Joint Spatial Channel and Coupling Impedance Matrices Estimation in Compact MIMO Systems: The Use of Adaptive Loads

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***Abstract*—In this paper we present a novel way of *jointly* estimating the coupling matrix of closely spaced receive antennas as well as the spatial channel of compact MIMO systems. Unlike conventional approaches, we do not assume any knowledge of the types and geometries of the antennas, or their separations, or having line of sight spatial channels. Our algorithm is based only on changing the load impedances of the antennas and measuring the voltages across such loads. We use the nonlinear relation between the load impedances and the compound channel estimates in order to estimate *both* the spatial channel and the coupling matrix. Our algorithm is of relevance to future communication systems incorporating heterogeneous networks in dense environments. In such networks different users/nodes may occasionally be closely spaced and hence, mutually coupled. Using our algorithm, they can estimate both the coupling matrix and the spatial channel matrix in real-time and thereby optimize their matching networks and signal processing in order to enhance their performance.**

I. INTRODUCTION

Future communication systems are expected to move in the direction of decentralization with lots of heterogeneous nodes communicating together and assisting each other in long range communications [1]. Such networks are going to span different concepts like device to device communication, user cooperation and virtual MIMO. On one hand, the number of antennas per device is reportedly expected to increase strongly, on the other hand the form factor of the devices is expected to decrease. In addition, the density of the wireless devices per physical area is expected to increase strongly due to the increase of wireless applications and users. All these reasons enhance the interest in the so called compact MIMO systems, in which the antennas of either the same device or other devices in the close vicinity are very close in terms of the operation wave length λ.

One of the main characteristics of closely spacing the antennas is antenna coupling. To incorporate the effects of mutual coupling, the multiport description of the antenna elements is usually used. A summary of this description is found in [2], [3]. In compact MIMO communications, usually the estimated channel is a compound channel which has two components. While the first accounts for the effects of the circuits and coupling, the other accounts for the spatial channel. Given full knowledge of the coupling matrix and either instantaneous or statistical knowledge of the spatial channel, different algorithms for adapting the matching network in order to enhance the performance of MIMO systems were proposed, e.g. in [4], [5]. These algorithms showed substantial boosts in the achievable rates of the compact MIMO. Such results triggered our interest in the coupling matrix estimation. Knowledge of the coupling matrix is important for other applications, such as direction of arrival estimation [6].

In [6], a method was proposed for estimating the so called receiver coupling matrix in which the coupling matrix is defined different from conventional methods. It is further refined in [7]. For more elaboration on the difference between conventional coupling matrix and the receiver coupling one, the reader is guided to [3]. In [8], the whole effect of the antenna coupling is modeled by a matrix $\mathbf{C}$ which is estimated. The main drawback of all these methods is that they all require line of sight (LOS) measurements, as well as some other conditions and constraints that vary from one algorithm to another. Such requirements are prohibitive for estimating the coupling matrix in real-time between different antennas that may be by chance near each other, for example in a dense sensor network. In circuit theory, different techniques based on adaptive loading were used in characterizing multiports, e.g. [9]. However such techniques were used with known and calibrated reference signals from network analyzers. Knowing such reference signals is analogue to full knowledge of the spatial channel in the case of coupled receive antennas.

Up to our knowledge, joint estimation of the coupling matrix and the spatial channel using variable load impedances was not proposed before. This is the gap we fill in this paper. Our idea is based on the nonlinear relationship between the load impedances and the estimated compound channel. We consider two sources of noise in our system, namely the antenna noise and the load noise. Using the circuit relations, we formulate the estimation problem as a linear least squares problem. Since the coupling matrix is constrained to be symmetric, we extend a linear algebraic algorithm presented in [10] to find the least squares solution. We believe that our algorithm is of interest to modern communication systems with high node density within limited space. The main advantage of our algorithm is that it enables real-time estimation of *both* the spatial channel and the coupling matrix without any need of LOS conditions or knowledge of antenna properties. Hence, it would be beneficial in communication systems with dense *cooperating* users.

