Opportunistic Relaying for Wireless Multicasting

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***Abstract*—In this paper, we consider relay-aided wireless multicasting where one source transmits the same information to multiple destinations. We assume that there is a half-duplex regenerative relay station (RS) which may cooperatively assist the source. Our underlying problems are two-fold: when and how should the source and the RS cooperate to improve the multicast rate? Firstly, we show the required conditions for using the RS for wireless multicasting and propose opportunistic relaying strategies where the RS, when in use, forwards only partial information of the source. Thus, the proposed opportunistic relaying strategies lead to an efficient use of the RS. Secondly, we explain how we can maximise the multicast rate by optimising both the source transmission rate and the time sharing between the source and the RS. It is shown that the proposed opportunistic relaying strategies improve the multicast rate of one-hop (direct links only) wireless multicasting and they outperform other benchmarked relaying strategies.**

I. Introduction

In some communication scenarios, a source needs to simultaneously transmit the same information to multiple destinations. Such wireless multicasting may appear, for example, in wireless sensor networks or in cellular networks. In wireless sensor networks, a lead or gateway sensor node needs to simultaneously send the same control information to a group of sensor nodes. In cellular networks, a base station needs to simultaneously transmit the same multimedia streaming contents to a group of mobile stations [1]–[3].

Since the common information needs to be decoded reliably by all destinations, the multicast rate is defined by the weakest link between the source and the destinations [4]. In this work, we assume that there is a half-duplex node which acts as a regenerative relay station (RS) and may assist the source with transmitting the common information to the destinations. The underlying problems which attract us are two-fold: 1). In wireless multicasting, when should the regenerative RS cooperatively assist the source? 2). How should the source and RS cooperation for wireless multicasting be performed?

Related works: In [2] a decode-and-forward strategy[1] is applied in two-hop cooperative wireless multicasting with multiple relays. The optimal power allocation and relays location strategies are analysed. In [3] a different relaying strategy is used where each destination coherently combines only its received signals from the relays in the second hop. Hence, the relays use the same codebooks, but they can be different to the codebook used by the source. In [3], the transmission rates of the source and the relays are optimised, as well as the relays location, and it is shown that the best strategy is for the source and the relays to transmit with the same rate.

In [5], the concept of superposition coding [6] is applied in two-hop communication for three node case, i.e., one source, one RS and one destination. The source information is divided into basic and superimposed information. The RS decodes both information, but forwards only the superimposed information. The destination decodes the superimposed information from the RS and subtracts it from the direct (source-destination) link received signal. Hence, the basic information can be decoded interference free from the direct link received signal.

Contributions: We consider relay-aided wireless multicasting where one source transmits the same information to multiple destinations and there is one regenerative RS which may assist the source. Different to [2], [3], we are interested in opportunistic strategies where the RS is used only if its use improves the multicast rate. Similar to [3], we consider relaying strategies where the source and the RS may use different codebooks and, hence, the destinations do not perform coherent combining to the received signals from the source and the RS. Different to [3], the RS, when in use, forwards only partial information of the source and, hence, is used efficiently. Moreover, we maximise the multicast rate by optimising the source transmission rate and the time sharing between the source and the RS. In networks with mobile destinations, optimising the time sharing between the source and the RS to maximise the multicast rate is more practical than optimising the RS location by moving the RS to the best location.

In this work, we firstly propose opportunistic partial-information relaying (OPIR) strategy for wireless multicasting and show the opportunistic conditions to use the RS, namely, both source-RS and RS-destination links have to be better than the weakest direct link between the source and the destinations. Secondly, using OPIR strategy, we show how we can maximise the multicast rate by optimising the source transmission rate and the time sharing between the source and the RS. Afterwards, we propose opportunistic superposition coding (OSC), which is an extension of [5] to the case of wireless multicasting. Finally, after showing the drawbacks of OSC, we propose opportunistic partial-superimposed superposition coding (OPSSC) strategy which closes the performance gap between OSC and OPIR.

II. System Model

We consider wireless multicasting with one source and N destinations, where one regenerative half-duplex RS may assist

[1]Decode-and-forward strategy: RS decodes source information, re-encodes it using the same codebook as by the source and transmits it to destination, which coherently combines the received signals from source and RS.

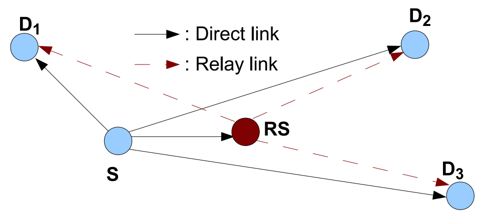

Fig. 1. Two-hop wireless multicasting: Source S transmits common information to destinations D_1, D_2 and D_3, and one RS may assist the source.

the source as shown in Figure 1 for an example scenario of $N = 3$. We assume that the source and the RS know the global channel state information (CSI) in the network. Our aim is to use the RS opportunistically to improve the multicast rate of one-hop (direct links only) wireless multicasting. The destinations which can decode the source information from their direct link do not need the RS. The RS only assists the destinations which cannot reliably decode the source information from their direct link. Both the source and the RS use different codebooks and the destinations which are assisted by the RS do not perform coherent combining.

