MIMO Relaying with Compact Antenna Arrays: Coupling, Noise Correlation and Superdirectivity

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***Abstract*—It is well known that compact arrays introduce spatial channel correlation, antenna coupling, superdirectivity and noise correlation. While these effects have been thoroughly investigated for point to point MIMO, only isolated results are available on the effect of such phenomena on performance of relaying systems. This paper tries to fill part of this void. Specifically we study the impact of lossless and lossy compact antenna arrays on multiuser amplify and forward (AF) relaying in the presence of different noise sources. We optimize the gain allocation for sum rate maximization under a *dissipated* power constraint. For compact arrays this constraint is more relevant than the commonly used radiated power constraint. We provide an extensive insight-oriented discussion of our results, some of which may appear counter intuitive at first glance. We show that for compact MIMO relaying the standard model, considering only channel correlation but ignoring antenna coupling and noise correlation, is insufficient and may lead to a wrong system characterization.**

I. Introduction

Future wireless communication systems face two major challenges: extremely high spectral efficiency and ubiquitous coverage. It is commonly accepted that multiple-input multiple-output (MIMO) signaling and relaying are key technologies in this context. The use of MIMO terminals allows for spatial multiplexing and thus an increase in spectral efficiency, which grows with the number of used antennas. Relaying on the other hand has shown to be an efficient means to increase coverage. Additionally, relays can also help to mitigate the impairment of interference in wireless networks, which further improves their performance [1]. In point-to-point MIMO communication as well as relaying, increasing the number of antennas boosts the performance. As a result, massive MIMO arrays have recently attracted considerable attention [2]. However, the (massive) antenna arrays are desired to be compact. Low transmit frequencies (e.g. TV white space [3]) as well as antenna form factor constraints require an antenna spacing of only a fraction of a wavelength. It is well known that compact arrays are associated with different physical effects. While these effects have been thoroughly investigated for point to point MIMO, only minor results are available for relaying systems. This present paper tries to fill part of this void.

Specifically there are four dominant physical effects that come with compact MIMO arrays: i) spatial channel correlation, ii) antenna coupling, which leads to iii) noise correlation as well as iv) superdirectivity. *Spatial correlation* describes the correlation between the coefficients of the physical channel matrix. For a given antenna array the spatial correlation is determined by the scattering profile of the propagation environment. The effect of spatial correlation on MIMO systems is well understood [4], [5]. *Antenna coupling* means that a voltage is produced in one antenna by the current flowing in a closely located antenna. Effects of antenna coupling on MIMO systems were studied in different works, such as [6], [7]. These papers only considered spatially uncorrelated noise. Nevertheless, the additive noise at the receiver comes from external sources [8], from the ohmic losses in the antennas and from the receiver hardware (mainly the low noise amplifier (LNA)). Due to coupling, *noise* from different sources has different correlation properties. This has been studied in [9], [10], [11]. Results in [10], [11] show that in case of *lossless* antennas and having only external noise, closely spaced antennas are beneficial in MIMO systems. In the transmit mode, such arrays have the so called *superdirective* property [12]. In spite of having promising simulation results [13], superdirective antennas are known to perform poorly in the presence of minor ohmic antenna losses [14]. In this case, ohmic losses decrease the directivity of the array and the radiated power. A comprehensive study about the relationship between circuit theory and MIMO communication theory can be found in [9].

The described physical effects also carry over to communication networks that are assisted by relays. While systems with decode and forward (DF) relays can be seen as concatenated point-to-point MIMO systems that are decoupled by the decoding and encoding function, non-regenerative relaying such as amplify and forward (AF) is fundamentally different. The difference comes from the fact that the relay noise is also amplified and transmitted to the destination; thus the first and second hop are not disjoint. A MIMO AF relay has to optimize its gain matrix taking into account its own noise properties and those of the destination noise. Relay gain allocation schemes that attempt to maximize performance are required to include these noise terms into the optimization. This has not been studied so far for compact MIMO and is the main focus of this paper.

Among different schemes for relay gain allocation, the gradient based ones are of particular relevance to our work [15], [16], [17]. While [15], [16] assume full channel state information knowledge, [17] uses the knowledge of statistical properties of the channels in the optimization. These studies do not consider the stated physical effects. Other related work such as [18], [19] studies the performance of AF relaying in presence of spatial correlation only. These results show that spatial correlation leads to performance degradation in most of the scenarios. In [20], the presence of correlated interference for *single antenna* AF relays is studied.

