Multiuser MIMO Two-way Relaying for Cellular Communications

Celal Eşli and Armin Wittneben
ETH Zurich, Communication Technology Laboratory, CH-8092 Zurich, Switzerland
Email: {cesli,wittneben}@nari.ee.ethz.ch

***Abstract*— Two-way relaying, which enables bidirectional simultaneous data transmission between two nodes, is an efficient means to reduce the spectral efficiency loss observed in conventional half-duplex relaying schemes. In this paper, we consider a multiuser cellular two-way relaying scenario with several mobile stations (MSs) at one end of the bidirectional link and a single base station (BS) serving all MSs at the other end. Both the BS and the MSs exchange private messages simultaneously via a single relay node, i.e., concurrent uplink and downlink, in only two time slots, independent of the number of MSs. In the downlink, while the relay separates different MSs spatially, e.g., using either zero-forcing beamforming or zero-forcing dirty paper coding, it benefits from XOR precoding followed by self-interference cancellation, in order to separate messages within a message pair to be exchanged between the BS and each MS. The corresponding sum rate optimization problem is solved with an iterative algorithm based on semidefinite programming.**

***Index Terms*— Two-way relaying, MIMO, multiuser cellular communications, precoding, convex optimization.**

I. Introduction

Two-way relaying [1] has been proposed to reduce the spectral efficiency loss in conventional half-duplex relaying schemes [2], where two time-slots are required for the transmission from source to destination, yielding a pre-log factor 1/2 in the corresponding rate expressions. Instead, in two-way relaying, a bidirectional transmission link is established between source and destination via a relay, employing only two time-slots. When compared to conventional *one-way* relaying schemes that need four time slots for the same transmission, the spectral efficiency doubles with the two-way relaying. The two-way protocol consists of two phases, i.e., time slots. In the first phase, both nodes transmit simultaneously via a multiple access channel scenario to the relay. In the second phase, the relay broadcasts a common message back to the nodes, which is obtained by combining the received messages. Since the nodes know their own transmitted signal, they subtract the back-propagated *self-interference* prior to decoding. The combination of the received messages can be either with superposition coding [1] or XOR precoding [3].

The broadcast capacity region of two-way relaying has been derived in [4], and furthermore, relay selection has been proposed in [5] for two-way relaying. In [6], the effect of transmit channel state information (CSI) is investigated at the decode-and-forward (DF) relay, which is motivated by the assumption that the relay has to estimate the multiple-input multiple-output (MIMO) channels for decoding in the first time slot anyway, and the channel stays the same in the second phase. Considering MIMO nodes, design and optimization of precoders have been presented for both the superposition coding and the XOR precoding. Moreover, XOR precoding has shown to provide higher sum rates than superposition coding if transmit CSI (CSIT) is used.

In this paper, we extend the MIMO two-way relaying scheme with XOR precoding to a multiuser cellular relaying scenario, where there are a base station (BS) and K mobile stations (MS) communicating via a single DF relay. While the BS has private messages for each MS, the MSs want to transmit their own messages to the BS; in other words, there are K message pairs which need to be exchanged. A similar scenario has been considered in [7], where, in contrary to our case, one-way relaying, i.e., transmission from the BS to MSs, is considered with an amplify-and-forward relay. In the context of conventional MIMO two-way relaying [6], the BS can serve each MS in a time-division multiple access fashion via the relay, which, consequently, needs $2K$ time slots. In contrary, we propose to serve all users only in 2 time slots coherently in the same frequency band, where the relay separates different messages intended for the BS and the MSs in two levels: individual messages for MSs are separated spatially, whereas the separation of each member of message pair between the BS and any MS is satisfied through XOR precoding followed by *self-interference* cancellation. In order to separate different MSs, we use either zero-forcing beamforming (ZFBF) [8] or zero-forcing dirty paper coding (ZFDPC) [9]. We aim at maximizing the sum rate of the bidirectional transmissions between the BS and the MSs for both schemes. A novel iterative semidefinite programming based algorithm is proposed for sum rate maximization.[1]

II. System and Signal Model

We consider a relay assisted wireless broadcasting scenario, where there are a single transmitter, K receivers and a single relay, each equipped with N_T, N_R, and M antennas, respectively. As the transmitter sends private messages to the individual receivers, also the receivers have private and independent messages to transmit to the transmitter. The aforementioned

[1] **Notation:** Boldface lowercase and capital letters indicate vectors and matrices, respectively. The superscripts $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ stand for complex conjugate, matrix transpose, complex conjugate transpose, respectively. The operators $\mathrm{E}\{\cdot\}$, $\mathrm{blkdiag}\{\mathbf{X}_1, \mathbf{X}_2, \ldots\}$, $\mathrm{Tr}(\mathbf{X})$, and $\succeq$ denote expectation, a block diagonal matrix with $\mathbf{X}_1$, $\mathbf{X}_2, \ldots$ on its diagonal, the trace of $\mathbf{X}$, and positive semidefiniteness, respectively. $\mathcal{CN}(0, \sigma^2)$ stands for a zero-mean complex normal distribution with variance σ^2.