II. SYSTEM MODEL

We consider a network with N widely spaced transmit antennas and closely spaced, slowly moving receiver terminals with total number of antennas M. When such terminals are

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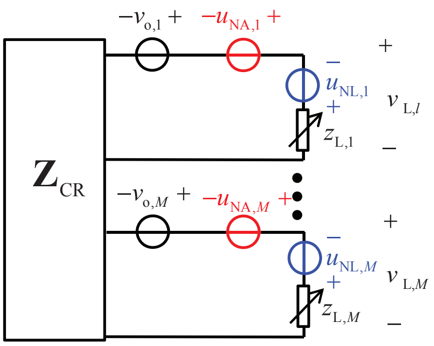

Fig. 1. Circuit model for the receiver of compact MIMO with adaptive load impedances.

near each other, they form a coupled antenna array. This system is nothing but a MIMO system with M closely spaced receive antennas and N transmit antennas that are uncorrelated and uncoupled. Such a system can be described by

$$\mathbf{v}_{\mathrm{L}} = \mathbf{A}\mathbf{v}_{\mathrm{g}} + \mathbf{u}_{\mathrm{n}}, \tag{1}$$

where the vectors $\mathbf{v}_{\mathrm{g}} \in \mathbb{C}^{N\times 1}$ and $\mathbf{v}_{\mathrm{L}} \in \mathbb{C}^{M\times 1}$ are the generator voltages and the load voltages, respectively. The equivalent channel matrix is denoted by $\mathbf{A} \in \mathbb{C}^{M\times N}$, while the noise vector $\mathbf{u}_{\mathrm{n}} \in \mathbb{C}^{M\times 1}$ which has the covariance matrix $\mathbf{K}_{\mathrm{u}} \in \mathbb{C}^{M\times M}$. We assume that all transmitters use identical RF front ends and same transmit power, precisely the mean of the generator voltage squared, i.e. $\mathrm{E}[\mathbf{v}_{\mathrm{g}}\mathbf{v}_{\mathrm{g}}^H] = P\mathbf{I}_N$ with $\mathbf{I}_N$ as the $N\times N$ identity matrix. In this section, we summarize the relationship between the circuit components of the receiver shown in Fig.1 and the matrices $\mathbf{A}$ and $\mathbf{K}_{\mathrm{u}}$. We use a simpler system model than the ones used in [2], [4], in which we assume that the antenna array is followed by adaptive load impedances directly.

A. Signals and Circuit Components at the Receiver

The vector of open circuit voltages $\mathbf{v}_{\mathrm{o}} \in \mathbb{C}^{M\times 1}$ is seen in Fig.1. Such voltages are related to the transmitter generator voltages by

$$\mathbf{v}_{\mathrm{o}} = \mathbf{Z}_{\mathrm{SRT}}\sqrt{\alpha}\mathbf{I}_N\mathbf{v}_{\mathrm{g}}, \tag{2}$$

where the scalar $\sqrt{\alpha}$ captures the effect of the transmit side circuits and has unit Siemens. The transimpedance matrix $\mathbf{Z}_{\mathrm{SRT}} \in \mathbb{C}^{M\times N}$ resembles the physical propagation channel that maps the currents flowing in the transmit antennas to the voltages $\mathbf{v}_{\mathrm{o}}$ at the receiver array. We model $\mathbf{Z}_{\mathrm{SRT}}$ using the Kronecker model as:

$$\mathbf{Z}_{\mathrm{SRT}} = \mathbf{R}_{\mathrm{RX}}^{\frac{1}{2}}\mathbf{Z}_{\mathrm{P}}. \tag{3}$$

The entries of the matrix $\mathbf{Z}_{\mathrm{P}}$ are independent identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. The matrix $\mathbf{R}_{\mathrm{RX}} \in \mathbb{C}^{M\times M}$ denotes the spatial correlation at the receiver.

The first component of the receiver is the antenna array. The antenna array is characterized as a multiport with impedance matrix $\mathbf{Z}_{\mathrm{CR}} \in \mathbb{C}^{M\times M}$, *which we call coupling matrix*. The matrix $\mathbf{Z}_{\mathrm{CR}}$ is a symmetric matrix due to the reciprocity theorem, as discussed in [11]. The vector $\mathbf{u}_{\mathrm{NA}}$ is the external noise collected by the radiation part of the antennas, which we call antenna noise. As discussed in [2] the noise $\mathbf{u}_{\mathrm{NA}}$ is assumed to be generated from a 3D isotropic noise source, hence its covariance matrix

$$\mathbf{R}_{\mathrm{NA}} = \mathrm{E}[\mathbf{u}_{\mathrm{NA}}\mathbf{u}_{\mathrm{NA}}^H] = 4\mathrm{k}_{\mathrm{B}}\mathrm{B}T_{\mathrm{A}}\Re\{\mathbf{Z}_{\mathrm{CR}}\}, \tag{4}$$

where k_{B} is the Boltzmann constant, B is the bandwidth and T_{A} is the equivalent noise temperature of the antennas. The $\Re\{.\}$ denotes the real part of the input matrix.