Let $i \in \mathcal{I}$ denotes the index of a destination, with $\mathcal{I} = \{1, \cdots, N\}$ the set of destinations indices. Let also h_{sd_i}, h_{sr} and h_{rd_i} denote the channel gains of source-destination i link, source-RS link and RS-destination i link, respectively. In the first hop, the source multicasts its information to destinations and RS. The received signal at destination i is given by

$$y_{\mathrm{sd}_i} = h_{\mathrm{sd}_i} x_{\mathrm{s}} + z_{\mathrm{sd}_i}, \tag{1}$$

and the received signal at the RS is given by

$$y_{\mathrm{sr}} = h_{\mathrm{sr}} x_{\mathrm{s}} + z_{\mathrm{sr}}, \tag{2}$$

with $x_{\mathrm{s}} \sim \mathcal{CN}(0, \sigma_{\mathrm{s}}^2)$ the source transmit symbol, $z_{\mathrm{sd}_i} \sim \mathcal{CN}(0, \sigma_{\mathrm{z}}^2)$ and $z_{\mathrm{sr}} \sim \mathcal{CN}(0, \sigma_{\mathrm{z}}^2)$ the additive white Gaussian noise (AWGN) at destination i and at RS in the first hop, respectively, and $\mathcal{CN}(0, v)$ the notation for circularly symmetric zero-mean complex normal distribution with variance v.

The RS, when in use, transmits to destinations in the second hop. The received signal at destination i is given by

$$y_{\mathrm{rd}_i} = h_{\mathrm{rd}_i} x_{\mathrm{r}} + z_{\mathrm{rd}_i}, \tag{3}$$

with $x_{\mathrm{r}} \sim \mathcal{CN}(0, \sigma_{\mathrm{r}}^2)$ the RS transmit symbol and $z_{\mathrm{rd}_i} \sim \mathcal{CN}(0, \sigma_{\mathrm{z}}^2)$ the AWGN at destination i in the second hop.

The signal to noise ratios (SNRs) of source-destination i link, source-RS link and RS-destination i link are given by

$$\gamma_{\mathrm{sd}_i} = \frac{\sigma_{\mathrm{s}}^2}{\sigma_{\mathrm{z}}^2} \left| h_{\mathrm{sd}_i} \right|^2, \gamma_{\mathrm{sr}} = \frac{\sigma_{\mathrm{s}}^2}{\sigma_{\mathrm{z}}^2} \left| h_{\mathrm{sr}} \right|^2 \text{ and } \gamma_{\mathrm{rd}_i} = \frac{\sigma_{\mathrm{r}}^2}{\sigma_{\mathrm{z}}^2} \left| h_{\mathrm{rd}_i} \right|^2, \tag{4}$$

respectively. We assume a single channel with a normalized bandwidth of 1 Hz and a link with SNR of γ can send up to

$$C(\gamma) = \log_2(1 + \gamma) \text{ [bit/s]}. \tag{5}$$

For notational simplicity, without loss of generality, let us assume in the following that

$$\gamma_{\mathrm{sd}_1} \leq \gamma_{\mathrm{sd}_2} \leq \cdots \leq \gamma_{\mathrm{sd}_N}, \tag{6}$$

such that the weakest link is the source-destination 1 link. This assumption still holds even when the channels changes, by simply renumbering the destinations such that (6) holds.

III. Opportunistic Cooperative Relaying

In this section, we explain three opportunistic cooperative relaying strategies for wireless multicasting. We first explain OPIR strategy, where we show the conditions for using the RS and the way to maximise the multicast rate. Afterwards, we explain OSC strategy and OPSSC strategy.

A. Opportunistic Partial-Information Relaying

By utilizing list decoding and random binning, we proof the achievability of OPIR strategy for wireless multicasting. Nonetheless, in order to have a better flow of exposition for explaining OPIR strategy, we provide the sketch of proof of the achievability in the Appendix. The encoding process at the source and the RS, and the decoding process at the RS and the destinations are also explained there.

1) When and how to use the RS: Let N_{b}, T_{s} and T_{r} denote the number of bits that can be delivered in the network, the source transmit time and the RS transmit time, respectively. Since in wireless multicasting the multicast rate is defined by the weakest direct link, we have to first consider the destination with the weakest direct link, i.e., $i = 1$, such that

$$N_{\mathrm{b}} \leq T_{\mathrm{s}} I_{\mathrm{sr}}, \tag{7}$$

$$N_{\mathrm{b}} \leq T_{\mathrm{s}} I_{\mathrm{sd}_1} + T_{\mathrm{r}} I_{\mathrm{rd}_1}, \tag{8}$$

with $I_{\mathrm{sr}} = C(\gamma_{\mathrm{sr}}), I_{\mathrm{sd}_1} = C\left(\gamma_{\mathrm{sd}_1}\right)$ and $I_{\mathrm{rd}_1} = C(\gamma_{\mathrm{rd}_1})$. We have (7) since the RS has to be able to decode the source information. We have (8) since destination 1 receives from the source during T_{s} in the first hop and from the RS during T_{r} in the second hop.