978-1-4244-2644-7/08/$25.00 ©2008 IEEE

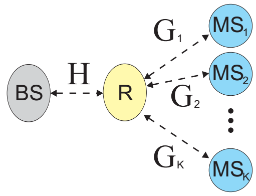

Fig. 1. The bidirectional connection between the BS and K MSs.

communication scenario fits well into a cellular network structure, in which the BS serves several MSs concurrently through the downlink, and receives data from MSs through the uplink. We establish a bidirectional connection between the BS and the MSs via a relay station. It is assumed that there is no direct link between the BS and MSs. The considered cellular network structure is summarized in Fig. 1.

We assume frequency-flat block fading channels between all nodes, where $\mathbf{H} \in \mathbb{C}^{M\times N_T}$, and $\mathbf{G}_i \in \mathbb{C}^{M\times N_R}, i \in \{1,\dots,K\}$ denote the uplink, i.e., transmission to the relay, channel matrices between the BS and the relay, and between the ith MS and the relay, respectively. Due to the reciprocity of the channels in the downlink, i.e., tranmission from the relay, the observed channel matrices at the BS and the ith MS are simply $\mathbf{H}^T$ and $\mathbf{G}_i^T$, respectively.

In the following, we present the principles of two-way relaying for the considered scenario and the corresponding transmission rate expressions. The communication protocol consists of uplink and downlink phases with equal durations, i.e., no time sharing. The channels are assumed to be constant over two phases.

A. Phase 1 - Multiple Access:

In the first phase, the BS and the MSs transmit simultaneously to the relay, whereby none of the transmit nodes have CSI knowledge. The BS wants to transmit the bit sequence $\mathbf{x}_{\text{BS}}^{(i)}$ to the ith MS and the ith MS wants to transmit the bit sequence $\mathbf{x}_{\text{MS}}^{(i)}$ to the BS. Defining the transmit signals of the BS and ith MS as $\left(\mathbf{x}_{\text{BS}}^{(1)}\cdots\mathbf{x}_{\text{BS}}^{(K)}\right) \rightarrow \mathbf{s}_{\text{BS}} \in \mathbb{C}^{N_T}$ and $\mathbf{x}_{\text{MS}}^{(i)} \rightarrow \mathbf{s}_{\text{MS}}^{(i)} \in \mathbb{C}^{N_R}$, respectively, the received signal $\mathbf{r}_{\text{R}}$ at the relay can be expressed as

$$\mathbf{r}_{\text{R}} = \mathbf{H}\mathbf{s}_{\text{BS}} + \sum_{i=1}^{K} \mathbf{G}_i \mathbf{s}_{\text{MS}}^{(i)} + \mathbf{n}_{\text{R}}, \tag{1}$$

where the noise term is assumed to be $\mathbf{n}_{\text{R}} \sim \mathcal{CN}\left(0, \sigma_R^2 \mathbf{I}_M\right)$, and the transmit signals are subject to the power constraints $\mathrm{E}\{\mathbf{s}_{\text{BS}}^H \mathbf{s}_{\text{BS}}\} \le P_{\text{BS}}$ and $\mathrm{E}\{(\mathbf{s}_{\text{MS}}^{(i)})^H \mathbf{s}_{\text{MS}}^{(i)}\} \le P_{\text{MS}}\ \forall i$.

The relay decodes the information from the BS and the MSs assuming that it has sufficient number of antennas for perfect decoding of $N_T + KN_R$ independent spatial substreams. Using a Gaussian codebook, the achievable rates of all nodes are theoretically described by the MIMO multiple access channel (MAC) rate region [10]. Defining the set of all nodes as $\mathcal{N} = \{\text{BS}, \text{MS}_1, \dots, \text{MS}_K\}$, the MAC rate region is given by the following inequalities:

$$R_{\mathcal{S}\to\text{R}} \le I_{\mathcal{S}\to\text{R}} = \log_2\left|\mathbf{I} + \frac{\delta P_{\text{BS}}}{N_T\sigma_R^2}\mathbf{H}\mathbf{H}^H + \sum_{i\in\tilde{\mathcal{S}}}\frac{P_{\text{MS}}}{N_R\sigma_R^2}\mathbf{G}_i\mathbf{G}_i^H\right| \tag{2}$$

for all $\mathcal{S} \subseteq \mathcal{N}$, where $\tilde{\mathcal{S}}$ is the index set of MSs in $\mathcal{S}$, and $\delta = \mathbf{1}_{\{\text{BS}\}\in\mathcal{S}}$ indicates the presence of BS in $\mathcal{S}$.

B. Phase 2 - Broadcast:

In the second phase, after decoding the received messages perfectly, the relay needs to broadcast the messages to the corresponding nodes through the reciprocal channels of the first phase. The BS is supposed to be supplied with all $\mathbf{s}_{\text{MS}}^{(i)}$'s, $i \in \{1,\dots,K\}$, whereas each MS intends to receive the corresponding $\mathbf{s}_{\text{BS}}^{(i)}$, $i \in \{1,\dots,K\}$.

