Diversity and Spatial Multiplexing of MIMO Amplitude Detection Receivers

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***Abstract*— We consider nonlinear MIMO systems that use amplitude-only (envelope) receivers. Such systems are interesting for low-power, low-complexity applications like sensor networks. We study different modulation schemes and the respective diversity order obtained by the maximum likelihood detector in an uncoded system with perfect channel state information. We show that nonlinear MIMO amplitude receivers loose half the available receive diversity when the uncoded transmission rate increases above 1 bit/s/Hz, or when spatial multiplexing is performed.**

I. Introduction

Nonlinear MIMO systems have been introduced in [1] and [2]. They are MIMO systems that employ amplitude-only or phase-only receivers on each receiver antenna. The complex-valued received signal undergoes a nonlinear processing, where either the amplitude or the phase is extracted. This receiver structure is significantly simpler and less power-consuming compared to the standard I/Q receiver. Combining an amplitude or phase receiver with multiple antennas aims at taking advantage of both, the high data rate gains provided by the spatial dimension, and the low-complexity, low power-consumption properties of amplitude or phase detection. Such systems can be employed in sensor networks or similar applications, where strict power and cost constraints demand simple and efficient receivers. However, nonlinear MIMO systems have not been considered in literature yet and little is known regarding their fundamental properties.

The information theoretic limits of nonlinear MIMO systems were investigated in [1] for the case of perfect channel state information (CSI) at the receiver, and in [2] for noisy CSI. It was shown that such systems also exploit the spatial dimension, however in a different way compared to linear MIMO systems [3]. Specifically, an $N \times N$ nonlinear MIMO system achieves $\frac{N}{2}$ spatial multiplexing gain at high SNR, which means that half the degrees of freedom are lost due to dimensionality loss induced by the nonlinearity. However, nonlinear MIMO systems increase their spatial multiplexing gain beyond $\frac{N}{2}$ when more receive antennas are employed ($N_{\mathrm{R}} > N$), contrary to a linear MIMO system [1]. Nevertheless, the resulting spatial multiplexing gain never reaches the limit (optimum) set by the linear MIMO system.

The information theoretic results presented in [1] and [2] reveal the potential of these systems. In this paper, we deal with a more realistic system and consider practical modulation schemes and the performance of the maximum likelihood (ML) detector. We explore the diversity order of the ML detector for uncoded transmission and for different transmission rates—with and without spatial multiplexing. In this framework, we only consider MIMO amplitude detection receivers and assume that the receiver has perfect knowledge of the fading channel. We consider on-off keying (OOK) and its extension to multiple transmit antennas as the basic modulation scheme, since it is a straightforward choice for amplitude detection. Contrary to conventional OOK which is always used in conjunction with non-coherent communication in literature [4], in our case we deal with a coherent system, in the sense of *channel knowledge availability at the receiver*. Finally, we consider modulation across the transmit antennas in one channel use (no STBC) and investigate only receive diversity.

The paper is structured as follows. In Section II we present the system model. In Section III we consider the pairwise error probability between any two transmit symbols, and analyze the problem using two types of scalar detection problems. We show that the diversity order of these detection problems is 1 and $\frac{1}{2}$, respectively. In Section IV we show that the diversity order of the corresponding vector detection problems is upper bounded by N_{R} and $\frac{N_{\mathrm{R}}}{2}$, respectively. The diversity loss of the second detection problem occurs whenever the uncoded transmission rate increases beyond 1 bit/s/Hz, or when spatial multiplexing is performed. We present simulation results in Section V which verify our theoretical results.

Notation: Throughout the paper, bold-faced italic lower and upper case letters stand for vectors and matrices, respectively. b_i is the ith element of vector $\boldsymbol{b}$. $(\cdot)^{\mathrm{H}}$ denotes complex-conjugate transposition. $\boldsymbol{I}_N$ is the $N \times N$ identity matrix. The circularly symmetric complex Gaussian distributed random variable $\boldsymbol{x}$ with mean $\boldsymbol{m}$ and covariance matrix $\boldsymbol{R}$ is denoted by $\boldsymbol{x} \sim \mathcal{CN}(\boldsymbol{m}, \boldsymbol{R})$. $\delta_{i,j}$ is the Kronecker delta.