Next components at the receivers' side are the *adaptive* load impedances. Since we are only dealing with passive elements the real part of any load $z_{\mathrm{L},i}$ is always *non-negative*, i.e.

$$z_{\mathrm{L},i} = |r_i| + jg_i, \tag{5}$$

where the operator $|.|$ denotes the absolute value of the input value. We define the diagonal matrix $\mathbf{Z}_{\mathrm{L}} \in \mathbb{C}^{M\times M}$, where the diagonal elements of $\mathbf{Z}_{\mathrm{L}}$ are the load impedances terminating the receive antennas as

$$\mathbf{Z}_{\mathrm{L}} = \begin{bmatrix} z_{\mathrm{L},1} & 0 & \dots & 0 \\ 0 & z_{\mathrm{L},2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & z_{\mathrm{L},M} \end{bmatrix}. \tag{6}$$

From basic circuit theory, each load $z_{\mathrm{L},i}$ has a series voltage source which models the noise it produces $u_{\mathrm{NL},i}$. The noise generated from different load impedances are uncorrelated. The vector of load noise voltage sources has the covariance matrix

$$\mathbf{R}_{\mathrm{NL}} = \mathrm{E}[\mathbf{u}_{\mathrm{NL}}\mathbf{u}_{\mathrm{NL}}^H] = 4\mathrm{k}_{\mathrm{B}}\mathrm{B}T_{\mathrm{L}}\Re\{\mathbf{Z}_{\mathrm{L}}\}, \tag{7}$$

where T_{L} is the equivalent noise temperature of the load impedances. We assume that T_{L} is equal for all load impedances.

B. Input-Output Relationship

From basic circuit theory, the measured voltage $\mathbf{v}_{\mathrm{L}}$ is

$$\mathbf{v}_{\mathrm{L}} = \mathbf{D}\mathbf{v}_o + \overbrace{\mathbf{D}\mathbf{u}_{\mathrm{NA}} + (\mathbf{D} - \mathbf{I}_M)\mathbf{u}_{\mathrm{NL}}}^{\mathbf{u}_{\mathrm{n}}}, \tag{8}$$

where matrix $\mathbf{D} \in \mathbb{C}^{M\times M}$ is the voltage divider matrix defined as

$$\mathbf{D} = \mathbf{Z}_{\mathrm{L}}(\mathbf{Z}_{\mathrm{L}} + \mathbf{Z}_{\mathrm{CR}})^{-1}. \tag{9}$$

The total covariance matrix of the noise $\mathbf{K}_{\mathrm{u}}$ is

$$\mathbf{K}_{\mathrm{u}} = \mathbf{D}\mathbf{R}_{\mathrm{NA}}\mathbf{D}^H + (\mathbf{D} - \mathbf{I}_M)\mathbf{R}_{\mathrm{NL}}(\mathbf{D} - \mathbf{I}_M)^H \tag{10}$$

For measuring the compound channel, we use N orthogonal training sequences that form a matrix $\mathbf{V}_{\mathrm{G}} = \sqrt{P}\mathbf{I}_N$. Hence, the elements of the open circuit voltage matrix $\mathbf{V}_{\mathrm{o}} = \sqrt{\alpha P}\mathbf{Z}_{\mathrm{SRT}}$. The load voltage matrix $\mathbf{V}_{\mathrm{L}} \in \mathbb{C}^{M\times N}$ is the estimated compound channel scaled by $\sqrt{\alpha P}$

$$\mathbf{V}_{\mathrm{L}} = \mathbf{D}\mathbf{V}_{\mathrm{o}} + \mathbf{U}_n, \tag{11}$$

where the N columns of the noise matrix $\mathbf{U}_{\mathrm{n}}$ are statistically independent.

III. Proposed Joint Estimation Algorithm

As discussed in the introduction, separately estimating the antenna coupling matrix $\mathbf{Z}_{\text{CR}}$ and the spatial channel is of great importance to compact MIMO systems [4]. In this section we formulate and explain the joint estimation problem. We assume that the spatial channel is constant over the time of load switching. *From equation* (11) *defined in subsection II-B, the matrix* $\mathbf{V}_o$ *is the spatial channel and the matrix* $\mathbf{V}_L$ *is the compound channel, both scaled by* $\sqrt{\alpha P}$.