We assume that the transmit signal of the relay $\mathbf{s}_{\text{R}} \in \mathbb{C}^M$ is determined by the decoded bit-sequences of the first phase, i.e., $\left(\mathbf{x}_{\text{BS}}^{(1)}\cdots\mathbf{x}_{\text{BS}}^{(K)}\mathbf{x}_{\text{MS}}^{(1)}\cdots\mathbf{x}_{\text{MS}}^{(K)}\right)$. As the relay transmits in the second phase, the BS and the MSs decode the signals they receive. Since they know the information they have transmitted in the first phase (*self-interference*), they can cancel this contribution and decode the other part of the information.

We use the XOR precoding scheme, which combines the two information bit sequences on bit-level prior to encoding. Specifically, the relay applies bitwise XOR operation on both the ith decoded bit-sequences $\mathbf{x}_{\text{BS}}^{(i)}$ and $\mathbf{x}_{\text{MS}}^{(i)}$, and codes the resulting bit-sequences $\mathbf{x}_{\text{R}}^{(i)}$, i.e., $\mathbf{x}_{\text{MS}}^{(i)} \oplus \mathbf{x}_{\text{BS}}^{(i)} = \mathbf{x}_{\text{R}}^{(i)} \rightarrow \mathbf{s}_{\text{R}}^{(i)}$. Hence, while broadcasting the message pair $\left(\mathbf{x}_{\text{MS}}^{(i)}, \mathbf{x}_{\text{BS}}^{(i)}\right)$, they do not cause interference to each other, i.e., the messages and the corresponding nodes are separated in bit-level. Next, in order to separate different MSs spatially, the transmit signal of the relay $\mathbf{s}_{\text{R}}$ is obtained by superposing $\mathbf{s}_{\text{R}}^{(i)}$'s after precoding each with the matrix $\mathbf{W}_i$: $\mathbf{s}_{\text{R}} = \sum_{i=1}^{K}\mathbf{W}_i\mathbf{s}_{\text{R}}^{(i)}$, where $\sum_{i=1}^{K}\mathrm{Tr}(\mathbf{W}_i\mathbf{W}_i^H) \le P_{\text{R}}$. We explain the structure and the optimization of the precoder matrices $\mathbf{W}_i$ in the following sections, and present here only the rate expressions for the general case.

The received signals at the BS and the MSs are given by

$$\mathbf{r}_{\text{BS}} = \mathbf{H}^T\sum_{i=1}^{K}\mathbf{W}_i\mathbf{s}_{\text{R}}^{(i)} + \mathbf{n}_{\text{BS}}, \text{ and } \mathbf{r}_{\text{MS}}^{(i)} = \mathbf{G}_i^T\sum_{i=1}^{K}\mathbf{W}_i\mathbf{s}_{\text{R}}^{(i)} + \mathbf{n}_{\text{MS}}^{(i)},$$

where the noise terms $\mathbf{n}_{\text{BS}} \sim \mathcal{CN}\left(0, \sigma_{BS}^2\mathbf{I}_{N_T}\right)$, and $\mathbf{n}_{\text{MS}}^{(i)} \sim \mathcal{CN}\left(0, \sigma_{MS}^2\mathbf{I}_{N_R}\right)$. Since the BS is interested in messages from all MSs, i.e., $\mathbf{s}_{\text{R}}^{(i)}\forall i$, it decodes them all, whereas the ith MS decodes only the $\mathbf{s}_{\text{R}}^{(i)}$, assuming the rest $\mathbf{s}_{\text{R}}^{(j)}, j \ne i, j \in \{1,\dots,K\}$ as noise. After decoding, the self-interference cancelation is done by applying a simple XOR operation, i.e., $\mathbf{x}_{\text{R}}^{(i)} \oplus \mathbf{x}_{\text{MS}}^{(i)} = \mathbf{x}_{\text{BS}}^{(i)}$, and $\mathbf{x}_{\text{R}}^{(i)} \oplus \mathbf{x}_{\text{BS}}^{(i)} = \mathbf{x}_{\text{MS}}^{(i)}$. Thus, the mutual information between the relay and the corresponding nodes are given by:

$$I_{\text{R}\to\text{BS}} = \log_2\left|\mathbf{I} + \frac{1}{\sigma_{BS}^2}\mathbf{H}^T\sum_{i=1}^{K}(\mathbf{W}_i\mathbf{W}_i^H)\mathbf{H}^*\right|, \tag{3}$$

$$I_{\text{R}\to\text{MS}}^{(i)} = \log_2\left|\mathbf{I} + \frac{\mathbf{G}_i^T\mathbf{W}_i\mathbf{W}_i^H\mathbf{G}_i^*}{\sigma_{MS}^2\mathbf{I} + \sum_{j=1,j\ne i}^{K}\mathbf{G}_j^T\mathbf{W}_j\mathbf{W}_j^H\mathbf{G}_j^*}\right|, \forall i. \tag{4}$$