II. System Model

We consider the MIMO system depicted in Fig. 1, with N_{T} transmit and N_{R} receive antennas. The transmitter emits the signal $\boldsymbol{s} \in \mathbb{C}^{N_{\mathrm{T}}}$ over the stationary memoryless flat fading channel $\boldsymbol{H} \in \mathbb{C}^{N_{\mathrm{R}} \times N_{\mathrm{T}}}$, with tap-gain H_{ij} from the jth transmit to the ith receive antenna. We use a block fading model for the channel $\boldsymbol{H}$. The channel remains constant for some (coherence) period, long enough to allow for accurate estimation, and changes to an independent realization in the next block. We assume that the channel is *perfectly known* at the

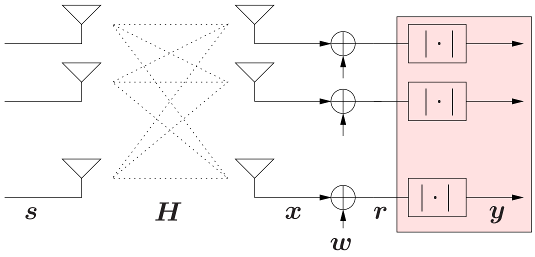

Fig. 1. MIMO System Reference Model

receiver. The received vector $\boldsymbol{x} \in \mathbb{C}^{N_\mathrm{R}}$ is perturbed by a zero-mean circularly symmetric Gaussian noise vector $\boldsymbol{w} \in \mathbb{C}^{N_\mathrm{R}}$, with autocorrelation function $\mathcal{E}[\boldsymbol{w}_k \boldsymbol{w}_l^\mathrm{H}] = \sigma_w^2 \boldsymbol{I}_{N_\mathrm{R}} \delta(k-l)$, where k and l are symbol instants. The envelope of the received signal is extracted on each antenna, producing the observation $y_i = |x_i + w_i|$ on the ith antenna. The detector has access only to $\boldsymbol{y}$. The common modulation scheme for the SISO case is OOK, i.e., the transmitter emits one of the symbols $s \in \{0, \sqrt{2E_b}\}$ with equal probability. OOK has been studied extensively in older publications for the AWGN channel [4].

III. Pairwise error probabilities

We are interested in the average pairwise error probability (PEP) between two equally likely symbols $\boldsymbol{s}^\mathrm{A}$ and $\boldsymbol{s}^\mathrm{B}$, where $\boldsymbol{s}^\mathrm{Y} = \left[s_1^\mathrm{Y}, \ldots, s_{N_\mathrm{T}}^\mathrm{Y}\right]^\mathrm{T}$, $\mathrm{Y} \in \{\mathrm{A}, \mathrm{B}\}$. Furthermore, $E_s = \|\boldsymbol{s}^\mathrm{Y}\|_2^2$ is the signal energy. For the time being we do not define the structure of the symbols. The corresponding received signal on the ith antenna is

$$y_i^\mathrm{Y} = \left|\sum_{j=1}^{N_\mathrm{T}} h_{ij} s_j^\mathrm{Y} + w_i\right| = \left|x_i^\mathrm{Y} + w_i\right|, \quad \mathrm{Y} \in \{\mathrm{A}, \mathrm{B}\}. \tag{1}$$

Since the receiver has perfect knowledge of $\boldsymbol{H}$, the distribution $\mathrm{f}(y_i|\boldsymbol{s} = \boldsymbol{s}^\mathrm{Y}) = \mathrm{f}(y_i|x_i^\mathrm{Y})$ is Ricean, given by

$$\mathrm{f}\left(y_i|\boldsymbol{s} = \boldsymbol{s}^\mathrm{Y}\right) = \frac{2y_i}{\sigma_w^2} e^{-\frac{y_i^2 + |x_i^\mathrm{Y}|^2}{\sigma_w^2}} \mathrm{I}_0\left(\frac{2y_i|x_i^\mathrm{Y}|}{\sigma_w^2}\right), \tag{2}$$

where $\mathrm{I}_0(\cdot)$ is the zeroth order modified Bessel function [5]. If $x_i^\mathrm{Y} = 0$ the distribution reduces to a Rayleigh distribution. As can be seen, x_i^Y appears in (2) only through its norm. Hence, the same distribution also describes the following detection problem

$$y_i^\mathrm{Y} = \left|\left|x_i^\mathrm{Y}\right| + w_i'\right|, \tag{3}$$

where $\boldsymbol{w}' \sim \mathcal{CN}\{\mathbf{0}, \sigma_w^2 \boldsymbol{I}\}$ has the same noise statistics as $\boldsymbol{w}$. That is, (1) and (3) are equivalent detection problems *with respect to the ML detector*. Eq. (3) is another manifestation that the phase of the received signal carries no information. We will consider detection problem (3) in the following.