A. Problem Definition

As it can be seen from (9) and (11), the matrix $\mathbf{V}_{\text{L}}$ is a nonlinear function of the load impedance matrix. This is a very important observation that leads to our proposed estimation method. If the spatial channel is constant for l time slots, we can change the adaptive impedance and hence have different impedance matrices $\mathbf{Z}_{\text{L}}^{(j)}$ for $j \in \{1, \dots, l\}$. For every load impedance matrix $\mathbf{Z}_{\text{L}}^{(j)}$ we are going to have the measured load voltage

$$\mathbf{V}_{\text{L}}^{(j)} = \overbrace{\mathbf{Z}_{\text{L}}^{(j)}(\mathbf{Z}_{\text{L}}^{(j)} + \mathbf{Z}_{\text{CR}})^{-1}}^{\mathbf{D}^{(j)}} \mathbf{V}_{\text{o}} + \mathbf{U}_{\text{n}}^{(j)}. \tag{12}$$

For the l changes of the load impedances we will have

$$\underbrace{\begin{bmatrix} \mathbf{V}_{\text{L}}^{(1)} \\ \mathbf{V}_{\text{L}}^{(2)} \\ \vdots \\ \mathbf{V}_{\text{L}}^{(l)} \end{bmatrix}}_{\mathbf{W}} = \underbrace{\begin{bmatrix} \mathbf{D}^{(1)} \\ \mathbf{D}^{(2)} \\ \vdots \\ \mathbf{D}^{(l)} \end{bmatrix}}_{\mathbf{E}} \mathbf{V}_{\text{o}} + \begin{bmatrix} \mathbf{U}_{\text{n}}^{(1)} \\ \mathbf{U}_{\text{n}}^{(2)} \\ \vdots \\ \mathbf{U}_{\text{n}}^{(l)} \end{bmatrix}. \tag{13}$$

For shortness of notations, we define the $\mathbf{W} \in \mathbb{C}^{lM \times N}$ and $\mathbf{E} \in \mathbb{C}^{lM \times M}$ as shown in (13). Note that the matrix $\mathbf{E}$ is a function of $\mathbf{Z}_{\text{CR}}$.

We want to estimate *both* the open circuit voltages $\mathbf{V}_{\text{o}}$ and the coupling matrix $\mathbf{Z}_{\text{CR}}$ from the load voltage matrices we observe. Since we do not know the statistics of the noise we will resort to the least squares estimation (LSE). At first glance, estimating $\mathbf{V}_{\text{o}}$ and $\mathbf{Z}_{\text{CR}}$ seems to be complicated as the observation matrix $\mathbf{W}$ is related to the matrix $\mathbf{Z}_{\text{CR}}$ by an inverse operation. This makes the estimation problem cumbersome.

B. Formulating The Estimation Problem

With a deeper look in equation (12), one can see that it can be written as

$$\mathbf{Z}_{\text{CR}}\mathbf{C}_{\text{L}}^{(j)} = \mathbf{V}_{\text{o}} - \mathbf{V}_{\text{L}}^{(j)} + \underbrace{\mathbf{D}^{(j)^{-1}}\mathbf{U}_{\text{n}}^{(j)}}_{\tilde{\mathbf{U}}_{\text{n}}^{(j)}}, \tag{14}$$

where the matrix $\mathbf{C}_{\text{L}}^{(j)} \in \mathbb{C}^{M \times N}$ resembles the load currents when the impedance matrix $\mathbf{Z}_{\text{L}}^{(j)}$ is used, i.e.

$$\mathbf{C}_{\text{L}}^{(j)} = \mathbf{Z}_{\text{L}}^{(j)^{-1}} \mathbf{V}_{\text{L}}^{(j)}. \tag{15}$$

Equation (14) can be further written as

$$\begin{bmatrix} \mathbf{Z}_{\text{CR}} & \mathbf{V}_{\text{o}} \end{bmatrix} \begin{bmatrix} \mathbf{C}_{\text{L}}^{(j)} \\ -\mathbf{I}_N \end{bmatrix} = -\mathbf{V}_{\text{L}}^{(j)} + \tilde{\mathbf{U}}_{\text{n}}^{(j)}. \tag{16}$$