In this work, we consider all the aforementioned physical effects in a MIMO AF relay network. We first summarize the relation between circuit theory and communication theory with regard to MIMO communications. This helps to develop a system model for AF relaying. Next, we discuss our relaying setup in which the relay optimizes its gain matrix to maximize the sum rate under a maximum *dissipated* power constraint. For compact array systems, the performance as well as the best relay location strongly depends on: i) relay superdirectivity , ii) antenna losses and iii) noise sources. In case of superdirective and lossless arrays, peak performance is very high if there is only external noise. Minor ohmic antenna losses cause a performance drop as well as a change in the best relay location. In case of having noise only from the LNAs, the performance is much worse than having only external noise. Interestingly, optimizing the relay gain matrix while ignoring the relay noise correlation causes greater rate degradation than ignoring the destination noise correlation. Our results give insights on how compact arrays can beneficially be used in relaying, and how different physical parameters, which are commonly ignored, play an important role in the system performance.

II. MIMO PHYSICAL SYSTEM MODEL

In communication theory, a MIMO system with N transmit antennas and M receive antennas, is often described by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{H}$ is the channel matrix, and $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{n}$ are the vectors of the transmit signals, received signals and noise, respectively. In the sequel, we summarize the equivalent relation between the transmitter generator voltages $\mathbf{v}_g$, and the receiver load voltages $\mathbf{v}_l$ and noise $\mathbf{u}_n$ at the LNA. Our exposition follows along the lines of [9].

The circuit models of the transmitter and the receiver are shown in Fig.1. The two purely imaginary valued, symmetric matrices $\mathbf{Z}_{\text{MT}}$ and $\mathbf{Z}_{\text{MR}}$ denote the *lossless* matching networks at the transmitter side and at the receiver side. Each of the matching network submatrices has the form of $\begin{bmatrix}\mathbf{Z}_{M,11} & \mathbf{Z}_{M,12}\\ \mathbf{Z}_{M,21} & \mathbf{Z}_{M,22}\end{bmatrix}$, with all the square submatrices having the same size. The matrices $\mathbf{Z}_{\text{CT}}$ and $\mathbf{Z}_{\text{CR}}$ denote the impedance matrices of the antenna arrays of the transmitter and the receiver, respectively. Each matrix $\mathbf{Z}_{\text{Ci}}$ (where i can either be T to denote the transmitter or R to denote the receiver), is formed from two components. The first is the radiation component $\mathbf{Z}_{\text{Ai}}$ and the second is the real valued, diagonal lossy ohmic component $\mathbf{R}_{\text{Ai}}$

$$\mathbf{Z}_{\text{Ci}} = \mathbf{Z}_{\text{Ai}} + \mathbf{R}_{\text{Ai}}. \tag{2}$$

For arrays of half wavelength dipoles, the matrix $\mathbf{Z}_{\text{Ai}}$ can be calculated using formulas in [12]. Such antennas are called *canonical minimum scattering* antennas. This means that an open circuit antenna does not interact with the electric field around it [9]. We denote the input impedance matrices looking into the matching networks at the transmitter side and at the receiver side by $\mathbf{Z}_T$ and $\mathbf{Z}_R$, respectively. The terms R and z_{11} denote the impedance of the source and the input impedance of the LNA, respectively.

We introduce the transimpedance matrix $\mathbf{Z}_{\text{SRT}}$ describing the effect of the physical propagation channel between the transmitter and the receiver. It relates the currents flowing in the transmitter antenna array $\mathbf{i}_{\text{AT}}$ to the open circuit voltage at the receiver antenna array $\mathbf{v}_o$. The physical channel is modeled using the Kronecker model [4]

$$\mathbf{Z}_{\text{SRT}} = Q_i \mathbf{R}_{\text{RX}}^{\frac{1}{2}} \mathbf{Z}_p \mathbf{R}_{\text{TX}}^{\frac{1}{2}}. \tag{3}$$

The entries of the matrix $\mathbf{Z}_P$ are independent identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. The two matrices $\mathbf{R}_{\text{TX}}$ and $\mathbf{R}_{\text{RX}}$ are the spatial correlation matrices at the transmit antenna array and, respectively, the receive antenna array. The elements of the correlation matrices are functions of the scattering profiles around the transmit and receive arrays as well as the antenna elements spacing in terms of wavelength. The term Q_i accounts for the large scale effects (path loss).