Theorem 1: The RS will be used only when

$$\min\left(\gamma_{\mathrm{sr}}, \gamma_{\mathrm{rd}_1}\right) > \gamma_{\mathrm{sd}_1}. \tag{9}$$

Proof: From (7) and (8) we have

$$T_{\mathrm{r}} = \frac{T_{\mathrm{s}}\left(I_{\mathrm{sr}} - I_{\mathrm{sd}_1}\right)}{I_{\mathrm{rd}_1}}. \tag{10}$$

Since the RS can only transmit with positive rate, we have

$$T_{\mathrm{s}}\left(I_{\mathrm{sr}} - I_{\mathrm{sd}_1}\right) > 0, \tag{11}$$

which leads to

$$\gamma_{\mathrm{sr}} > \gamma_{\mathrm{sd}_1}. \tag{12}$$

The RS cooperation improves the direct link rate when

$$\frac{T_{\mathrm{s}} I_{\mathrm{sr}}}{T_{\mathrm{s}} + T_{\mathrm{r}}} > \frac{T_{\mathrm{s}} I_{\mathrm{sd}_1}}{T_{\mathrm{s}}}. \tag{13}$$

By inserting (10) into (13) we have

$$\gamma_{\mathrm{rd}_1} > \gamma_{\mathrm{sd}_1}. \tag{14}$$

From (12) and (14) we have (9). ■

Equation (10) shows the transmission time required by the RS to transmit $T_{\mathrm{s}}\left(I_{\mathrm{sr}} - I_{\mathrm{sd}_1}\right)$ to destination 1 through RS-destination 1 link which has a capacity of I_{rd_1}. Hence, the RS only forwards partial information of the source $T_{\mathrm{s}}\left(I_{\mathrm{sr}} - I_{\mathrm{sd}_1}\right)$ instead of the complete information $T_{\mathrm{s}} I_{\mathrm{sr}}$.

2) Multicast Rate Maximisation: In the following, we explain how we can maximise the multicast rate by optimising the source transmission rate R_{s} and the time sharing between the source and the RS, i.e., T_{s} and T_{r}. We note that, given (7), destinations i with $I_{\mathrm{sd}_i} \geq I_{\mathrm{sr}}$ will be able to decode the source information. Hence, the RS is used to assist only destinations $m \in \mathcal{M} \subseteq \mathcal{I}, \mathcal{M} = \{m | I_{\mathrm{sd}_m} < I_{\mathrm{sr}}\}$. In general, for multiple destinations $m \in \mathcal{M}$ we have inequalities given by

$$N_{\mathrm{b}} \leq T_{\mathrm{s}} I_{\mathrm{sd}_m} + T_{\mathrm{r}} I_{\mathrm{rd}_m}, \forall m \in \mathcal{M}, \tag{15}$$

with $I_{\mathrm{sd}_m} = C(\gamma_{\mathrm{sd}_m})$ and $I_{\mathrm{rd}_m} = C(\gamma_{\mathrm{rd}_m})$.

Without loss of generality, in order to ease the explanation and for notation simplicity, we rewrite (7) into

$$N_{\mathrm{b}} \leq T_{\mathrm{s}} I_{\mathrm{sd}_0} + T_{\mathrm{r}} I_{\mathrm{rd}_0}, \tag{16}$$

with $I_{\mathrm{sd}_0} = I_{\mathrm{sr}}$ and $I_{\mathrm{rd}_0} = 0$. We use Figure 2, which provides an example when there are multiple inequalities, to show the reason why we need to optimise the source transmission rate and the time sharing between the source and the RS, and how we optimise them to maximise the multicast rate. Clearly, given all inequalities, we first have to find the feasible region of N_b. All solid lines in Fig. 2 are the lines of all inequalities when they become equalities. The bold solid lines are the lines from four inequalities which define the feasible region, with the light green line the inequality of destination 1 (which is assumed to be the weakest), the brown line the inequality of destination k, the dark green line the inequality of destination l and the red line the inequality of the RS. When the source transmits with rate $R_{\mathrm{s}} = I_{\mathrm{sr}}$, i.e., the maximum rate that the RS can decode reliably, the lowest N_b is given by point C which is the N_b received by destination l. This is even less than the N_b of one-hop wireless multicasting with direct link only transmission ($T_{\mathrm{r}} = 0$ in Fig. 2). Therefore, when the RS is used, the source has to optimise R_{s} in order to maximise N_b. Hence, we have to find the maximum N_b which is obtained from the optimum crossing point of two inequalities in the feasible region. Point B in Fig. 2 is the intersection of two inequalities of destinations k and l (in general case $\{k, l\} \in \{0, \mathcal{M}\}$), which gives us the optimum time sharing between the source and the RS and, hence, the maximum N_b. The next step is now to find the optimum R_{s} which leads to point B. By equating the inequalities of destinations k and l, we have

$$T_{\mathrm{r}} = \frac{T_{\mathrm{s}} \left(I_{\mathrm{sd}_k} - I_{\mathrm{sd}_l}\right)}{I_{\mathrm{rd}_l} - I_{\mathrm{rd}_k}}. \tag{17}$$

Since the source transmits with rate R_{s}, we have

$$N_{\mathrm{b}} \leq T_{\mathrm{s}} R_{\mathrm{s}}. \tag{18}$$

By equating (18) with the inequality of either destination l or k, for example here we use destination k, we have

$$T_{\mathrm{r}} = \frac{T_{\mathrm{s}} \left(R_{\mathrm{s}} - I_{\mathrm{sd}_k}\right)}{I_{\mathrm{rd}_k}}, \tag{19}$$

and both (17) and (19) lead to

$$R_{\mathrm{s}} = \frac{\left(I_{\mathrm{sd}_k} - I_{\mathrm{sd}_l}\right) I_{\mathrm{rd}_k}}{I_{\mathrm{rd}_l} - I_{\mathrm{rd}_k}} + I_{\mathrm{sd}_k}. \tag{20}$$

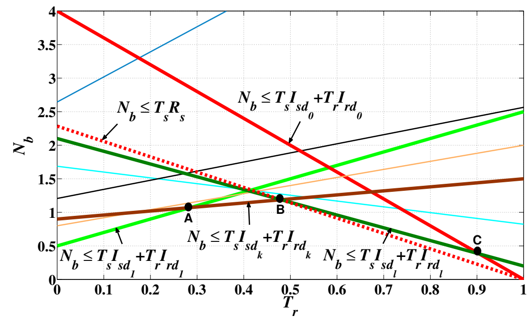

Fig. 2. Example of optimum decision region with $T_{\mathrm{s}} + T_{\mathrm{r}} = 1$

The bold dashed red line in Fig. 2 shows the line of (18) when it becomes an equality and the source transmits with rate R_{s} as given by (20), which maximises N_{b} at point B. Hence, being able to find the optimum time sharing between the source and the RS allows us to find the optimum source transmission rate R_{s} which maximises the multicast rate.