This leads to the following equation using all the load voltage matrices observed for different load matrices

$$\begin{bmatrix} \mathbf{Z}_{\text{CR}} & \mathbf{V}_{\text{o}} \end{bmatrix} \overbrace{\begin{bmatrix} \mathbf{C}_{\text{L}}^{(1)} & \dots & \mathbf{C}_{\text{L}}^{(l)} \\ & -\mathbf{I}_N^l & \end{bmatrix}}^{\mathbf{\Gamma}} = -\overbrace{\begin{bmatrix} \mathbf{V}_{\text{L}}^{(1)} & \dots & \mathbf{V}_{\text{L}}^{(l)} \end{bmatrix}}^{\mathbf{\Upsilon}} + \underbrace{\begin{bmatrix} \tilde{\mathbf{U}}_{\text{n}}^{(1)} & \dots & \tilde{\mathbf{U}}_{\text{n}}^{(l)} \end{bmatrix}}_{\tilde{\mathbf{U}}}. \tag{17}$$

The matrix $\mathbf{I}_N^l$ is the Kronecker product of an all ones row vector of length l and the $M \times M$ identity matrix, i.e.

$$\mathbf{I}_N^l = \begin{bmatrix} 1 & \dots & 1 \end{bmatrix} \otimes \mathbf{I}_N. \tag{18}$$

The matrices $\mathbf{\Gamma}$ and $\mathbf{\Upsilon}$ have sizes of $(M+N) \times Nl$ and $M \times Nl$, respectively.

With the new description of the system equations in (17), we can formulate our LSE problem as

$$\begin{aligned} \underset{\mathbf{Z}_{\text{CR}}, \mathbf{V}_{\text{o}}}{\text{minimize}} \quad & f(\mathbf{Z}_{\text{CR}}, \mathbf{V}_{\text{o}}) = \left\| \begin{bmatrix} \mathbf{Z}_{\text{CR}} & \mathbf{V}_{\text{o}} \end{bmatrix} \mathbf{\Gamma} - \mathbf{\Upsilon} \right\|^2 \\ \text{subject to} \quad & \mathbf{Z}_{\text{CR}} = \mathbf{Z}_{\text{CR}}^T. \end{aligned} \tag{19}$$

We were able to get rid of the problem of the inverses in $\mathbf{W}$ and we can write the LSE problem as linear problem which is a convex one. By inspecting (17), we can see that the estimation problem will surely not have a unique solution if $M+N > Nl$. In this case the matrix $\mathbf{\Gamma}$ will not have full rank thus it will not be invertible. This means that the minimum value of l is 2.

C. Proposed Solution

If there was no constraint that the matrix $\mathbf{Z}_{\text{CR}}$ is a symmetric matrix, the LSE solution would have been straightforward. However, due to this symmetry constraint we choose to solve the problem in a two stage fashion. We first estimate $\hat{\mathbf{Z}}_{\text{CR}}$ and then use our estimate to calculate $\hat{\mathbf{V}}_{\text{o}}$. For a given $\mathbf{Z}_{\text{CR}}$, the LSE of $\mathbf{V}_{\text{o}}$ is

$$\hat{\mathbf{V}}_{\text{o}} = (\mathbf{Z}_{\text{CR}}\mathbf{\Gamma}_{\text{c}} - \mathbf{\Upsilon})\, \mathbf{I}_N^{l^H} \underbrace{\left(\mathbf{I}_N^l \mathbf{I}_N^{l^H}\right)^{-1}}_{1/l \quad \mathbf{I}_N}, \tag{20}$$

where

$$\mathbf{\Gamma}_{\text{C}} = \begin{bmatrix} \mathbf{C}_{\text{L}}^{(1)} & \dots & \mathbf{C}_{\text{L}}^{(l)} \end{bmatrix}. \tag{21}$$

For $\hat{\mathbf{V}}_{\text{o}}$, we can write the system equations only in term of $\mathbf{Z}_{\text{CR}}$ as

$$\begin{bmatrix} \mathbf{Z}_{\text{CR}} & \frac{1}{l}(\mathbf{Z}_{\text{CR}}\mathbf{\Gamma}_{\text{C}} - \mathbf{\Upsilon})\,\mathbf{I}_N^{l^H} \end{bmatrix} \begin{bmatrix} \mathbf{\Gamma}_{\text{C}} \\ -\mathbf{I}_N^l \end{bmatrix} = \mathbf{\Upsilon} + \tilde{\mathbf{U}}. \tag{22}$$

