{0}\end{aligned} \quad (25)$$

We obtain the second defining equation for the equalizer vector from the orthogonal condition (16).

$$\begin{aligned}\mathcal{E}\left\{\vec{\hat{s}}_{0,i}\cdot e_i^*\right\} &= \mathbf{R}_{\hat{s}_{0,i}r}\cdot\vec{g}_1+\mathbf{R}_{\hat{s}_{0,i}\hat{s}_{0,i}}\cdot\vec{g}_{2,i}\\ &- \mathcal{E}\left\{\vec{\hat{s}}_{0,i}s_i^*\right\} = \vec{0}.\end{aligned} \quad (26)$$

Considering $\hat{s}_i = 0$, we get the third term of (26) as

$$\mathcal{E}\left\{\vec{\hat{s}}_{0,i}s_i^*\right\} = \vec{0}. \quad (27)$$

Taking this result and $\mathbf{R}_{\hat{s}_{0,i}r} = \mathbf{R}_{r\hat{s}_{0,i}}^H = \mathbf{D}_{P_{e_{0,i}}}\cdot\mathbf{\Lambda}_{ISI}^H$ as well as $\mathbf{R}_{\hat{s}_{0,i}\hat{s}_{0,i}} = \mathcal{E}\left\{\vec{\hat{s}}_{0,i}\cdot\vec{\hat{s}}_{0,i}^H\right\} = \mathbf{I}_{0,i}$, where $\mathbf{I}_{0,i}$ is an unit matrix with a zero at the ith position at the main diagonal, (26) can be written as

$$\mathcal{E}\left\{\vec{\hat{s}}_{0,i}\cdot e_i^*\right\} = \mathbf{D}_{P_{e_{0,i}}}\cdot\mathbf{\Lambda}_{ISI}^H\cdot\vec{g}_{1,i}+\mathbf{I}_{0,i}\cdot\vec{g}_{2,i} = \vec{0} \quad (28)$$

The term $\mathbf{I}_{0,i}\cdot\vec{g}_{2,i}$ generates a zero at the ith position of the equalizer vector. Thus, the equalizer matrix has only zeros at the main diagonal. With this knowledge (28) can be written as

$$\boxed{\vec{g}_{2,i} = -\mathbf{D}_{P_{e_{0,i}}}\cdot\mathbf{\Lambda}_{ISI}^H\cdot\vec{g}_{1,i}.} \quad (29)$$

To determine the equalizer matrix $\mathbf{G}_2$ we have to solve (29) N times with different $\vec{g}_{1,i}$ and $\mathbf{D}_{P_{e_{0,i}}}$. We can reduce the calculation effort as follows: The equalizer matrix $\mathbf{G}_2$ can be calculated directly with a filter (Nulling) matrix and a complete $\mathbf{D}_{P_e}$ matrix as

$$\boxed{\mathbf{G}_2 = -(\mathbf{D}_{P_e}\cdot\mathbf{\Lambda}_{ISI}^H\cdot\mathbf{G}_1)\odot\begin{bmatrix}0 & \cdots & 1\\ \vdots & \ddots & \vdots\\ 1 & \cdots & 0\end{bmatrix}} \quad (30)$$

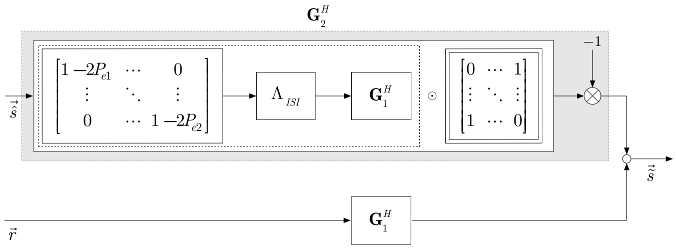


Fig. 9. Structure of the equalizer matrix $\mathbf{G}_2$

The operator $\odot$ is equivalent to an element-wise multiplication of two matrices.

$$\mathbf{A} \odot \mathbf{B} = [a_{ij} \cdot b_{ij}]_{i,j=1,\ldots,N} \tag{31}$$