The distribution of x_i^Y given $\boldsymbol{s}^\mathrm{Y}$ is Gaussian $x_i^\mathrm{Y} \sim \mathcal{CN}\{0, \sigma_h^2 E_s\}$, $\forall i$. Furthermore, x_i^Y is independent across different antennas

$$\mathcal{E}\left[x_i^\mathrm{Y} x_j^{\mathrm{Y},*}\right] = \delta_{i,j} \cdot \sigma_h^2 E_s \tag{4}$$

for the same symbol Y. This is in general not true among different symbols at the ith receive antenna

$$\mathcal{E}\left[x_i^\mathrm{A} x_i^{\mathrm{B},*}\right] = \sigma_h^2 \sum_{j=1}^{N_\mathrm{T}} s_j^\mathrm{A} s_j^{\mathrm{B},*}, \tag{5}$$

where the correlation depends on the structure of the transmitted signals.

Since the noise is independent between different receive antennas, the distribution of $\mathrm{f}(\boldsymbol{y}|\boldsymbol{x}^\mathrm{Y})$ can be factored like

$$\mathrm{f}\left(\boldsymbol{y}|\boldsymbol{x}^\mathrm{Y}\right) = \prod_{i=1}^{N_\mathrm{R}} \mathrm{f}\left(y_i|x_i^\mathrm{Y}\right). \tag{6}$$

Consequently, the log-likelihood ratio can be written as

$$\mathrm{LLR} = \ln \frac{\mathrm{f}\left(\boldsymbol{y}|\boldsymbol{x}^\mathrm{A}\right)}{\mathrm{f}\left(\boldsymbol{y}|\boldsymbol{x}^\mathrm{B}\right)} = \sum_{i=1}^{N_\mathrm{R}} \ln \frac{\mathrm{f}\left(y_i|x_i^\mathrm{A}\right)}{\mathrm{f}\left(y_i|x_i^\mathrm{B}\right)}, \tag{7}$$

which is the sum of the LLRs at each receive antenna. We will treat the detection problem at each antenna separately. The ML detector will optimally combine the likelihood ratios at each antenna before taking the final decision. The performance of the vector detection problem is closely connected to the detection problem at each antenna.

Let us now consider the scalar detection problem at the ith receive antenna. We distinguish between two cases: either $x_i^\mathrm{Y} = 0$, when $\boldsymbol{s}^\mathrm{Y} = \mathbf{0}$, or $x_i^\mathrm{Y} \neq 0$ when at least one entry of $\boldsymbol{s}^\mathrm{Y}$ is non-zero. This separation leads to two distinct scalar detection problems[1]:

$$\mathcal{D}_1^i: \quad \begin{cases} y_i^\mathrm{A} = |w_i'|, & \boldsymbol{s}^\mathrm{A} = \mathbf{0}, \\ y_i^\mathrm{B} = \left||x_i^\mathrm{B}| + w_i'\right|, & \boldsymbol{s}^\mathrm{B} \neq \mathbf{0}, x_i^\mathrm{B} \neq 0 \end{cases} \tag{8}$$

$$\mathcal{D}_2^i: \quad \begin{cases} y_i^\mathrm{A} = \left||x_i^\mathrm{A}| + w_i'\right|, & \boldsymbol{s}^\mathrm{A} \neq \mathbf{0}, x_i^\mathrm{A} \neq 0 \\ y_i^\mathrm{B} = \left||x_i^\mathrm{B}| + w_i'\right|, & \boldsymbol{s}^\mathrm{B} \neq \mathbf{0}, x_i^\mathrm{B} \neq 0 \end{cases} \tag{9}$$

The receiver has to decide between a Rayleigh and a Rice distribution for the two hypotheses in $\mathcal{D}_1^i$ (see (2)), and between two Rice distributions in $\mathcal{D}_2^i$. Evaluating the probability of error for $\mathcal{D}_1^i$ and $\mathcal{D}_2^i$ analytically is not possible to the best of our knowledge. An approximate analytic expression exists for $\mathcal{D}_1^i$ for the AWGN channel. For $\mathcal{D}_2^i$, we will use an auxiliary detection problem $\mathcal{L}_2^i$ whose error probability is a lower bound to $\mathcal{D}_2^i$.