The data transferred from the relay to the BS comprises the private messages from all MSs' intended for the BS, whereas the mutual information (3) does not give explicit information about the rates of the individual messages sent from different MSs, i.e., $I_{\text{R}\to\text{BS}}^{(i)}$, $\forall i \in 1,\dots,K$, but only the sum rate of total data transmitted from the relay to the BS, i.e., $I_{\text{R}\to\text{BS}} = \sum_{i=1}^{K} I_{\text{R}\to\text{BS}}^{(i)}$. Hence, the data rate of the individual

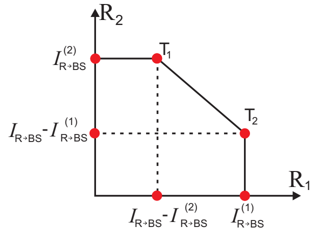

Fig. 2. The rate region of the individual messages of each MS through the transmission from the relay to the BS.

information sent from each MS to the BS, in so much as the corresponding rate region needs to investigated.

In the following, we consider the $K = 2$ case, and present the rate region for the data of individual MSs throughout the transmission from the relay to the BS. Afterwards, the findings will be generalized to an arbitrary value of K. Assume that the decoder at the BS uses successive interference cancellation to decode messages $\mathbf{s}_{\mathrm{R}}^{(1)}$ and $\mathbf{s}_{\mathrm{R}}^{(2)}$, with a decoding order ψ. When $\psi_1 = [1, 2]$, i.e., decode $\mathbf{s}_{\mathrm{R}}^{(1)}$ assuming $\mathbf{s}_{\mathrm{R}}^{(2)}$ is noise, and then subtract the effect of $\mathbf{s}_{\mathrm{R}}^{(1)}$ from the signal and decode $\mathbf{s}_{\mathrm{R}}^{(2)}$, the achievable rates for both messages are expressed with

$$I_{\text{R→BS}}^{(2)} = \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{BS}^2} \mathbf{H}^T \mathbf{W}_2 \mathbf{W}_2^H \mathbf{H}^* \right|,\ I_{\text{R→BS}}^{(1)} = I_{\text{R→BS}} - I_{\text{R→BS}}^{(2)},$$

which represents point T_1 in Fig. 2, and likewise, when $\psi_2 = [2, 1]$, the achievable rates become

$$I_{\text{R→BS}}^{(1)} = \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{BS}^2} \mathbf{H}^T \mathbf{W}_1 \mathbf{W}_1^H \mathbf{H}^* \right|,\ I_{\text{R→BS}}^{(2)} = I_{\text{R→BS}} - I_{\text{R→BS}}^{(1)},$$

which is represented by point T_2 in Fig. 2. The points between T_1 and T_2 can be achieved by time-sharing between different decoding orders, i.e., ψ_1 and ψ_2. The described rate region resembles the conventional MAC rate region, except the fact that time sharing is applied between the decoding orders, but not between individual users. Basically, one can obtain the aforementioned rate region also through rate-splitting or joint decoding. The extension to any $K > 2$ case is trivial and follows the same approach that can be followed for multiple users MAC region, and hence, details are dropped here. Finally, defining the set of messages belonging to different MSs as $\mathcal{M} = \{m_1, m_2, \ldots, m_K\}$, the achievable rate region for the MS messages transmitted from the relay to the BS is described by the inequalities

$$\breve{R}_{\text{R→BS}_\mathcal{W}} \leq \breve{I}_{\text{R→BS}_\mathcal{W}} = \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{BS}^2} \mathbf{H}^T \sum_{i \in \mathcal{W}} (\mathbf{W}_i \mathbf{W}_i^H) \mathbf{H}^* \right|, \quad (5)$$

for all $\mathcal{W} \subseteq \mathcal{M}$, where the subscript $\text{BS}_\mathcal{W}$ denotes the set of MS messages transmitted from the relay to the BS, and with this new notation $\breve{I}_{\text{R→BS}_{m_i}}$ stands for $\tilde{I}_{\text{R→BS}}^{(i)}$ for all $i \in \{1, \ldots, K\}$. Hence, with (5), $I_{\text{R→BS}} \subset \{\breve{I}_{\text{R→BS}_\mathcal{W}}\ \forall \mathcal{W} \subseteq \mathcal{M}\}$.

In contrary to the aforementioned case, the transmission rate of the messages from the BS to the individual MSs is not an issue, since we optimize for sum rate of the BS. Depending on the individual transmission rates of the messages from the relay to each MS, i.e., $I_{\text{R→MS}}^{(i)}$, the BS can allocate its total rate accordingly to the individual private messages to the MSs.