B. Opportunistic Superposition Coding

In this subsection, we explain OSC strategy which is a generalisation of superposition coding relaying of [5] to wireless multicasting with multiple destinations. Using OSC, once (9) is fulfilled, the source transmits

$$x_{\mathrm{s}} = \sqrt{1 - \alpha} x_{\mathrm{s_B}} + \sqrt{\alpha} x_{\mathrm{s_{SI}}}, \tag{21}$$

with $x_{\mathrm{s_B}}$ and $x_{\mathrm{s_{SI}}}$ the source basic and superimposed information, respectively, and α the power allocation coefficient for the superimposed information. When in use, the RS first decodes $x_{\mathrm{s_B}}$ by treating $x_{\mathrm{s_{SI}}}$ as noise. Then, it subtracts $x_{\mathrm{s_B}}$ from y_{sr} and decodes $x_{\mathrm{s_{SI}}}$ interference free. Using OSC strategy, the source transmits with rate R_{s} as given by (20), which relates to an SNR of

$$\gamma_{\mathrm{s}} = 2^{R_{\mathrm{s}}} - 1, \tag{22}$$

where now $R_{\mathrm{s}} = R_{\mathrm{s_B}} + R_{\mathrm{s_{SI}}}$. The RS and all nodes i with $I_{\mathrm{sd}_i} \geq R_{\mathrm{s}}$ are able to decode $x_{\mathrm{s_B}}$ with rate

$$R_{\mathrm{s_{B_r}}} = C\left(\frac{(1-\alpha)\gamma_{\mathrm{s}}}{\alpha\gamma_{\mathrm{s}} + 1}\right), \tag{23}$$

and $x_{\mathrm{s_{SI}}}$ with rate

$$R_{\mathrm{s_{SI}}} = C\left(\alpha\gamma_{\mathrm{s}}\right). \tag{24}$$

The RS re-encodes $x_{\mathrm{s_{SI}}}$ into x_{r} and transmits x_{r} to destinations $m \in \mathcal{M}$. Destination m decodes x_{r} from y_{rd_m} to obtain $x_{\mathrm{s_{SI}}}$. Afterwards, it subtracts $x_{\mathrm{s_{SI}}}$ from y_{sd_m} and decodes $x_{\mathrm{s_B}}$ interference free. Assuming that (6) holds and destinations $m \in \mathcal{M}$ are able to decode $x_{\mathrm{s_{SI}}}$ reliably from the RS and subtract $x_{\mathrm{s_{SI}}}$ from their direct link received signal, the basic information at destinations $m \in \mathcal{M}$ can be decoded with rate

$$R_{\mathrm{s_{B_m}}} = I_{\mathrm{s_{B_1}}} = C\left((1-\alpha)\gamma_{\mathrm{sd_1}}\right), \forall m \in \mathcal{M}. \tag{25}$$

The basic information has to be decoded reliably by the RS and the destinations from their direct link received signal, hence,

$$R_{\mathrm{s_B}} = \max_{0 \leq \alpha \leq 1} \min\left(R_{\mathrm{s_{B_r}}}, I_{\mathrm{s_{B_1}}}\right), \tag{26}$$

which leads to

$$\alpha = \min\left(1, \frac{\gamma_{\mathrm{s}} - \gamma_{\mathrm{sd}_1}}{\gamma_{\mathrm{s}}\gamma_{\mathrm{sd}_1}}\right). \quad (27)$$

C. Opportunistic Partial-Superimposed Superposition Coding

Using OSC strategy, the destinations only exploit some portions of their direct link received signal power, namely, $(1-\alpha)$ which is used to decode the basic information. The idea of OPSSC strategy is to exploit the α portions of the direct link received signal power and try to decode the superimposed information. For a given α in (27), destination $m \in \mathcal{M}$ could reliably decode the superimposed information from the direct link received signal with rate

$$I_{\mathrm{s_{SI}}_m} = C\left(\frac{\alpha\gamma_{\mathrm{sd}_m}}{(1-\alpha)\gamma_{\mathrm{sd}_m} + 1}\right). \quad (28)$$

Since $I_{\mathrm{s_{SI}}_m} < R_{\mathrm{s_{SI}}}$, the RS will help destinations $m \in \mathcal{M}$ to obtain the source superimposed information by forwarding the partial-superimposed information with rate $\left(R_{\mathrm{s_{SI}}} - I_{\mathrm{s_{SI}}_1}\right)$. Since destinations m, $\forall m > 1$, have better direct link observations than destination 1, they simply need to decode the partial-superimposed information from the RS with rate $\left(R_{\mathrm{s_{SI}}} - I_{\mathrm{s_{SI}}_m}\right)$. Hence, the encoding and decoding schemes explained in the Appendix for OPIR strategy should be applied. The difference is that OPSSC strategy only considers the source superimposed information with rate $R_{\mathrm{s_{SI}}}$, while OPIR strategy considers the complete source information with rate R_{s}. Thus, OPSSC requires lower complexity codebook at the RS. Nonetheless, for wireless multicasting with $N \geq 2$, OPSSC strategy is inferior to OPIR strategy since the rate of the basic information, which is decoded by all destinations from their direct link received signals, is still defined by the destination with the weakest direct link.