A. Diversity order of $\mathcal{D}_1^i$

We will perform an approximate analysis for $\mathcal{D}_1^i$. Since in this case $\boldsymbol{s}^\mathrm{A} = \mathbf{0}$, we set $\|\boldsymbol{s}^\mathrm{B}\|_2 = \sqrt{2E_s}$ to keep the total energy of both symbols normalized. Consequently the variance of x_i^B is $2\sigma_h^2 E_s$. We use the normalized variable $\bar{x}_i^\mathrm{B} = x_i^\mathrm{B}/\sqrt{2E_s}$, where $\bar{x}_i^\mathrm{B} \sim \mathcal{CN}(0, \sigma_h^2)$. A decision threshold for the ML detector can not be computed analytically due to the non invertibility of the Bessel function. Furthermore, the optimum threshold is a function of the SNR. A good approximation for the optimum threshold is given in [4]. For high SNR, the threshold converges to the mean of the two

[1] We denote with $\mathcal{D}_1$ and $\mathcal{D}_2$ the respective vector detection problems.

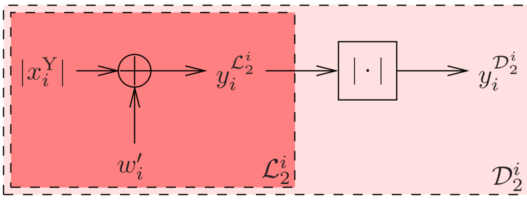

Fig. 2. Detection Problems $\mathcal{D}_2^i$ and $\mathcal{L}_2^i$.

noiseless received symbols, $|x_i^{\mathrm{B}}|/2$. An approximation to the error probability for high SNR is given in [6]

$$P_e^{\mathcal{D}_1^i}\left(\bar{x}_i^{\mathrm{B}}\right) \simeq \frac{1}{(4\pi|\bar{x}_i^{\mathrm{B}}|^2\gamma)^{1/4}} e^{-\frac{|\bar{x}_i^{\mathrm{B}}|^2}{2}\gamma}, \tag{10}$$

by substituting γ with $2|\bar{x}_i^{\mathrm{B}}|^2\gamma$ in order to take fading into account, and γ is the SNR per receive antenna $\gamma = \frac{E_s}{\sigma_w^2}$. Averaging over $|\bar{x}_i^{\mathrm{B}}|$, which incorporates the fading channel coefficients, we obtain the average probability of error

$$\bar{P}_e^{\mathcal{D}_1^i} \simeq \int_0^\infty \mathrm{f}_{|\bar{x}_i^{\mathrm{B}}|}\left(|\bar{x}_i^{\mathrm{B}}|\right) P_e^{\mathcal{D}_1}\left(\bar{x}_i^{\mathrm{B}}\right) \mathrm{d}\,|\bar{x}_i^{\mathrm{B}}| = \frac{\left(\frac{2}{\pi}\right)^{1/4}\Gamma\left(\frac{3}{4}\right)}{(\gamma(2+\gamma)^3)^{1/4}}, \tag{11}$$

where $\mathrm{f}_{|\bar{x}_i^{\mathrm{B}}|}(|\bar{x}_i^{\mathrm{B}}|)$ is a Rayleigh distribution and we used $\sigma_h^2 = 1$. The Taylor series expansion of (11) for high SNR yields

$$\bar{P}_e^{\mathcal{D}_1^i} \simeq \frac{\left(\frac{2}{\pi}\right)^{1/4}\Gamma\left(\frac{3}{4}\right)}{\gamma} + O\left(\frac{1}{\gamma^2}\right), \tag{12}$$

which proves that the diversity order of $\mathcal{D}_1^i$ is 1.

B. Diversity order of $\mathcal{D}_2^i$

Let us define the detection problem

$$\mathcal{L}_2^i: \quad \begin{cases} y_i^{\mathrm{A}} = |x_i^{\mathrm{A}}| + w_i', & \boldsymbol{s}^{\mathrm{A}} \neq \mathbf{0}, x_i^{\mathrm{A}} \neq 0 \\ y_i^{\mathrm{B}} = |x_i^{\mathrm{B}}| + w_i', & \boldsymbol{s}^{\mathrm{B}} \neq \mathbf{0}, x_i^{\mathrm{B}} \neq 0 \end{cases} \tag{13}$$

$\mathcal{D}_2^i$ is obtained from $\mathcal{L}_2^i$ by taking the norm of $y_i^{\mathcal{L}_2^i}$ as depicted in Fig. 2. Since $\mathcal{D}_2^i$ is a processed version of $\mathcal{L}_2^i$, the average probability of error of $\mathcal{L}_2^i$ is always lower or equal to that of $\mathcal{D}_2^i$.

$$\bar{P}_e^{\mathcal{L}_2^i} \leqslant \bar{P}_e^{\mathcal{D}_2^i}. \tag{14}$$

In fact processing will either lead to a sufficient statistic, and hence to identical error performance, or will destroy information, and thus deteriorate error performance.