The antenna currents at the transmitter $\mathbf{i}_{\text{AT}}$ are related to the generator voltages $\mathbf{v}_g$ by [9]

$$\mathbf{i}_{\text{AT}} = \mathbf{T}_T (R\mathbf{I}_N + \mathbf{Z}_T)^{-1} \mathbf{v}_g = \mathbf{M}_T \mathbf{v}_g, \tag{4}$$

where $\mathbf{T}_T = (\mathbf{Z}_{\text{MT},22} + \mathbf{Z}_{CT})^{-1} \mathbf{Z}_{\text{MT},21}$. The matrix $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. The total *dissipated power* at the transmitter is given by $P_T = \text{E}[\frac{1}{2}\mathbf{i}_{\text{AT}}^H \Re\{\mathbf{Z}_{\text{CT}}\}\mathbf{i}_{\text{AT}}]$. As the matching network $\mathbf{Z}_{\text{MT}}$ is lossless, the total power dissipated at the transmitter is

$$P_T = \text{E}[\frac{1}{2}\mathbf{i}_{\text{GT}}^H \Re\{\mathbf{Z}_{\text{T}}\}\mathbf{i}_{\text{GT}}] = \text{E}[\text{Tr}(\frac{1}{2}\mathbf{v}_g \mathbf{v}_g^H \mathbf{B})] \tag{5}$$

The operators Tr(.) and $\Re\{.\}$ denote the trace and the real parts of the input matrices, respectively. The *effective power admittance* matrix $\mathbf{B}$ is

$$\mathbf{B} = (R\mathbf{I}_N + \mathbf{Z}_T)^{-1^H} \Re\{\mathbf{Z}_{\text{T}}\} (R\mathbf{I}_N + \mathbf{Z}_T)^{-1}. \tag{6}$$

The first two noise components come from the antenna. The antenna noise $\mathbf{n}_{\text{AR}}$ in Fig.1 is the sum of two uncorrelated voltages, i.e. $\mathbf{n}_{\text{AR}} = \mathbf{n}_{\text{ext}} + \mathbf{n}_{\text{l}}$. The noise vector $\mathbf{n}_{\text{ext}}$ is the external noise collected by the radiation component of the antenna array ($\mathbf{Z}_{\text{AR}}$), while $\mathbf{n}_l$ is the vector of the noise generated by the ohmic losses $\mathbf{R}_{\text{AR}}$ in the antennas. Both $\mathbf{n}_{\text{ext}}$ and $\mathbf{n}_{\text{l}}$ are assumed to be Gaussian and generated from 3D isotropic noise sources with temperatures T_{AE} and T_{AL}, respectively . According to [8],

$$\mathbf{R}_{na} = \text{E}[\mathbf{n}_{\text{AR}}\mathbf{n}_{\text{AR}}^H] = 4\text{k}_{\text{B}} B_W (T_{\text{AE}}\Re\{\mathbf{Z}_{\text{AR}}\} + T_{\text{AL}}\mathbf{R}_{\text{AR}}), \tag{7}$$

where k_{B} is the Boltzmann constant, B_W is the bandwidth.

The third noise source is the LNA. As discussed in [9], [10], the noise of each LNA is modeled by a series voltage source and a parallel current source at the input of the LNA. Such sources have the statistical properties:

$$\begin{aligned} \text{E}[\mathbf{i}_n\mathbf{i}_n^H] &= \beta\mathbf{I}_M, \quad \text{E}[\mathbf{v}_n\mathbf{v}_n^H] = \beta R_N^2 \mathbf{I}_M \\ \text{E}[\mathbf{v}_n\mathbf{i}_n^H] &= \rho\beta R_N \mathbf{I}_M. \end{aligned} \tag{8}$$

For narrow bandwidth, the parameters in (8) are considered to be constant. Each LNA has an optimal matching impedance z_{opt} for which its noise contribution is minimal. Note that there is no correlation between noise generated from different LNAs.

As discussed in [9], the relationship between the open circuit voltages $\mathbf{v}_o$ and the load voltages $\mathbf{v}_l$ at the receiver LNA is

$$\mathbf{v}_l = \mathbf{D}\mathbf{T}_R \mathbf{v}_o = \mathbf{M}_R \mathbf{v}_o, \tag{9}$$