For OPSSC strategy, using the same encoding and decoding as explained in the Appendix, the destinations $m \in \mathcal{M}$ find the list of possible bin indices from the RS (see Appendix). However, the RS can also assist all destinations $m \in \mathcal{M}$ by ensuring that they can find the true bin index, i.e., all destinations $m \in \mathcal{M}$ decode with rate $\left(R_{\mathrm{s_{SI}}} - I_{\mathrm{s_{SI}}_1}\right)$. This will lower the decoding complexity at the destinations with the penalty of having longer RS transmission time T_{r}.

IV. Sum Rate Expressions

In the following, we provide the sum rate expressions of wireless multicasting with different transmission strategies. For benchmarking, we consider the direct extension of two different relaying strategies, namely, two-hop and decode-and-forward, to the case of wireless multicasting.

A. OPIR Strategy

The sum rate of OPIR strategy is given by

$$R_{\mathrm{opir}}^{\mathrm{sum}} = NR = \frac{NR_{\mathrm{s}}}{1 + \max_{m\in\mathcal{M}}\left(\frac{R_{\mathrm{s}} - I_{\mathrm{sd}_m}}{I_{\mathrm{rd}_m}}\right)}, \quad (29)$$

with R the achievable rate of OPIR strategy derived in Appendix. The factor N in the numerator is due to N destinations receive the same information from the source.

B. OSC Strategy

The sum rate of OSC strategy is given by

$$R_{\mathrm{osc}}^{\mathrm{sum}} = \frac{NR_{\mathrm{s}}}{1 + \max_{m\in\mathcal{M}}\left(\frac{R_{\mathrm{s_{SI}}}}{I_{\mathrm{rd}_m}}\right)}. \quad (30)$$

The highest ratio between $R_{\mathrm{s_{SI}}}$ and I_{rd_m} for all $m \in \mathcal{M}$ defines the additional required resources since the RS has to simultaneously deliver $x_{\mathrm{s_{SI}}}$ with rate $R_{\mathrm{s_{SI}}}$.

C. OPSSC Strategy

The sum rate of OPSSC strategy is given by

$$R_{\mathrm{opssc}}^{\mathrm{sum}} = \frac{NR_{\mathrm{s}}}{1 + \max_{m\in\mathcal{M}}\left(\frac{R_{\mathrm{s_{SI}}} - I_{\mathrm{s_{SI}}_m}}{I_{\mathrm{rd}_m}}\right)}. \quad (31)$$

If the RS assists destinations $m \in \mathcal{M}$ such that they can decode the true bin index from the RS, the sum rate becomes

$$R_{\mathrm{opssc_{low}}}^{\mathrm{sum}} = \frac{NR_{\mathrm{s}}}{1 + \max_{m\in\mathcal{M}}\left(\frac{R_{\mathrm{s_{SI}}} - I_{\mathrm{s_{SI}}_1}}{I_{\mathrm{rd}_m}}\right)}. \quad (32)$$

D. Two-Hop Strategy without Direct Link

Using two-hop strategy, the direct link will not be used [5]. Therefore, the RS decodes the complete source information and forwards it to the destinations. It is assumed that the RS knows the global CSI such that it can optimise its transmission time. The sum rate of two-hop strategy is given by

$$R_{\mathrm{th}}^{\mathrm{sum}} = \frac{NI_{\mathrm{sr}}}{1 + \max_{i\in\mathcal{I}}\left(\frac{I_{\mathrm{sr}}}{I_{\mathrm{rd}_i}}\right)}. \quad (33)$$

E. Decode-and-Forward Strategy with Direct Link

The sum rate of decode-and-forward strategy is given by

$$R_{\mathrm{df}}^{\mathrm{sum}} = \frac{N}{2}\min\left(I_{\mathrm{sr}}, \min_{i\in\mathcal{I}} C\left(\gamma_{\mathrm{sd}_i} + \gamma_{\mathrm{rd}_i}\right)\right). \quad (34)$$

The pre-log factor $\frac{1}{2}$ is due to both the source transmission time and the RS transmission time are fixed and equal.

F. Direct Link

The sum rate of direct link only transmission is given by

$$R_{\mathrm{direct}}^{\mathrm{sum}} = N\min_{i\in\mathcal{I}} I_{\mathrm{sd}_i}. \quad (35)$$

V. Simulation Results

We consider a scenario as shown in Fig. 3. The source and destination 1 have unit distance, i.e., $d_{\mathrm{sd}_1} = 1$. The other destinations are located between the source and destination 1 with equal distance, i.e., $d_{\mathrm{sd}_i} = \frac{(N-i)+1}{N}, \forall i \in \mathcal{I}$. The RS is located between the source and destination 1 with distance d_{sr} from the source and d_{rd_i} from node i, with $d_{\mathrm{rd}_i} = |d_{\mathrm{sd}_i} - d_{\mathrm{sr}}|$. The channel gains are modeled as $h_{\mathrm{sd}_i} = \frac{\xi}{d_{\mathrm{sd}_i}^{(\beta/2)}}$, $h_{\mathrm{sr}} = \frac{\xi}{d_{\mathrm{sr}}^{(\beta/2)}}$ and $h_{\mathrm{rd}_i} = \frac{\xi}{d_{\mathrm{rd}_i}^{(\beta/2)}}$, with independent and identically distribution $\xi \sim \mathcal{CN}(0,1)$, where $\beta = 3$ is the path loss exponent. We set $\sigma_{\mathrm{s}}^2 = 1$, $\sigma_{\mathrm{r}}^2 = 1$ and $\sigma_{\mathrm{z}}^2 = 1$. We vary d_{sr} and compute the corresponding sum rate.