This further simplifies to

$$\underbrace{\mathbf{Z}_{\text{CR}}\mathbf{\Gamma}_{\text{C}}\left(\mathbf{I}_{Nl} - \frac{1}{l}\mathbf{I}_N^{l^H}\mathbf{I}_N^l\right)}_{\mathbf{G}} = \underbrace{\mathbf{\Upsilon}\left(\mathbf{I}_{Nl} - \frac{1}{l}\mathbf{I}_N^{l^H}\mathbf{I}_N^l\right)}_{\mathbf{B}} + \tilde{\mathbf{U}}. \tag{23}$$

This leads to the reduction of the estimation problem to

$$\begin{aligned} \underset{\mathbf{Z}_{\text{CR}}}{\text{minimize}} \quad & f(\mathbf{Z}_{\text{CR}}) = \|\mathbf{Z}_{\text{CR}}\mathbf{G} - \mathbf{B}\|^2 \\ \text{subject to} \quad & \mathbf{Z}_{\text{CR}} = \mathbf{Z}_{\text{CR}}^T. \end{aligned} \tag{24}$$

There are several ways to solve the LSE problem for a symmetric $\hat{\mathbf{Z}}_{\text{CR}}$. From literature of linear algebra, this problem is very similar to the so-called symmetric Procrustes problem [10]. An elegant algorithm was proposed to solve a similar problem with real values only in [10]. We slightly modify this algorithm and use it here for the estimation of the complex impedance matrix.

The main idea of the algorithm is based on the fact that the Frobenious norm in (19) is unchanged under a unitary transformation. The algorithm would be summarized as follows: Let the singular value decomposition of the matrix $\mathbf{G}$ be defined as $\mathbf{G} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$. The matrices $\mathbf{U}$ and $\mathbf{V}$ are both unitary matrices, while the matrix $\boldsymbol{\Sigma}$ is a *rectangular diagonal matrix* having the form

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_M & 0 & \dots & 0 \end{bmatrix}. \tag{25}$$

The values of $\sigma_i \quad \forall i \in \{1, .., M\}$ are all real. Note that the matrix $\mathbf{U}^T$ is also a unitary matrix. The Frobenious norm in (24) can be written as

$$f(\mathbf{Z}_{\text{CR}}) = \left\| \underbrace{\mathbf{U}^T\mathbf{Z}_{\text{CR}}\mathbf{U}}_{\mathbf{T}}\boldsymbol{\Sigma} - \underbrace{\mathbf{U}^T\mathbf{B}\mathbf{V}}_{\mathbf{Q}} \right\|^2. \tag{26}$$

The matrix $\mathbf{T}$ is symmetric for a symmetric $\mathbf{Z}_{\text{CR}}$. Since the matrix $\boldsymbol{\Sigma}$ is a diagonal matrix, the estimation problem becomes very simple. As discussed in [10], the symmetric matrix $\hat{\mathbf{T}}$ that minimizes the function $f(\mathbf{Z}_{\text{CR}})$ has the following elements:

$$\hat{t}_{i,j} = \frac{q_{i,j}\sigma_i + q_{j,i}\sigma_j}{\sigma_j^2 + \sigma_i^2}, \tag{27}$$

where $q_{i,j}$ is the element of the matrix $\mathbf{Q}$ at the i^{th} row and j^{th} column. Note that in order to have a unique solution, all the singular values σ_i for $i \in \{1, .., M\}$ have to be non-zero. Finally, the least squares estimate of the coupling matrix is

$$\hat{\mathbf{Z}}_{\text{CR}} = \mathbf{U}^*\hat{\mathbf{T}}\mathbf{U}^H. \tag{28}$$

The matrix $\mathbf{U}^*$ is the conjugate complex of the matrix $\mathbf{U}$.

One of the main beauties of our approach is the fact that we can use only a single transmitter at a fixed location for estimating the coupling matrix. In this case the size of $\mathbf{V}_{\text{o}}$ is $1 \times M$. This is conditioned on the ability to change the loads in a way such that the rank of matrix $\mathbf{G}$ is M.