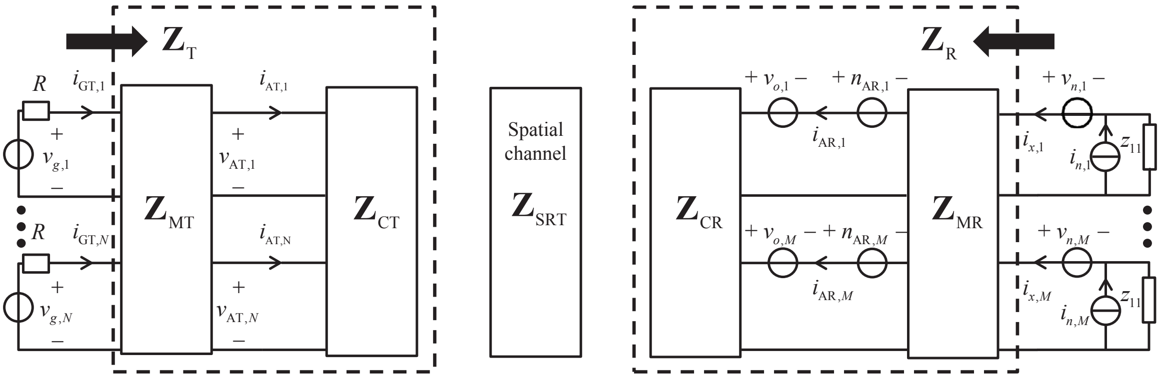

Fig. 1. Circuit model for MIMO communications. The transmitter is on the left and the receiver is on the right.

where $\mathbf{D} = z_{11}\mathbf{I}_M(z_{11}\mathbf{I}_M + \mathbf{Z}_R)^{-1}$ and $\mathbf{T}_R = \mathbf{Z}_{\text{MR},12}(\mathbf{Z}_{\text{MR},22} + \mathbf{Z}_{CR})^{-1}$. This leads to the final input-output relation of the MIMO system similar to (1)

$$\mathbf{v}_l = \mathbf{A}\mathbf{v}_g + \mathbf{u}_n. \tag{10}$$

with equivalent channel matrix $\mathbf{A} = \mathbf{M}_R\mathbf{Z}_{\text{SRT}}\mathbf{M}_T$ and noise vector $\mathbf{u}_n = \mathbf{D}(\mathbf{Z}_R\mathbf{i}_n - \mathbf{v}_n + \mathbf{T}_R\mathbf{n}_{\text{AR}})$. The covariance matrix of the noise voltages $\mathbf{u}_n$ is given as

$$\mathbf{R}_{un} = \text{E}[\mathbf{u}_n\mathbf{u}_n^H] = \beta\mathbf{D}\Phi\mathbf{D}^H, \tag{11}$$

with

$$\Phi = \overbrace{\mathbf{Z}_R\mathbf{Z}_R^H - 2R_N\Re\{\rho^*\mathbf{Z}_R\} + R_N^2\mathbf{I}_N}^{\text{LNA noise}} + \overbrace{\frac{1}{\beta}\mathbf{T}_R\mathbf{R}_{na}\mathbf{T}_R^H}^{\text{Ant. noise}}, \tag{12}$$

where $(.)^*$ denotes the complex conjugate operation.

III. AF Relaying

In our AF relaying scenario, we have N_S single antenna sources with identical radio front ends, that communicate with a destination which has N_D antennas. The communication is done through a relay equipped with N_R antennas. There is no direct link between the sources and the destination. In the first time slot, the relay receives its load voltage

$$\mathbf{v}_{l,R} = \mathbf{A}_{SR}\mathbf{v}_{g,S} + \mathbf{u}_{n,R}, \tag{13}$$

where $\mathbf{A}_{\text{SR}}$, $\mathbf{v}_{g,S}$ and $\mathbf{u}_{n,R}$ are the equivalent channel matrix between the sources and the relay, the source voltages and the relay noise voltages, respectively. The relay directly multiplies the received voltage vector $\mathbf{v}_{l,R}$ with its gain matrix $\mathbf{G}$, and transmits the resulting voltages to the destination in the second time slot. The voltage received at the destination is

$$\mathbf{v}_{l,D} = \mathbf{A}_{\text{RD}}\mathbf{G}\mathbf{A}_{\text{SR}}\mathbf{v}_{g,S} + \mathbf{A}_{\text{RD}}\mathbf{G}\mathbf{u}_{n,R} + \mathbf{u}_{n,D}, \tag{14}$$

where $\mathbf{A}_{\text{RD}}$ and $\mathbf{u}_{n,D}$ are the equivalent channel matrix between the relay and the destination, and the destination noise, respectively. The covariance matrices of the noise voltages at the relay and at the destination are denoted by $\mathbf{R}_{\text{un},R}$ and $\mathbf{R}_{\text{un},D}$, respectively.