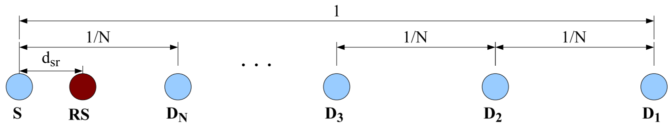

Fig. 3. Simulation set up

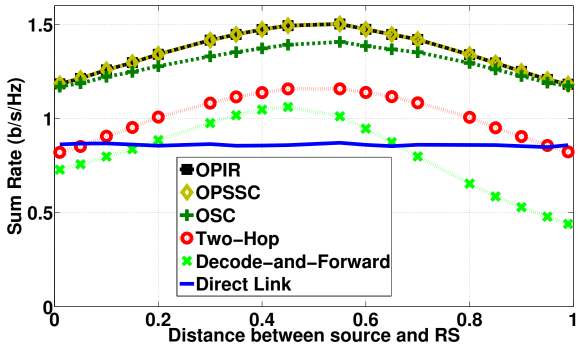

Fig. 4. Sum rate performance for $N = 1$

Figure 4 and 5 show the sum rate performance of wireless multicasting with $N = 1$ and $N = 4$, respectively. OPIR, OSC and OPSSC strategies improve the sum rate of one-hop wireless multicasting with direct link only transmission, and outperform other relaying strategies. For $N = 1$, OPSSC strategy performs similar to OPIR strategy, and it outperforms OSC strategy. This happens since OPSSC strategy also exploits the α portion of the direct link received signal power. For $N = 4$, OPSSC strategy is slightly worse than OPIR strategy since the rate of the basic information, which is decoded by all destinations from their direct link received signal, is defined by the weakest destination. It can be seen that OPSSC strategy with low-complexity also outperforms OSC strategy. In all cases, OSC is inferior to OPIR, and OPSSC is able to close the performance gap between OSC and OPIR.

VI. Conclusion

In this paper, we consider relay-aided wireless multicasting where one source transmits the same information to multiple destinations and there is one regenerative half-duplex RS which may assist the source. We propose opportunistic cooperative relaying strategies, namely, OPIR, OSC and OPSSC strategies. When in use, the RS forwards only partial information of the source to the destinations. Hence, it is used efficiently. We explain when and how to use the RS opportunistically, and how to maximise the multicast rate by optimising the source transmission rate and the time sharing between the source and the RS. It is shown that the proposed opportunistic cooperative relaying strategies improve the multicast rate of one-hop wireless multicasting with direct link only transmission and outperform other relaying strategies.

VII. Appendix

The sketch of proof of the achievability of OPIR strategy is derived using the same argument as in [7] (see Appendix

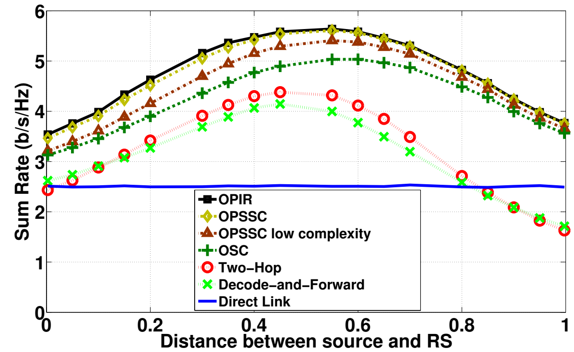

Fig. 5. Sum rate performance for $N = 4$

I and II of [7]) and [8]. In the following, we assume that $T_{\mathrm{s}} + T_{\mathrm{r}} = 1$.

A. Discrete Memoryless Channels

1) Outline: Assume that the source wants to send an i.i.d. message $w, w \in \{1, \cdots, 2^{nR}\}$, to the destinations, with n the code length which is assumed to be sufficiently large. We have

$$R = T_{\mathrm{s}} R_{\mathrm{s}} \le T_{\mathrm{s}} I_{\mathrm{sr}}, \tag{36}$$

$$R \le T_{\mathrm{s}} I_{\mathrm{sd}_m} + (1 - T_{\mathrm{s}}) \left(T_{\mathrm{s}} \left(R_{\mathrm{s}} - I_{\mathrm{sd}_m} \right) \right), \forall m \in \mathcal{M}. \tag{37}$$

Equation (36) guarantees that the RS will be able to decode w. Equation (37) comes due to the fact that destination m has to be able to decode w after receiving from the source in the first hop, i.e., $(T_{\mathrm{s}} I_{\mathrm{sd}_m})$, and from the RS in the second hop, i.e., $T_{\mathrm{r}} \left(T_{\mathrm{s}} \left(R_{\mathrm{s}} - I_{\mathrm{sd}_m} \right) \right)$, with $T_{\mathrm{r}} = (1 - T_{\mathrm{s}})$.