From (14) we observe that the diversity order of $\mathcal{D}_2^i$ can not exceed the diversity order of $\mathcal{L}_2^i$ and hence $\mathcal{L}_2^i$ provides an upper bound. We can perform the same lower bounding to the error probability of $\mathcal{D}_1^i$ too, however this is not necessary since (12) provides the exact diversity order.

The probability of error of $\mathcal{L}_2^i$ is readily obtained, since it is a standard real-valued detection problem in Gaussian noise. We use again the normalized variables $\bar{x}_i^{\mathrm{Y}} = x_i^{\mathrm{Y}}/\sqrt{E_s}$, $\mathrm{Y} = \{\mathrm{A}, \mathrm{B}\}$, such that $\bar{x}_i^{\mathrm{Y}} \sim \mathcal{CN}(0, \sigma_h^2)$, and obtain [7]:

$$P_e^{\mathcal{L}_2^i}\left(\bar{x}_i^{\mathrm{A}}, \bar{x}_i^{\mathrm{B}}\right) = \mathrm{Q}\left(\left||\bar{x}_i^{\mathrm{A}}| - |\bar{x}_i^{\mathrm{B}}|\right|\sqrt{\frac{\gamma}{2}}\right). \tag{15}$$

At high SNR, the Rice distributions in $\mathcal{D}_2^i$ can be approximated by Gaussian distributions[2] (cf. [8]) and the probability of error of $\mathcal{D}_2^i$ is well approximated by (15). Hence, the bound is tight at high SNR. In this regime, the detection threshold is the mean of $|\bar{x}_i^{\mathrm{A}}|$ and $|\bar{x}_i^{\mathrm{B}}|$. Furthermore, the norm $|\bar{x}_i^{\mathrm{Y}}|$ is important, meaning that the distribution of the power of $\boldsymbol{s}^{\mathrm{Y}}$ across the transmit antennas does not change the probability of error, which is only affected by the total power of $\boldsymbol{s}^{\mathrm{Y}}$.

The average probability of error is computed by averaging over $\bar{x}_i^{\mathrm{A}}$ and $\bar{x}_i^{\mathrm{B}}$

$$\bar{P}_e^{\mathcal{L}_2^i} = \iint_0^\infty \mathrm{f}_{|\bar{x}_i^{\mathrm{A}}|,|\bar{x}_i^{\mathrm{B}}|}\left(|\bar{x}_i^{\mathrm{A}}|, |\bar{x}_i^{\mathrm{B}}|\right) P_e^{\mathcal{L}_2^i}\left(\bar{x}_i^{\mathrm{A}}, \bar{x}_i^{\mathrm{B}}\right) \mathrm{d}|\bar{x}_i^{\mathrm{A}}|\mathrm{d}\,|\bar{x}_i^{\mathrm{B}}|. \tag{16}$$

As we saw earlier, x_i^{A} and x_i^{B} are independent only if $\sum_{j=1}^{N_{\mathrm{T}}} s_j^{\mathrm{A}} s_j^{\mathrm{B},*} = 0$. In this case, their joint pdf is decomposed in the product of two Rayleigh distributions, and (16) can be computed analytically in (17) at the bottom of this page. The Taylor series expansion for high SNR yields

$$\bar{P}_e^{\mathcal{L}_2^i} \simeq \frac{1}{\sqrt{2}\gamma^{1/2}} + O\left(\frac{1}{\gamma^{3/2}}\right), \tag{18}$$

where we used $\sigma_h^2 = 1$. Thus, the diversity order of $\mathcal{L}_2^i$ is only $\frac{1}{2}$, when x_i^{A} and x_i^{B} are independent. Otherwise, the joint pdf is a two dimensional Rayleigh distribution, which is known analytically [9]. However, the average probability of error can not be computed analytically anymore. Simulations show that the diversity order in this case is $\frac{1}{2}$, meaning that the correlation does not affect the diversity.