Each source transmits with power P_S, while for the relay the total *dissipated power* should not exceed a certain value P_R. Note that the sources are assumed to have a spatial separation that is large enough for coupling and correlation between them to be ignored. This leads to the effective power admittance matrix $\mathbf{B}_S$ of the sources in equation (6) to be a scaled identity matrix, i.e. $\mathbf{B}_S = 2\alpha\mathbf{I}_{N_S}$, where α is an admittance in (6). This means that $\text{E}[\mathbf{v}_{g,s}\mathbf{v}_{g,s}^H] = \frac{1}{\alpha}P_S\mathbf{I}_{N_S}$.

The system topology is shown in Fig. 2. The separation distance between two adjacent antennas at the destination is denoted by d_d. In a practical system, the second hop may a have different frequency than the first hop. We adapt our model by differentiating between the separation distance between two adjacent antennas at the relay in the receive mode $d_{r,r}$ and in the transmitter mode $d_{r,t}$. This will also help in understanding the consequences of having superdirectivity when the relay is transmitting.

For antenna separation distance $d_{r,t}$ at the relay, there is a corresponding effective power admittance matrix $\mathbf{B}_R$. This leads to the total power dissipated in the relay to be

$$\tilde{P}_R(\mathbf{G}) = \text{Tr}\left(\tfrac{1}{2}\left(\tfrac{1}{\alpha}P_S\mathbf{G}\mathbf{A}_{\text{SR}}\mathbf{A}_{\text{SR}}^H\mathbf{G}^H + \mathbf{G}\mathbf{R}_{\text{un},R}\mathbf{G}^H\right)\mathbf{B}_R\right). \tag{15}$$

The instantaneous sum rate of the system described by (14) in [bits/channel use], as a function of the signal and noise covariance matrices $\mathbf{K}_s$ and $\mathbf{K}_u$, is given by

$$r(\mathbf{G}) = \frac{1}{2}\log_2\det(\mathbf{K}_u + \mathbf{K}_s) - \frac{1}{2}\log_2\det(\mathbf{K}_u), \tag{16}$$

where

$$\mathbf{K}_s = \frac{1}{\alpha}P_S\mathbf{A}_{\text{RD}}\mathbf{G}\mathbf{A}_{\text{SR}}\mathbf{A}_{\text{SR}}^H\mathbf{G}^H\mathbf{A}_{\text{RD}}^H \tag{17}$$

$$\mathbf{K}_u = \mathbf{A}_{\text{RD}}\mathbf{G}\mathbf{R}_{\text{un},R}\mathbf{G}^H\mathbf{A}_{\text{RD}}^H + \mathbf{R}_{\text{un},D}. \tag{18}$$

The $\frac{1}{2}$ factor in (16) comes from the two time slots required to transfer packets from the sources to the destination.

The relay optimizes its gain matrix $\mathbf{G}$ to maximize (16). The optimization problem can be stated formally as:

$$\underset{\mathbf{G}}{\text{maximize}} \quad r(\mathbf{G}), \quad \text{subject to} \quad \tilde{P}_R(\mathbf{G}) \leq P_R. \tag{19}$$

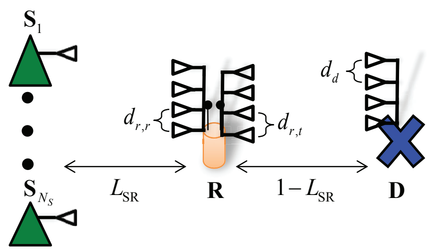

Fig. 2. AF relaying system topology.

For solving the optimization problem in (19), we use a gradient-search algorithm similar to the one in [15]. Due to the different power constraint, the Jacobian matrix in [15] is modified to incorporate (15).

IV. System Setup and Performance

A. System Setup

In our system we have four sources, four relay antennas and four destination antennas, i.e. $N_S = N_R = N_D = 4$. The sources are widely spaced in terms of wavelength, i.e. uncoupled and uncorrelated, but their physical distance to the relay is approximately the same. Without loss of generality, we normalize the source-destination physical distance to unity. We use a path loss exponent of 3.7, which is valid for communication in urban environments [21], leading to Q_i in (3) to be $Q_i = L^{-1.85}$, where L is the physical distance between the transmitter and the receiver. The source-relay distance is denoted by L_{SR}.

For the LNA parameters in (8), we choose $z_{11} = R_N = z_{\text{opt}} = 75\Omega$, and $\rho = 0$. We scale the parameter β such that in (12) the effect of LNA noise is equal to the effect of antenna noise in the case of a single antenna system:

$$\Gamma = \beta(|z_{\text{opt}}|^2 - 2R_N \Re\{\rho z_{\text{opt}}\} + R_N^2) = 4\text{k}_\text{B} T_A B_W \Re\{z_{\text{opt}}\}. \quad (20)$$

Also, the source impedance is chosen to be $R = 75\Omega$. We define the signal to noise ratio to be $\text{SNR} = \frac{\tilde{P}_R}{\Gamma}$.