IV. Performance Evaluation

In this section we are going to evaluate the performance of the proposed algorithm. We set $\sqrt{\alpha P} = 1$, hence $\mathbf{V}_{\text{o}} = \mathbf{Z}_{\text{SRT}}$. At the receivers, we consider a 2×2 uniform planar array, with $d_r = 0.1\lambda$. We assume that all the antenna elements used are $\lambda/2$ dipoles which are coplanar and with parallel axes. The mutual coupling matrix $\mathbf{Z}_{\text{CR}}$ can be calculated using formulas in [11]. For this configuration the matrix $\mathbf{Z}_{\text{CR}}$ is approximately

$$\mathbf{Z}_{\text{CR}} \approx \begin{bmatrix} 73.1+42.5j & 67.3+7.5j & 67.3+7.5j & 61.7-4.8j \\ 67.3+7.5j & 73.1+42.5j & 61.7-4.8j & 67.3+7.5j \\ 67.3+7.5j & 61.7-4.8j & 73.1+42.5j & 67.3+7.5j \\ 61.7-4.8j & 67.3+7.5j & 67.3+7.5j & 73.1+42.5j \end{bmatrix}.$$

Similar to [4], we assume wide-sense stationary uncorrelated scattering in 2D at the receiver antennas. The entries of the correlation matrix $\mathbf{R}_{\text{RX}}$ are given as $R_{\text{RX},i,j} = J_0(2\pi d_{i,j})$, where $d_{i,j}$ is the separation between the i^{th} and j^{th} antenna, and J_0 is the first kind Bessel function of order 0. The elements of the matrix $\mathbf{Z}_{\text{P}}$ of the transimpedance matrix $\mathbf{Z}_{\text{SRT}}$ have *unit* variance. The signal to noise ratio (SNR) γ is defined as

$$\gamma = \frac{1}{4\text{K}_\text{B}\text{B}(\text{T}_\text{A} + \text{T}_\text{L})73.1} \quad , \tag{29}$$

where the two equivalent noise temperatures T_A and T_L are defined in section II. This means that we define the SNR for the case of having a single receiver antenna. For the upcoming results we set T_A and T_L to be equal.

We model the adaptive load impedances as random variables. For any load impedance $z_{\text{L},i}$, the two variables r_i and g_i defined in (5) are both independent Gaussian random variables with zero mean and variances β_r^2 and β_g^2, respectively. To evaluate the performance of our algorithms, we use the root mean square error (RMSE) between the estimated matrices and the true ones, i.e.

$$f_1 = \frac{\left\|\hat{\mathbf{Z}}_{\text{CR}} - \mathbf{Z}_{\text{CR}}\right\|}{\sqrt{(M(M+1)/2)}} \text{ and } f_2 = \frac{\left\|\hat{\mathbf{V}}_{\text{o}} - \mathbf{V}_{\text{o}}\right\|}{\sqrt{(MN)}}. \tag{30}$$

To evaluate the performance of the algorithm we first define the $m\%$ RMSE to be the value for which we guarantee an RMSE below it for $m\%$ of the estimates. We plot the cumulative distribution function (CDF) of f_1 for $N = 2$, $\beta_r = 5$, $\beta_g = 250$ and for two SNR values. Namely, $\gamma = 25$dB and $\gamma = 28$dB. As it can be seen in Fig. 2 the performance becomes better as the number of load impedance realizations (l) used increase. By comparing the results in Fig. 2 a very interesting observation can be witnessed. Using $l = 4$ at $\gamma = 28$dB leads to a 70% RMSE of around 9.44Ω which is higher than the one when using $l = 8$ at $\gamma = 25$dB, which is around 5.24Ω. The former can be achieved by having two channel measurements per load matrix at $\gamma = 25$dB and averaging them. For $l = 8$ at $\gamma = 28$dB and $l = 16$ at $\gamma = 25$dB we find that the two 70% RMSE are both approximately 3.65Ω. However, for higher values of l we find the contrary. For $\gamma = 25$dB and $l = 64$, we get 70% RMSE of 2.01Ω. While for $\gamma = 28$dB $l = 32$ we have a 70% RMSE of 1.636Ω. In other words, it is better to have smaller number of impedance realizations at higher SNR than having higher number of impedance realizations at lower SNR. We plot the CDFs of f_2 in Fig. 3 where we can see the same trend as for f_1 in Fig. 2. *Both figures show that our joint estimation algorithm works with high accuracy, reaching* 80%

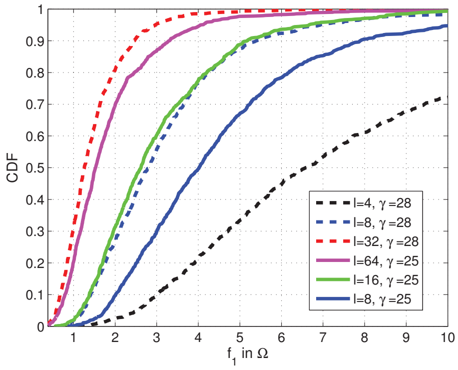

Fig. 2. CDF of the coupling matrix estimate RMSE (f_1) for $M = 4$, $N = 2$, $\gamma = 25$dB and number of impedance matrices $l \in \{8, 16, 64\}$, $\gamma = 28$dB and number of impedance matrices $l \in \{4, 8, 32\}$.