IV. Diversity and Spatial Multiplexing

So far we considered the scalar detection problem at each antenna separately. However, the ML decoder jointly detects the transmitted signal using the whole received vector and (7). Unlike for linear MIMO receivers, coming up with a combination scheme, e.g. maximum ratio combining, is not possible in this case. This would significantly simplify the ML expression. However, we know that the ML, or any combination scheme for that matter, will achieve at most a diversity equal to the number of receive antennas (independent

[2]The Gaussian approximation is in this case tightly concentrated around the mean, and the negative part of the distribution can be neglected.

$$\bar{P}_e^{\mathcal{L}2} = \frac{\sqrt{8\gamma} + \gamma^2 - \gamma^{3/2}\sqrt{4+\gamma} - 2\left(-1 + \sqrt{\frac{\gamma}{4+\gamma}}\right)(4 + \sqrt{8+2\gamma}) + \gamma\left(6 - \sqrt{\frac{4\gamma}{4+\gamma}} + \sqrt{8+2\gamma}\right)}{2(2+\gamma)\sqrt{4+\gamma}(\sqrt{2} + \sqrt{4+\gamma})} \tag{17}$$

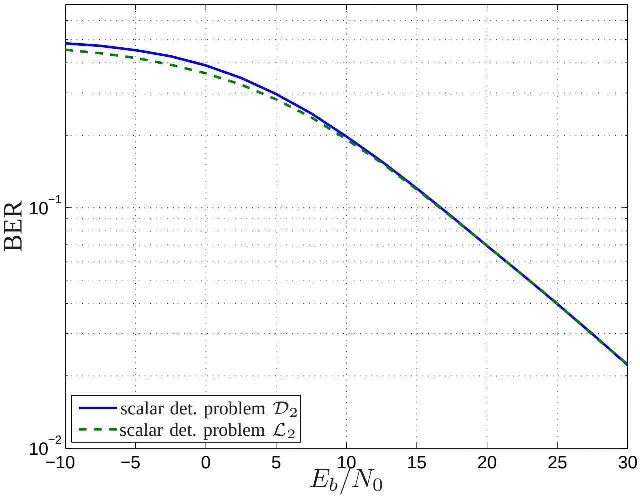

Fig. 3. BER of scalar $\mathcal{D}_2^i$ and $\mathcal{L}_2^i$.

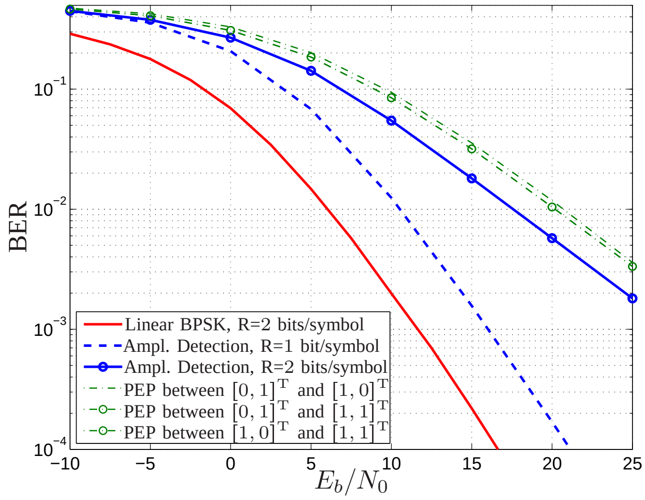

Fig. 4. BER plot for a 2×2 MIMO system.

branches) times the diversity order of the individual detection problems. This means, that for the vector detection problem $\mathcal{D}_1$, the ML detector will achieve a diversity of N_R, while for $\mathcal{D}_2$, the diversity will not be larger than $\frac{N_\mathrm{R}}{2}$. Notice that (7) is the same in the case of a $1 \times N$ receive diversity scheme. The achievable diversity in that case equals N times the diversity of the detection problem on each receive branch.

When it comes to evaluating the diversity of a modulation scheme with more than two symbols, the poorest PEPs dominate the performance. As we saw, the fundamental difference between problems $\mathcal{D}_1$ and $\mathcal{D}_2$ lies in the achieved diversity. However, $\mathcal{D}_1$ corresponds to $\boldsymbol{s}^\mathrm{A} = \mathbf{0}$, which means that the diversity of the PEP between any two other symbols, not equal to $\mathbf{0}$, will be at most $\frac{N_\mathrm{R}}{2}$. This occurs whenever we perform spatial multiplexing, since in this case we will have at least 2^{N_T} symbols, and $2^{N_\mathrm{T}} - 1$ of them will not be equal to the all-zero symbol. Hence, performing spatial multiplexing with a MIMO amplitude detection receiver reduces diversity to one half of the maximum available diversity. The only way to achieve the maximum diversity is by constraining the rate to only two symbols, $\boldsymbol{s}^\mathrm{A} = \mathbf{0}$ and $\boldsymbol{s}^\mathrm{B} \neq \mathbf{0}$, i.e. to a rate of 1 bit/s/Hz. In this case, the multiple antennas at the transmitter are of no use, and a single-input multiple-output system suffices.