All the antennas used are half wavelength dipoles. The relay and the destination have uniform linear arrays. We assume that the arrays are in thermal equilibrium, which means that the antenna noise temperatures are the same, i.e. $T_{\text{AL}} = T_{\text{AE}}$. The scattering profile at the relay and at the destination is assumed to be wide-sense stationary uncorrelated scattering in 2D. As discussed in [4], the entries of the correlation matrices are given as $R_{x,i,j} = J_0(2\pi d_{i,j})$, where $d_{i,j}$ is the separation between the i^{th} and j^{th} antenna, and J_0 is the first kind Bessel function of order 0. As stated in [22] multiport matching is very hard to realize in practical systems and also suffers from having extremely small bandwidth at low separation distances [23]. We thus use conventional single port noise matching and power matching for receiving and transmitting, respectively. This means that the block matrices of the matching networks are diagonal ones [9], [22]. Since the optimization function in (19) is non-convex, we use multiple random initializations for each optimization, and select the one giving the highest rate. Throughout the rest of this section we assess performance in terms of ergodic sum rate, i.e. the mean of instantaneous sum rate across many channel realizations.

B. Effects of Superdirectivity and Losses

As discussed in [14], placing the transmit antennas very close to each other has two opposing consequences. The first one is the very high (super) directivity of the array which is beneficial, while the second is the detrimental high correlation of the entries of the transimpedance matrix. In other words, this is similar to having higher received power for an equivalent channel with a high largest eigenvalue but smaller other eigenvalues, i.e. a high eigenvalue spread. In this subsection we examine the effect of changing the transmit

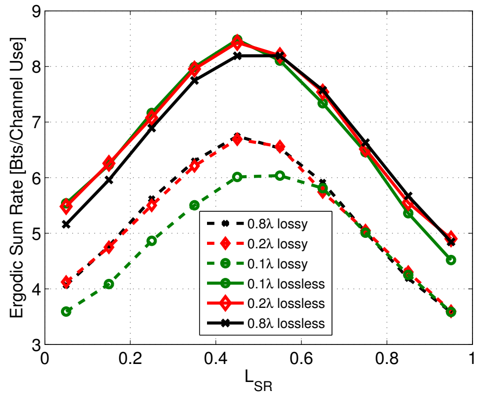

Fig. 3. Effects of transmitter superdirectivity and antenna losses for the case of noiseless LNAs for $d_{r,t} = 0.1\lambda, 0.2\lambda, 0.8\lambda$, $d_{r,r} = d_d = 0.1\lambda$ and SNR = 12 dB.

antenna separation of the relay $d_{r,t}$. The two distances $d_{r,r}$ and d_d are set to 0.1λ. We evaluate the performances when all the antennas are i) lossless and ii) lossy. For the latter case, we choose an ohmic impedance of 0.73Ω. Note that such a small value does not change the performance when all the antennas are widely spaced. We assume that the *antennas* are the only noise sources.

1) *Effect of superdirectivity*: We can make very interesting observations by looking at the lossless curves in Fig.3. In the case of having superdirectivity (i.e. low $d_{r,t}$), the peak performance is better than without it. Furthermore, the best relay location (where the average qualities of the two links are the same) gets closer to the sources. When the relay is closer to the sources, the second hop suffers from higher path loss effects than the first hop. In this case (low received SNR regime), the average number of channel eigenvalues used in a transmission is low. Thus, the higher power at the destination (effect of superdirectivity) using lower number of eigenvalues (effect of spatial correlation) is beneficial. When the relay gets closer to the destination, the curve with the lowest $d_{r,t}$ becomes the worst. As the path loss effect starts to decrease, the power received at the destination becomes higher (high received SNR regime). In this case, transmitting using more eigenvalues and having lower power at the receiver is *more beneficial* than transmitting using less eigenvalues and having higher power at the receiver. This result shows the *tradeoff* between the power gain (*superdirectivity*) brought by closely spacing the relay transmit antennas and the *spatial multiplexing* loss that happens due to strong signal correlation. Note that all the curves merge close to the destination, as the first hop becomes the bottle neck, thus the impact of the second hop becomes negligible.