RMSE of approximately 2Ω *for* $\mathbf{Z}_{CR}$ *estimation at* $l = 32$ *and* $\gamma = 28$*dB.*

The performance dependency on the selection of the β_g is shown in Fig. 4. We fix $\beta_r = 5\Omega$ as the simulations in Fig. 2 to have a fair comparison with regards to the produced noise. The mean of the RMSE is plotted for $\gamma = 28$dB and β_g in the range between 80Ω and 2000Ω. As it can be seen, the performance is non-monotonic with respect to β_g. We can divide the curve in to two parts:

- *For* β_g *between* 80Ω *and* 500Ω: The higher the β_g becomes, the better the performance is. This can be explained intuitively by the following argument: At low β_g, the correlation between the load impedance values is high, hence the load voltages measured using different load impedances become correlated, thus leading to a matrix $\mathbf{\Gamma}_\mathrm{C}$ with correlated columns. This leads to poor performance. However, at higher β_g, the load impedances are uncorrelated resulting in good performance, which reaches its best in our scenario for $\beta_g = 500\Omega$ with a mean of RMSE approximately 1.045Ω.
- *For* $\beta_g > 500\Omega$: the performance starts to become worse. This also has an intuitive explanation. At very high β_g the probability of having very high load impedances increases which also leads to highly correlated currents. The following constructed example could elaborate better: If all the entries in the matrix $\mathbf{Z}_\mathrm{L}$ are very high leads to some how an open circuit measurements, i.e. the matrix $\mathbf{D}$ becomes an identity matrix. Hence, the measured voltages will become always the same (or highly correlated), which leads to the worsening of the performance.

V. Conclusion and Future work

We presented a novel way of jointly estimating the coupling matrix between close antennas as well as the spatial channel based on load switching at the receivers. We managed to formulate the problem as a constrained least squares problem for which we modified an algorithm from linear algebra literature to solve. Our results show that the algorithm works with satisfactory results. The algorithm does not need any information about the structure of the antennas or restricts the spatial channel to be LOS. Thus, it can be done in real-time. Hence, it is applicable in dense sensor networks with cooperating heterogeneous devices. For future work, we consider deeper analysis of the load impedance selection.

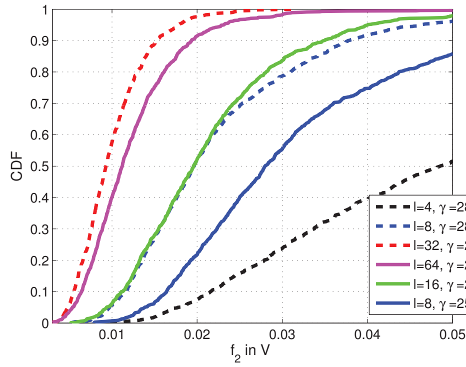

Fig. 3. CDF of the spatial channel estimate RMSE (f_2) for $M = 4$, $N = 2$, $\gamma = 25$dB and number of impedance matrices $l \in \{8, 16, 64\}$, $\gamma = 28$dB and number of impedance matrices $l \in \{4, 8, 32\}$.

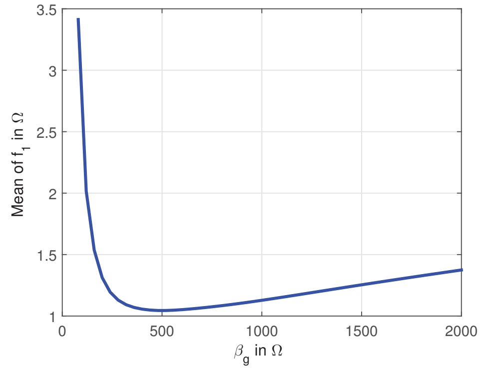

Fig. 4. Mean of the coupling matrix estimate RMSE f_1 for $M = 4$, $N = 2$, $\gamma = 28$dB, $\beta_r = 5\Omega$, β_g between 80Ω and 2000Ω and for $l = 64$.

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