Example: Let us consider the extension of OOK to multiple transmit antennas. We use 2^{N_T} symbols of the form $\boldsymbol{s}_i \in \{0, \sqrt{2E_s/N_\mathrm{T}}\}^{N_\mathrm{T}}$ (see the Appendix). The uncoded rate of the modulation alphabet is N_T bits/s/Hz, achieved through spatial multiplexing. The PEP between the all-zero symbol and any other symbol has the form of $\mathcal{D}_1$, and diversity order equal to N_R. However, the PEP of all other combinations of symbols has the form of $\mathcal{D}_2$. Those combinations where at least one symbol has a zero for every $j = 1, \dots, N_\mathrm{T}$, fulfill $\sum_{j=1}^{N_\mathrm{T}} s_j^\mathrm{A} s_j^{\mathrm{B},*} = 0$ and the corresponding upper bound has been computed in $\mathcal{L}_2$. The rest of the symbols, where at least for one j both symbols equal $\sqrt{2E_s/N_\mathrm{T}}$, do not fulfill the above condition. In any case, the achievable diversity can not be higher than $\frac{N_\mathrm{R}}{2}$. Since the diversity loss occurs whenever we have more than 2 symbols, there is no modulation alphabet with rate higher than 1 bit/s/Hz that can achieve full receive diversity.

V. Simulation Results

In this section we present BER results for amplitude detection MIMO receivers and compare them to a linear MIMO receiver that uses uncoded BPSK and ML detection. The reference system uses spatial multiplexing, hence transmitting N_T bits/s/Hz, and exploits the full diversity of the MIMO channel, equal to N_R. For the amplitude detection MIMO system, we use two modulation schemes: OOK without spatial multiplexing (rate $R = 1$ bit/s/Hz) and extended OOK with spatial multiplexing (rate $R = N_\mathrm{T}$ bits/s/Hz). The first modulation scheme consist of the symbols $[0, \dots 0]^\mathrm{T}$ and $[\sqrt{2E_s/N_\mathrm{T}}, \dots, \sqrt{2E_s/N_\mathrm{T}}]^\mathrm{T}$ and is equal to using only one transmit antenna and the symbols $\{0, \sqrt{2E_s}\}$. For a fair comparison we use the SNR per bit, given by $E_b/N_0 = \gamma/R$. The channel has i.i.d. Gaussian entries with unit variance.

Fig. 3 depicts the average error rate of the scalar detection problem $\mathcal{D}_2^i$ and the corresponding lower bound $\mathcal{L}_2^i$. The lower bound exhibits better error performance at low SNR, and the performance is practically identical beyond 15 dB. Simulation shows that the diversity order of $\mathcal{D}_2^i$ is in practice equal to that of the lower bound, namely $\frac{1}{2}$.

Fig. 4 depicts the average BER for a 2×2 system. Amplitude detection exhibits diversity equal to $N_\mathrm{R} = 2$ when $R = 1$, and diversity equal to $\frac{N_\mathrm{R}}{2} = 1$ when $R = 2$. In the case of spatial multiplexing, the diversity is destroyed by the pairs:

$$\{\boldsymbol{s}^\mathrm{A}, \boldsymbol{s}^\mathrm{B}\} \in \left\{ \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}, \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}, \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\} \right\},$$

which correspond to detection problems of type $\mathcal{D}_2$. The first pair exhibits the property that x_i^A and x_i^B are independent, while this is not the case for the other two pairs. The average PEPs of these pairs is also shown in Fig. 4. We see that

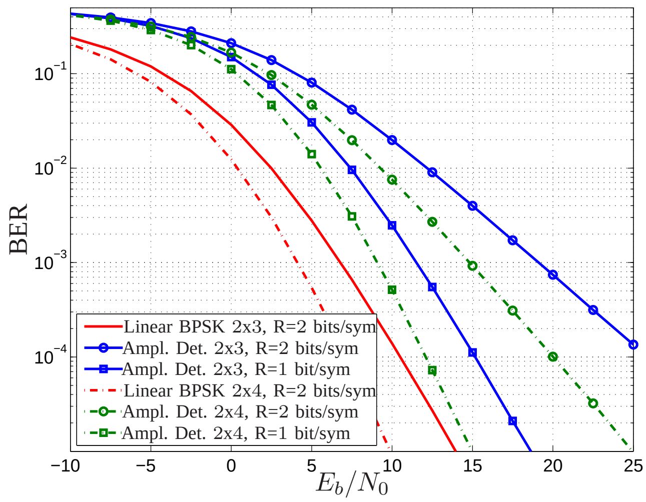

Fig. 5. BER plot for a 2×3 and 2×4 MIMO system.