- In the first phase, the source transmits w. The RS can decode w reliably since $I_{\mathrm{sr}} \ge R_{\mathrm{s}}$. Destinations $m \in \mathcal{M}$ could not decode w since $I_{\mathrm{sd}_m} < R_{\mathrm{s}}$.
- In the second phase, the RS distributes the source messages into a set of bins B_b, with $b \in \left\{1, \cdots, 2^{n(1-T_{\mathrm{s}})\left(T_{\mathrm{s}}\left(R_{\mathrm{s}} - I_{\mathrm{sd}_1}\right)\right)}\right\}$ the bin index, and transmits to the destinations the bin index b where the message w lies. Since in the first hop, destination m has observed $T_{\mathrm{s}} I_{\mathrm{sd}_m} \ge T_{\mathrm{s}} I_{\mathrm{sd}_1}$, it has side information $T_{\mathrm{s}} \left(I_{\mathrm{sd}_m} - I_{\mathrm{sd}_1} \right)$. Thus, within the RS transmission phase $(1 - T_{\mathrm{s}})$, destination m has to observe

$$T_{\mathrm{s}} \left((R_{\mathrm{s}} - I_{\mathrm{sd}_1}) - (I_{\mathrm{sd}_m} - I_{\mathrm{sd}_1}) \right) = T_{\mathrm{s}} (R_{\mathrm{s}} - I_{\mathrm{sd}_m}). \tag{38}$$

2) Random Code Generation: Fix $p(\mathbf{x}_{\mathrm{s}})$ and $p(\mathbf{x}_{\mathrm{r}})$.

- Source node
  - Generate $2^{nT_{\mathrm{s}} R_{\mathrm{s}}}$ i.i.d. length-nT_{s} sequence $\mathbf{x}_{\mathrm{s}}$ each with probability

$$p(\mathbf{x}_{\mathrm{s}}) = \prod_{j}^{nT_{\mathrm{s}}} p(x_{\mathrm{s}j}). \tag{39}$$

  Label them as $\mathbf{x}_{\mathrm{s}}(w)$, $w \in \{1, \cdots, 2^{nT_{\mathrm{s}} R_{\mathrm{s}}}\}$.
- RS

– Generate $2^{n(1-T_\mathrm{s})\left(T_\mathrm{s}\left(R_\mathrm{s}-I_{\mathrm{sd}_1}\right)\right)}$ i.i.d. length-$n(1-T_\mathrm{s})$ sequence $\mathbf{x}_\mathrm{r}$ each with probability

$$p\left(\mathbf{x}_\mathrm{r}\right)=\prod_{j}^{n(1-T_\mathrm{s})} p\left(x_{\mathrm{r}j}\right). \tag{40}$$

Label them as $\mathbf{x}_\mathrm{r}\left(b\right), b \in \left\{1,\cdots,2^{n(1-T_\mathrm{s})\left(T_\mathrm{s}\left(R_\mathrm{s}-I_{\mathrm{sd}_1}\right)\right)}\right\}$.

– Randomly partition $2^{nT_\mathrm{s}R_\mathrm{s}}$ source messages indices w into a set of $2^{n(1-T_\mathrm{s})\left(T_\mathrm{s}\left(R_\mathrm{s}-I_{\mathrm{sd}_1}\right)\right)}$ bins $B_b, b \in \left\{1,\cdots,2^{n(1-T_\mathrm{s})\left(T_\mathrm{s}\left(R_\mathrm{s}-I_{\mathrm{sd}_1}\right)\right)}\right\}$.

3) Encoding: Let w be sent by the source.

- First hop (source transmission): The source node sends $\mathbf{x}_\mathrm{s}\left(w\right)$.
- Second hop (RS transmission): RS gets an estimation of the source message $\hat{w}$. It knows that the source message $\hat{w}$ is in bin B_b. Then, it sends $\mathbf{x}_\mathrm{r}\left(b\right)$ to the destinations to inform them about the bin index b.

4) Decoding: In the following, code length n is chosen sufficiently large.

- At the end of the first hop
  – At the RS: The RS receives $\mathbf{y}_\mathrm{sr}$ and it estimates w by finding a unique $\hat{w}$ such that $\left(\mathbf{x}_\mathrm{s}\left(\hat{w}\right),\mathbf{y}_\mathrm{sr}\right)$ are jointly typical. $\hat{w}=w$ with high probability if

$$T_\mathrm{s}R_\mathrm{s} \le T_\mathrm{s} I\left(X_\mathrm{s};Y_\mathrm{sr}\right). \tag{41}$$

  – At the destinations: Destination m received $\mathbf{y}_{\mathrm{sd}_m}$ and could only estimate a list $\mathcal{L}\left(\mathbf{y}_{\mathrm{sd}_m}\right)$ such that $\hat{w}\in\mathcal{L}\left(\mathbf{y}_{\mathrm{sd}_m}\right)$ if $\left(\mathbf{x}_\mathrm{s}\left(\hat{w}\right),\mathbf{y}_{\mathrm{sd}_m}\right)$ are jointly typical. We can have $\hat{w}\in\mathcal{L}\left(\mathbf{y}_{\mathrm{sd}_m}\right)$ with high probability if

$$T_\mathrm{s}I_{\mathrm{sd}_m} \le T_\mathrm{s} I\left(X_\mathrm{s};Y_{\mathrm{sd}_m}\right). \tag{42}$$