We define R_{BS} and R_{MS_i} as the sum rate that the BS broadcast to the MSs and the rate that the ith MS can transmit data to the BS, respectively. Hence, the overall rate region of the presented multiuser MIMO two-way relaying scheme can be defined by combining all constraints related with the MAC phase, i.e., $I_{\mathcal{S}\text{→R}}\ \forall \mathcal{S} \subseteq \mathcal{N}$, and the broadcast phase, i.e., $I_{\text{R→MS}}^{(i)}\ \forall i, \breve{I}_{\text{R→BS}_\mathcal{W}}\ \forall \mathcal{W} \subseteq \mathcal{M}$, such that we have

$$\{R_{\text{BS}}, R_{\text{MS}_1}, \ldots, R_{\text{MS}_K}\} \in I_{\mathcal{S}\text{→R}} \cup I_{\text{R→MS}}^{(i)} \cup \breve{I}_{\text{R→BS}_\mathcal{W}},$$

for all $\mathcal{S} \subseteq \mathcal{N}$, $\mathcal{W} \subseteq \mathcal{M}$, and $i = \{1, \ldots, N\}$. Note that the final overall transmission rates should be multiplied with 1/2 in order to introduce the effect of two time-slots needed for the relaying traffic pattern.

III. Precoding at the Relay

In the previous section we have derived the rate expressions for two-way relaying but not specified the structure of the precoding matrices at the relay. In this section, we present how to design these precoders with the knowledge of CSIT.

Since we assume that there is no CSI at the transmitters in the first phase, the maximum achievable rates are readily available by (2). However, the achievable rates through the downlink in the second phase are totally dependent on the chosen precoding matrices and given by (3)-(4). The downlink scenario we are considering is nothing but a modified broadcast channel, where the relay broadcasts private messages to the MSs, i.e., $\mathbf{s}_{\mathrm{R}}^{(i)}$ to the ith MS, and there is a node, different from the conventional broadcast channel, namely the BS, which intends to receive all private messages, i.e., $\mathbf{s}_{\mathrm{R}}^{(i)} \forall i$. There are several MIMO broadcast (BC) transmission schemes proposed in the literature, e.g. beamforming (BF), ZFBF, dirty paper coding (DPC). Although the DPC is proven to be the optimal in terms of maximizing the sum rate of the conventional broadcast channel, it is not practical because of its nonlinearity. Instead, we investigate two simpler but suboptimal schemes, which are based on the ZFBF [8], and the successive ZFDPC [9].

A. Zero-Forcing Beamforming

Neglecting the BS for the moment, we apply zero-forcing between the MSs, and choose the precoding matrices, $\mathbf{W}_i$, such that they satisfy the zero-forcing condition $\mathbf{G}_j^T \mathbf{W}_i = 0$ for $j \neq i, i = 1, \ldots, K$. In other words, defining $\tilde{\mathbf{G}}_i = [\mathbf{G}_1 \cdots \mathbf{G}_{i-1}\ \mathbf{G}_{i+1} \cdots \mathbf{G}_K]^T$, $\mathbf{W}_i$ is forced to lie in the nullspace of $\tilde{\mathbf{G}}_i$. In order to transmit data to the ith MS, the nullspace should not be empty, i.e., $M > \text{rank}(\tilde{\mathbf{G}}_i)$. Hence, the condition $M > \max_{1 \leq i \leq K} \{\text{rank}(\tilde{\mathbf{G}}_i)\}$ must be satisfied to zero-force and transmit data concurrently to all MSs. Using the zero-forcing $\mathbf{W}_i^{\text{zf}}$, the received signal at the ith MS becomes

$$\mathbf{r}_{\text{MS}}^{(i)} = \mathbf{G}_i^T \mathbf{W}_i^{\text{zf}} \mathbf{s}_{\mathrm{R}}^{(i)} + \mathbf{n}_{\text{MS}}^{(i)}, \quad (6)$$

and substituting $\mathbf{W}_i^{\text{zf}}$ into (4), it modifies to

$$\begin{aligned} I_{\text{R→MS}}^{\text{zf},(i)} &= \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{MS}^2} \mathbf{G}_i^T \mathbf{W}_i^{\text{zf}} (\mathbf{W}_i^{\text{zf}})^H \mathbf{G}_i^* \right|, \\ &= \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{MS}^2} \mathbf{G}_i^T \mathbf{V}_i \mathbf{\Lambda}_i \mathbf{V}_i^H \mathbf{G}_i^* \right|, \quad (7) \end{aligned}$$

where $\mathbf{V}_i = \text{null}(\tilde{\mathbf{G}}_i)$ and can be computed through the singular value decomposition (SVD) of $\tilde{\mathbf{G}}_i$, and $\mathbf{\Lambda}_i$ is the covariance matrix of the ith message $\mathbf{s}_{\text{R}}^{(i)}$. Next, the sum rate of the transmitted data from the relay to all MSs becomes

$$R_{\text{R}\to\text{MS}}^{\text{sum}} \leq \sum_{i=1}^{K} I_{\text{R}\to\text{MS}}^{\text{zf},(i)} \;=\; \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{MS}^2} \mathbf{G}^T \mathbf{\Lambda} \mathbf{G}^* \right|, \quad (8)$$