2) *Effect of antenna losses*: How dramatic the presence of minor ohmic losses can be, is also shown in Fig.3. While having the transmit antennas nearer to each other in the lossless antennas case yields the highest performance, the performance of such configuration turns out to be the worst when the antennas are lossy. Furthermore, the best relay location shifts more towards the destination. The performance degradation can be explained as follows. The coupling matrices of the lossless antenna arrays have some very low eigenvalues that are beneficial in the case of transmitter superdirectivity and the case of having external noise only. Minor diagonal

ohmic losses change the eigenvalue properties of the coupling matrices. In other words, the presence of ohmic losses in the antennas, regularizes the coupling matrices and hence causes a great performance reduction [14].

C. Different Noise Sources

In the upcoming discussions we study the effects of closely spacing the *lossless* antennas in the receive mode. To remove the effects of the transmitter superdirectivity, we assume *large* $d_{r,t}$. The relay is midway between the sources and the destination. To gain insight into the relevance of the different noise sources, we consider the hypothetical cases of having only antenna noise (Ant) or only LNA noise (LNA). Note that due to the normalization in (20) both cases would lead to identical results in the widely spaced antennas case.

1) *Effect of antenna separation*: In Fig. 4 we see how the performance changes with changing the inter-element spacing. The LNA result can be attributed to two main reasons. *The first* is the fact that at low antenna separation, single-port matching is not optimal for the LNA, as the off-diagonal elements increase in the coupling matrices [22], [9], hence the LNA noise contribution increases. *The second* is that due to coupling, the amount of the sources signal power absorbed by the receiver decreases. When the antennas are the dominant noise source, the rate varies within a small range. It reaches 8.4 [bit/Channel use] at both separation distances of 0.1λ. This is higher than the rate for the case of the antennas being widely separated, which is 8.0 [bit/Channel use]. This is because when the antennas are very near to each other, the noise collected by the antennas correlates even more than the signal.

2) *Rate scaling against SNR*: How the sum rate changes as a function of the SNR is very important to evaluate. At high SNR the curves tend to become linear, with a slope equivalent to the degrees of freedom (DOF) of the channel [5]. In Fig.5 we plot the results for the cases of having only external noise, and only LNA noise. The antenna separation distances at the relay and the destination are both 0.1λ. Asymptotically, both curves achieve 2 DOFs, which is the maximum number due to the prelog factor $\frac{1}{2}$ in (16). The performance difference between both cases is high, reaching around 15 dB in the high SNR regime.

3) *Rate against relay location*: How the sum rate changes as a function of the source-relay separation is shown in Fig. 6. When the relay and the destination have the same noise properties (Ant-Ant or LNA-LNA), the optimal position is near the middle. If the destination has LNA noise and the relay has antenna noise, the best position moves towards the destination, and vice versa. The optimal position is close to the point where the two hops have equal rates on average. We also plot the curve for the widely spaced antennas (IID), i.e. the case of neither coupling nor correlation. Even though relay and destination have identical radio front heads, the curve is not completely symmetric, this is because the relay does beam forming, while the sources transmit spatially white.

D. Effect of Ignoring Noise Correlation

In conventional MIMO systems, noise is assumed to be white. A related simplification in our setup is using only the diagonal elements of the noise covariance matrices in

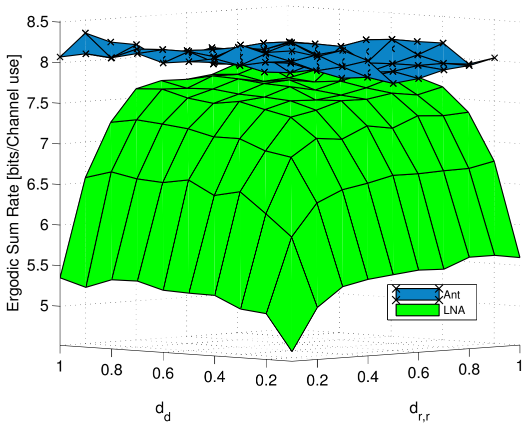

Fig. 4. Rate development as a function of the antenna separation at $L_{\text{SR}} = 0.5$ and SNR $= 12$ dB.