- At the end of the second hop
  Destination m receives $\mathbf{y}_{\mathrm{rd}_m}$ and estimates a list $\mathcal{L}\left(\mathbf{y}_{\mathrm{rd}_m}\right)$ such that $\hat{b}\in\mathcal{L}\left(\mathbf{y}_{\mathrm{rd}_m}\right)$ if $\left(\mathbf{x}_\mathrm{r}(\hat{b}),\mathbf{y}_{\mathrm{rd}_m}\right)$ are jointly typical. We have $\hat{b}\in\mathcal{L}\left(\mathbf{y}_{\mathrm{rd}_m}\right)$ with high probability if

$$\begin{aligned}&\left(1-T_\mathrm{s}\right)\left(T_\mathrm{s}\left(R_\mathrm{s}-I_{\mathrm{sd}_1}\right)\right)\\ &\le\left(1-T_\mathrm{s}\right)\left(I\left(X_\mathrm{r};Y_{\mathrm{rd}_m}\right)+T_\mathrm{s}\left(I_{\mathrm{sd}_m}-I_{\mathrm{sd}_1}\right)\right), &(43)\\ &\equiv\left(1-T_\mathrm{s}\right)\left(T_\mathrm{s}(R_\mathrm{s}-I_{\mathrm{sd}_m})\right)\le\left(1-T_\mathrm{s}\right)I\left(X_\mathrm{r};Y_{\mathrm{rd}_m}\right). &(44)\end{aligned}$$

  Destination m finds $\hat{w}$ if there is a unique $\hat{w}\in B_{\mathcal{L}\left(\mathbf{y}_{\mathrm{rd}_m}\right)}\cap\mathcal{L}\left(\mathbf{y}_{\mathrm{sd}_m}\right)$. From (37), (42) and (44), $\hat{w}=w$ with high probability if

$$R\le T_\mathrm{s}I\left(X_\mathrm{s};Y_{\mathrm{sd}_m}\right)+\left(1-T_\mathrm{s}\right)I\left(X_\mathrm{r};Y_{\mathrm{rd}_m}\right). \tag{45}$$

  Since (36) ensures that the RS can decode w reliably, thus, if (36), (41) and (45) are fulfilled for all $m\in\mathcal{M}$, there exist a channel code that makes the decoding error at each destination m less than ϵ.

B. Gaussian Channels

We examine the constraint in (41) under the Gaussian inputs

$$\begin{aligned}I\left(X_\mathrm{s},Y_\mathrm{sr}\right)&=H\left(Y_\mathrm{sr}\right)-H\left(Y_\mathrm{sr}|X_\mathrm{s}\right)\\ &=\frac{1}{2}\log_2\left(1-\frac{\mathrm{E}^2\left(\left(h_\mathrm{sr}x_\mathrm{s}+z_\mathrm{sr}\right)x_\mathrm{s}\right)}{\mathrm{Var}\left(\left(h_\mathrm{sr}x_\mathrm{s}+z_\mathrm{sr}\right)\right)\mathrm{Var}\left(x_\mathrm{s}\right)}\right)^{-1}\\ &=\frac{1}{2}\log_2\left(1-\frac{\left(h_\mathrm{sr}\sigma_\mathrm{s}^2\right)^2}{\left(\left|h_\mathrm{sr}\right|^2\sigma_\mathrm{s}^2+\sigma_\mathrm{z}^2\right)\left(\sigma_\mathrm{s}^2\right)}\right)^{-1}\\ &=\frac{1}{2}\log_2\left(1+\frac{\left|h_\mathrm{sr}\right|^2\sigma_\mathrm{s}^2}{\sigma_\mathrm{z}^2}\right)=C\left(\gamma_\mathrm{sr}\right).\end{aligned} \tag{46}$$

Similarly, for the constraint (42) we have

$$I\left(X_\mathrm{s};Y_{\mathrm{sd}_m}\right)=\frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{sd}_m}\right|^2\sigma_\mathrm{s}^2}{\sigma_\mathrm{z}^2}\right)=C\left(\gamma_{\mathrm{sd}_m}\right) \tag{47}$$

and for the constraint (44) we have

$$I\left(X_\mathrm{r};Y_{\mathrm{rd}_m}\right)=\frac{1}{2}\log_2\left(1+\frac{\left|h_{\mathrm{rd}_m}\right|^2\sigma_\mathrm{r}^2}{\sigma_\mathrm{z}^2}\right)=C\left(\gamma_{\mathrm{rd}_m}\right) \tag{48}$$

Hence, we have

$$R=T_\mathrm{s}R_\mathrm{s}\le T_\mathrm{s}C\left(\gamma_\mathrm{sr}\right) \tag{49}$$

$$R\le T_\mathrm{s}C\left(\gamma_{\mathrm{sd}_m}\right)+(1-T_\mathrm{s})C\left(\gamma_{\mathrm{rd}_m}\right),\forall m\in\mathcal{M}. \tag{50}$$

By having $I_\mathrm{sr}=C\left(\gamma_\mathrm{sr}\right)$, $I_{\mathrm{sd}_m}=C\left(\gamma_{\mathrm{sd}_m}\right)$ and $I_{\mathrm{rd}_m}=C\left(\gamma_{\mathrm{rd}_m}\right)$, the source transmission rate R_s in (20) has been derived to ensure that both (49) and (50) are fulfilled. This proves the achievability of the proposed OPIR strategy.

The optimum source transmission time can be found by the intersections of (49) and (50), which leads to

$$T_\mathrm{s}=\frac{I_{\mathrm{rd}_m}}{R_\mathrm{s}-I_{\mathrm{sd}_m}+I_{\mathrm{rd}_m}}. \tag{51}$$

Hence, from (49), (50) and (51), the achievable rate is given by

$$R=\frac{R_\mathrm{s}}{1+\max_m\frac{\left(R_\mathrm{s}-I_{\mathrm{sd}_m}\right)}{I_{\mathrm{rd}_m}}}. \tag{52}$$

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