where $\mathbf{G} = \text{blkdiag}\{\mathbf{V}_1^T \mathbf{G}_1, \ldots, \mathbf{V}_K^T \mathbf{G}_K\}$, and $\mathbf{\Lambda} = \text{blkdiag}\{\mathbf{\Lambda}_1, \ldots, \mathbf{\Lambda}_K\}$. Deciding on the precoder matrices, the mutual information between the relay and the BS is obtained by substituting the corresponding $\mathbf{W}_i^{\text{zf}}$ into (3):

$$I_{\text{R}\to\text{BS}}^{\text{zf}} \;=\; \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{BS}^2} \mathbf{H}^T \sum_{i=1}^{K} (\mathbf{V}_i \mathbf{\Lambda}_i \mathbf{V}_i^H) \mathbf{H}^* \right|. \quad (9)$$

B. Successive Zero-Forcing Dirty Paper Coding

Zero-forcing beamforming based orthogonalization schemes suffer from the restrictions on the necessary number of transmit antenna, which in return limits the maximum number of users that can be supported concurrently. Moreover, a complete block diagonalization reduces the sum rate considerably when compared with that of DPC based schemes. Hence, the proposition of [9] to combine zero-forcing combined with DPC, will be adopted to our scenario.

Neglecting again the BS, we define an ordered set of users $\mathcal{U}$ with an order π and $|\mathcal{U}| = K$, and force the precoding matrix $\mathbf{W}_{\pi(i)}^{\text{zfdpc}}$ to lie in the null space of

$$\breve{\mathbf{G}}_{i-1} = [\mathbf{G}_{\pi(1)}\ \mathbf{G}_{\pi(2)}\ \cdots\ \mathbf{G}_{\pi(i-1)}]^T \ \text{ for } \ i \in \{2, 3, \ldots, K\}.$$

For $i = 1$, $\breve{\mathbf{G}}_0$ is assumed to be a zero matrix. Thus, the received signal at the $\pi(i)$th MS becomes

$$\mathbf{r}_{\text{MS}}^{(\pi(i))} \;=\; \mathbf{G}_{\pi(i)}^T \mathbf{W}_{\pi(i)}^{\text{zfdpc}} \mathbf{s}_{\text{R}}^{(\pi(i))} + \mathbf{n}_{\text{MS}}^{(\pi(i))}$$
$$+ \underbrace{\mathbf{G}_{\pi(i)}^T \textstyle\sum_{j=1}^{i-1} \mathbf{W}_{\pi(i)}^{\text{zfdpc}} \mathbf{s}_{\text{R}}^{(\pi(i))}}_{\text{cancelled by DPC}} + \underbrace{\mathbf{G}_{\pi(i)}^T \textstyle\sum_{j=i+1}^{K} \mathbf{W}_{\pi(i)}^{\text{zfdpc}} \mathbf{s}_{\text{R}}^{(\pi(i))}}_{\text{cancelled by ZF}},$$

where the third summand, i.e., $\mathbf{G}_{\pi(i)}^T \sum_{j=i+1}^{K} \mathbf{W}_{\pi(i)}^{\text{zfdpc}} \mathbf{s}_{\text{R}}^{(\pi(i))}$, is cancelled by choosing $\mathbf{W}_{\pi(i)}^{\text{zfdpc}}$ in the nullspace of $\breve{\mathbf{G}}_{i-1}$, and the second summand, i.e., $\mathbf{G}_{\pi(i)}^T \sum_{j=1}^{i-1} \mathbf{W}_{\pi(i)}^{\text{zfdpc}} \mathbf{s}_{\text{R}}^{(\pi(i))}$, can be assumed to not exist by applying successive dirty-paper coding with noncausal knowledge of the interfering signals [9].

Substituting $\mathbf{W}_{\pi(i)}^{\text{zfdpc}}$ into (4), we obtain

$$I_{\text{R}\to\text{MS}}^{\text{zfdpc},(\pi(i))} \;=\; \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{MS}^2} \mathbf{G}_{\pi(i)}^T \mathbf{W}_{\pi(i)}^{\text{zfdpc}} (\mathbf{W}_{\pi(i)}^{\text{zfdpc}})^H \mathbf{G}_{\pi(i)}^* \right|,$$
$$= \;\log_2 \left| \mathbf{I} + \frac{1}{\sigma_{MS}^2} \mathbf{G}_{\pi(i)}^T \mathbf{Q}_{i-1} \mathbf{\Gamma}_{\pi(i)} \mathbf{Q}_{i-1}^H \mathbf{G}_{\pi(i)}^* \right|,$$

where $\mathbf{Q}_{i-1} = \text{null}(\breve{\mathbf{G}}_{i-1})$ and can be computed through the SVD of $\breve{\mathbf{G}}_{i-1}$, and $\mathbf{Q}_0$ is an identity matrix for $i = 1$; and $\mathbf{\Gamma}_{\pi(i)}$ is the covariance matrix of the $\pi(i)$th message. Next, the sum rate of the transmitted data from the relay to all MSs becomes