Figure 9 shows the structure of the equalizer matrix at the branch with the estimated symbols. Using (29) and (25), the determination of the equalizer vector $\vec{g}_{1,i}$ is given as

$$\boxed{\vec{g}_{1,i} = \left[\mathbf{\Lambda}_{ISI} \cdot (\mathbf{I} - \mathbf{D}^2_{P_{e_{0,i}}}) \cdot \mathbf{\Lambda}^H_{ISI} + \mathbf{R}_{nn}\right]^{-1} \cdot \vec{\lambda}_{ISI,i}.} \tag{32}$$

To determine the equalizer matrix $\mathbf{G}_1$ we must solve (32) N times with different $\mathbf{D}_{P_{e_{0,i}}}$.

IV. Results

In this section, results which are based on simulations using the outer code according to [3] in a Rayleigh fading environment are presented. Perfect channel knowledge at the receiver is assumed. The symbol error probability P_{ei} is a–priori estimated from the signal–to–noise ratio at the receiver.

Figure 10 and 11 show the excellent performance of the new decoder. They present the symbol error rate (SER) of 4-QAM versus E_s/N_0 at the receiver for a Rayleigh fading channel and block length 32 ($\vec{r}$ consists of 32 elements) and block length 128 respectively. The decoder has a very good performance, much better than a simple MMSE filter for ISI compensation and clearly better than a decoder according to [1]. The performance of the coding and the new decoder increases with growing block length.

This effect can be seen when comparing Figure 10 to Figure 11.

V. Conclusion

A new intersymbol interference method for space time codes has been presented. This decoder has an excellent performance which is proved by simulation results. The decoder can be used in any application for interference compensation such as multi-user interference cancellation for CDMA-systems.

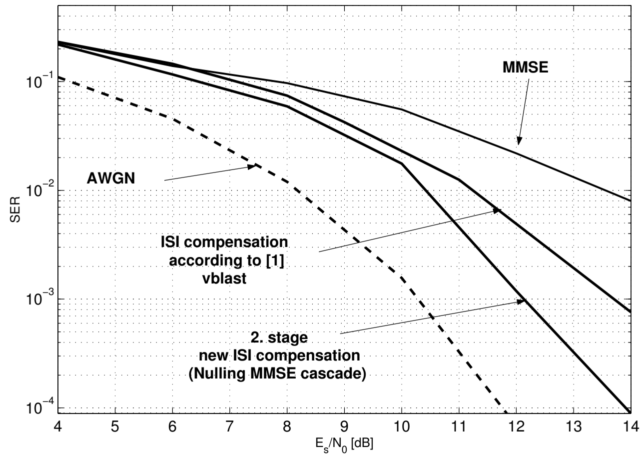


Fig. 10. SER performance compensation of different decoders (4QAM, block length 32)

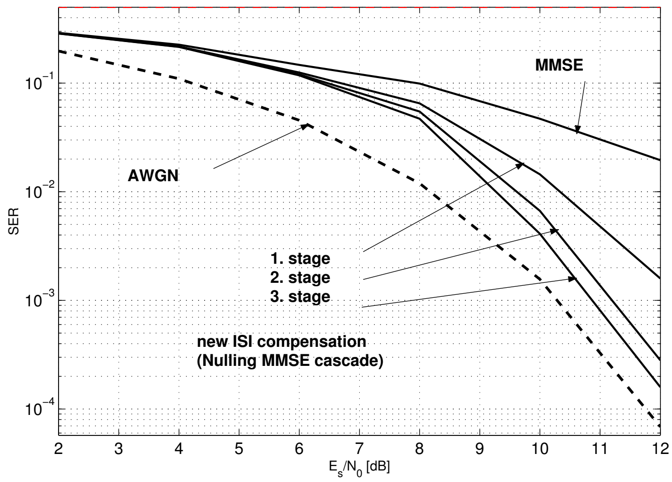


Fig. 11. SER performance (4QAM, block length 128)

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