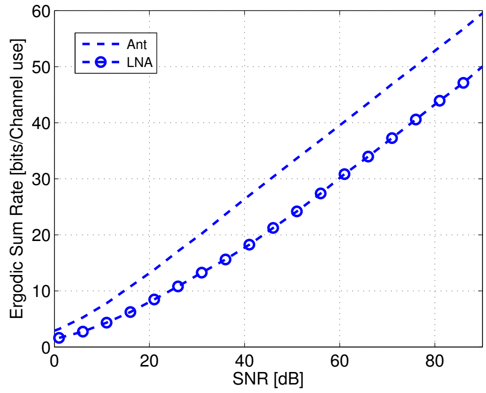

Fig. 5. Rate vs. SNR at $L_{\text{SR}} = 0.5$ with perfect noise covariance matrices knowledge at $d_{r,r} = d_d = 0.1\lambda$ and SNR $= 12$ dB.

the optimization. This case occurs for example, if the relay and the destination perform simple per-antenna noise power estimation. In Fig. 7 we study the impact of this simplification if there is only antenna noise. In each case the relay gain matrix $\hat{\mathbf{G}}$ is optimized according to (16), using the assumed covariance matrices. In all cases we assume full knowledge of the covariance matrices at the destination. For all the curves in Fig. 7, the antenna separation distances at the relay and at the destination are 0.1λ. For reference, we plotted the rates given that the relay uses spatially white transmission, i.e. $\hat{\mathbf{G}} = c\mathbf{I}_{N_S}$, where c is selected to satisfy the relay power constraint in (15).

The curve labeled correct shows the achievable sum rate without covariance mismatch. Interestingly enough there is a substantial performance difference between assuming a diagonal covariance matrix at the relay (R Diag, D Correct) or only at the destination (R Correct, D Diag). In the first case the performance loss peaks at $L_{\text{SR}} = 0.65$ with 1.4[bit/Channel use]. On the contrary assuming the destination noise to be white has minor effect on the performance. The maximum rate loss is around 0.6[bit/Channel use]. This can be attributed to the fact that the relay noise properties affect both the first and the second hop, while the destination noise properties affect only the latter. When the relay is near the destination ignoring the relay noise correlation leads to a performance similar to using a scaled identity matrix as the matrix $\hat{\mathbf{G}}$. At those points, if there is no full knowledge of the noise covariance matrices,

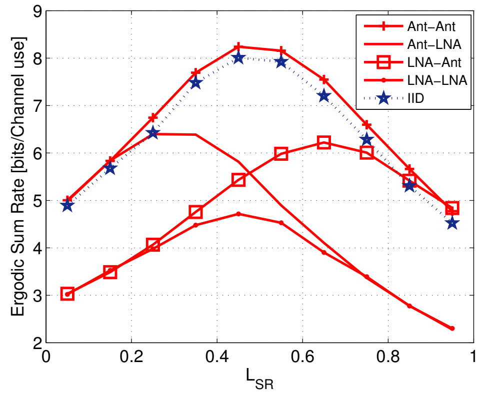

Fig. 6. Rate vs. relay position, for perfect noise covariance matrices knowledge at $d_{r,r} = d_d = 0.1\lambda$ and SNR = 12 dB.

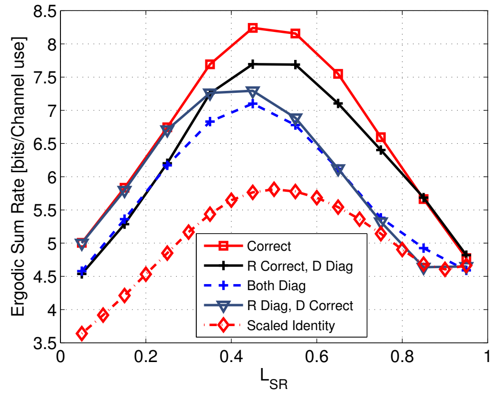

Fig. 7. Rate vs. relay position for using different combinations of noise covariance matrices at $d_{r,r} = d_d = 0.1\lambda$ and SNR = 12 dB.

it is better to save the computation power and time and just transmit spatially white.

When there is LNA noise only, ignoring the noise correlation has negligible effect on the performance. As the correlation of the noise is less than when the antennas are the dominant noise sources, it does not strongly influence the gain allocation.

V. Conclusion

In this work, we proposed a framework for looking at AF relaying with compact antennas. We first presented a system model for AF relaying based on circuit theory. Such a model captures antenna coupling and the physical aspects that accompany it. Our results showed that, in contrast to the case of widely spaced antennas, noise generated from different sources affects the system performance differently. With compact arrays, very high rates are possible if the noise is only collected by the *lossless* antennas and having superdirective arrays. However drastic performance degradation occurs in case of having minor antenna losses. The performance difference between having only LNA noise and only external antenna noise was very high, reaching more than 15 dB at high SNR. Following the conventional approach of assuming the noise to be white was shown to cause great losses, especially when the relay is near the destination. The system performance was shown to be more sensitive to ignoring the correlation of the relay noise than ignoring the one of the destination. Furthermore, the optimum relay position strongly depends on the dominant noise sources, the transmitter superdirectivity properties of the relay and the ohmic losses in the antennas.

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