$$R_{\text{R}\to\text{MS}}^{\text{sum}} \leq \sum_{i=1}^{K} I_{\text{R}\to\text{MS}}^{\text{zf},(i)} \;=\; \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{MS}^2} \mathbf{G}_{\pi}^T \mathbf{\Gamma} \mathbf{G}_{\pi}^* \right|, \quad (10)$$

where $\mathbf{G}_{\pi} = \text{blkdiag}\{\mathbf{Q}_0^T \mathbf{G}_{\pi(1)}, \ldots, \mathbf{Q}_{K-1}^T \mathbf{G}_{\pi(K)}\}$, and $\mathbf{\Gamma} = \text{blkdiag}\{\mathbf{\Gamma}_{\pi(1)}, \ldots, \mathbf{\Gamma}_{\pi(K)}\}$. Substituting the corresponding precoding matrices $\mathbf{W}_i^{\text{zfdpc}}$ into (3), the mutual information between the relay and the BS can be expressed as

$$I_{\text{R}\to\text{BS}}^{\text{zfdpc}} \;=\; \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{BS}^2} \mathbf{H}^T \sum_{i=1}^{K} (\mathbf{Q}_{i-1} \mathbf{\Gamma}_{\pi(i)} \mathbf{Q}_{i-1}^H) \mathbf{H}^* \right|.$$

It should be noted that the aforementioned rate expressions are for a given user order π; hence, the throughput can be improved by searching over different user orders.

IV. Maximizing the Sum Rate

In this section, we optimize the precoding matrices for both ZFBF and ZFDPC such that the sum rate, i.e., $R_{\text{BS}} + \sum_{i=1}^{K} R_{\text{MS},i}$, is maximized. In the following, we propose an iterative algorithm which exploits the geometry of the intersection of the uplink and the downlink rate regions.

A. The Problem Formulation

The general sum rate optimization problem is formulated as

$$\begin{aligned} &\max_{R_{\text{BS}}, R_{\text{MS},i}, \mathbf{\Omega}_i \forall i} \Big(R_{\text{BS}} + \sum_{i=1}^{K} R_{\text{MS},i}\Big) \\ &\text{subject to} \quad \{R_{\text{BS}}, R_{\text{MS},1}, \ldots, R_{\text{MS},K}\} \in \mathcal{C}_{\text{MAC}} \\ &\qquad\qquad\qquad \{R_{\text{MS},1}, \ldots, R_{\text{MS},K}\} \in \mathcal{C}_{\text{BS}} \\ &R_{\text{BS}} \leq \textstyle\sum_{i=1}^{K} I_{\text{R}\to\text{MS}}^{(i)},\ \sum_{i=1}^{K} \text{Tr}(\mathbf{\Omega}_i) \leq P_{\text{R}}, \mathbf{\Omega}_i \succeq 0 \forall i, \end{aligned} \quad (11)$$

where $\mathcal{C}_{\text{MAC}}$ is the set of all constraints related to the uplink, as given in (2), and $\mathcal{C}_{\text{BS}}$ is the set of all constraints related to the transmission from the relay to the BS in the second phase, as given in (5). The third constraint in (11) ensures the total transmission rate of the BS, and finally, the fourth constraint guarantees that the total relay power is bounded. The optimization problem (11) is in its most general form in the sense that both broadcasting schemes can be treated by substituting the corresponding mutual information expressions and $\mathbf{\Omega}_i$, i.e., $\mathbf{\Omega}_i := \mathbf{\Lambda}_i$ for ZFBF as defined in Section III-A and $\mathbf{\Omega}_i := \mathbf{\Gamma}_{\pi(i)}$ for ZFDPC as defined in Section III-B.

The sum rate formulation suggests the fact that the transmission rate of the ith common message $\mathbf{s}_{\text{R}}^{(i)}$ to the BS and the ith MS, and consequently, R_{BS} and $\sum_{i=1}^{K} R_{\text{MS}}^{(i)}$, need not to be equal to each other. In order to support unbalanced rates for $\mathbf{x}_{\text{R}}^{(i)}$, zero padding can be used for the lower rated bit sequence, i.e., for $\mathbf{x}_{\text{BS}}^{(i)}$ or $\mathbf{x}_{\text{MS}}^{(i)}$. Let us explain this feature with an example and say that $I_{\text{R}\to\text{BS}}^{(i)} > I_{\text{R}\to\text{MS}}^{(i)}$. Then, the relay pads zeros to the bit sequence $\mathbf{x}_{\text{MS}}^{(i)}$, and chooses the rate of its codebook to be $\max\{I_{\text{R}\to\text{BS}}^{(i)}, I_{\text{R}\to\text{MS}}^{(i)}\} = I_{\text{R}\to\text{BS}}^{(i)}$. Likewise, for decoding the ith message, the BS uses a codebook of size proportional to $I_{\text{R}\to\ \text{BS}}^{(i)}$. On the other hand, the ith MS has the apriori knowledge for decoding, which informs the node that there are some intentional and redundant zeros padded to the received bit stream, i.e., the rate of information is reduced correspondingly, and the node can shrink the size of its codebook to be proportional to $I_{\text{R}\to\text{MS}}^{(i)}$. Thus, by using zero padding and apriori knowledge at corresponding nodes, two-way relaying can support unbalanced rate tuples for each ith common message.