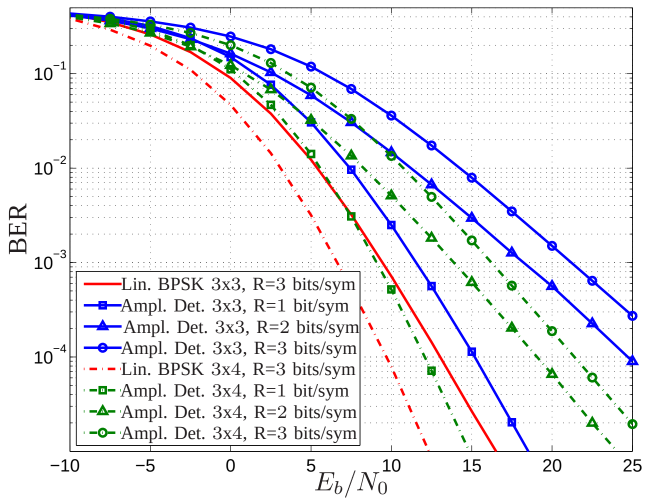

Fig. 6. BER plot for a 3×3 and 3×4 MIMO system.

correlation has no impact on the diversity order. The small SNR gain of the second and third pair is due to the higher transmitted energy of the respective symbols, compared to the first pair. Furthermore, the upper bound to the diversity of $\mathcal{D}_2$ is actually achieved in practice. In both cases, the performance is clearly inferior to the linear MIMO receiver, which however requires higher power consumption and hardware complexity.

Fig. 5 depicts the average BER of a 2×3 and 2×4 system, with and without spatial multiplexing. As predicted by the analysis of the PEPs, the diversity order with spatial multiplexing is only $\frac{N_R}{2} = 1.5$ and $\frac{N_R}{2} = 2$, respectively. Without spatial multiplexing, the full diversity of $N_R = 3$ and $N_R = 4$, respectively, is achieved. Once again, the upper bound to the diversity of $\mathcal{D}_2$ is achieved in practice for both systems.

Fig. 6 finally depicts the same comparison for a 3×3 and 3×4 system. This time, we have three levels of spatial multiplexing, yielding the uncoded rates $R = 1, 2, 3$ bits/s/Hz. The observation for the $R = 1$ and $R = 3$ cases are as expected—the diversity order equals N_R and $\frac{N_R}{2}$, respectively. The modulation for the $R = 2$ case has the form $\boldsymbol{s} = [a_1, a_1, a_2]^T$, $a_i \in \{0, \sqrt{2E_s/N_T}\}$, such that two independent data streams are transmitted. The resulting diversity order is again $\frac{N_R}{2}$. Note that the diversity loss occurs immediately when the rate increases from 1 to 2 bits/s/Hz. However, further increasing the spatial multiplexing and the rate have no impact on the diversity, which remains at $\frac{N_R}{2}$.

VI. Conclusions

We analyzed the performance of the ML detector of a MIMO receiver with amplitude-only detection. Contrary to linear MIMO systems, the nonlinear MIMO system looses half the available receive diversity when the uncoded rate is higher than 1 bit/s/Hz, or when spatial multiplexing is performed. This result does not imply that multiple transmit antennas are useless for such a system, but it rather shows that a performance penalty is induced by the nonlinearity of the receiver. Future works includes the extension to MIMO receivers with phase-only detection, where the same behavior can been observed.

Appendix
Average Energy of Extended OOK

Assume the set of equiprobable transmit symbols $\boldsymbol{s}_i \in \{0, \alpha\}^{N_T}$. We compute the average energy of the symbols. There is one all-zero symbol, N_T symbols with a single entry equal to α, $\binom{N_T}{2}$ symbols with two entries equal to α, etc. Hence, one symbol will have zero energy, N_T symbols will have energy equal to α^2, $\binom{N_T}{2}$ energy $2\alpha^2$, etc. Thus,

$$\begin{aligned} E_s &= \frac{1}{2^{N_T}} \sum_{k=1}^{N_T} \binom{N_T}{k} k\alpha^2 = \frac{N_T \alpha^2}{2^{N_T}} \sum_{k=0}^{N_T-1} \binom{N_T - 1}{k} \\ &= \frac{N_T \alpha^2}{2^{N_T}} 2^{N_T - 1} = \alpha^2 \frac{N_T}{2}, \end{aligned} \tag{19}$$

where we used identity (3.1.6) from [10]. Finally, we obtain $\alpha = \sqrt{2E_s/N_T}$.

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