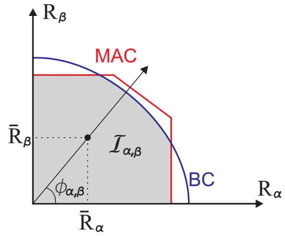

Fig. 3. The illustration of achievable rate region for any pair of rate tuples.

B. Sum Rate Maximization Algorithm

Having $K+1$ nodes exchanging information, i.e., one BS and K MSs, we search for $K+1$ rate tuples inside a space of $K+1$ dimensions, which is defined by the constraints in (11). This $K+1$ dimensional space is the intersection of the MAC and BC rate regions through the first and the second phase, respectively, and convex by definition, i.e., it is the convex hull of all achievable $K+1$ rate tuples.

In the following, before presenting the principles of the general maximization algorithm, we firstly give a simple motivating example for $K=2$ case, and then, building on this framework, the general sum rate maximization algorithm for arbitrary K is presented.

1) 2-Dimensional Case: Each rate pair within the $K+1$ rate tuple, i.e., $(\bar{R}_\alpha, \bar{R}_\beta)$, where $\alpha, \beta \in \{\text{BS}, \text{MS}_1, \ldots, \text{MS}_K\}$ and $\alpha \neq \beta$, is defined in a 2-dimensional convex rate region $\mathcal{I}_{\alpha,\beta}$ as depicted in Fig. 3 with the shaded area. The relation between any tuple within this region can be expressed through an angle $\phi_{\alpha,\beta} \in [0, \pi/2]$ as $\bar{R}_\beta = \bar{R}_\alpha \tan(\phi_{\alpha,\beta})$. Hence, the maximum possible sum rate of this pair, i.e., $\bar{R}_\alpha + \bar{R}_\beta = \bar{R}_\alpha(1+\tan(\phi_{\alpha,\beta}))$, for a given ϕ can be easily found through the optimization

$$\max \ \tau \quad \text{subject to} \ \left(\tau, \tau\tan(\phi_{\alpha,\beta})\right) \in \mathcal{C}_{\text{MAC}}^{(\alpha,\beta)}, \mathcal{C}_{\text{BC}}^{(\alpha,\beta)}, \quad (12)$$

where $\mathcal{C}_{\text{MAC}}^{(\alpha,\beta)}$ and $\mathcal{C}_{\text{BC}}^{(\alpha,\beta)}$ represent the convex MAC and BC region constraints associated with the rate pair (α, β), respectively; and we omit the power and semidefiniteness constraints for notational simplicity. For a given apriori τ, (12) turns out to be a convex semidefinite feasibility check problem. Hence, the problem (12) can be efficiently and optimally solved by a bisection method (over τ) combined with semidefinite feasibility checks [6], [11], [12], but we drop the explicit expressions for the brevity of the paper. Computing the optimal maximal $\tau^\star$, we find the optimal rate pair $(\bar{R}_\alpha^\phi, \bar{R}_\beta^\phi) = (\tau^\star, \tau^\star\tan(\phi_{\alpha,\beta}))$ which maximizes the sum rate of the pair in the direction of $\phi_{\alpha,\beta}$. Next, since the rate region $\mathcal{I}_{\alpha,\beta}$ is convex in terms of $\phi_{\alpha,\beta}$, we can search over the optimal $\phi_{\alpha,\beta}$ that maximizes $R_\alpha + R_\beta$, using an unconstrained minimization method, e.g., the gradient descent method. To sum up, for each iteration of the descent algorithm, i.e., for each chosen $\phi_{\alpha,\beta}$, we solve (12), and iterate $\phi_{\alpha,\beta}$, until iterating further does not induce significant change in sum rate.

2) The General Case: Previously, we only consider the constraints related to the individual chosen (α, β) pair, and disregard the constraints enforced through the relation of the chosen (α, β) pair with other nodes in the set $\mathcal{N}\backslash\{\alpha, \beta\}$. Hence, in the following, we extend the maximization optimization to arbitrary number of K case by considering all $K+1$ dimensional MAC and BC phases' constraints.

TABLE I

OVERALL SUM RATE MAXIMIZATION ALGORITHM

initiate: $\rightarrow \boldsymbol{\phi} \in [0, \pi/2]$
repeat: $\rightarrow$ ***solve*** $\mathcal{P}_\phi$ ***for given*** $\boldsymbol{\phi}$
  initiate: $\rightarrow R_\nu^{\min}, R_\nu^{\max}$
  repeat: $\rightarrow R_\nu = (R_\nu^{\min} + R_\nu^{\max})/2$
    $\rightarrow$ ***solve the feasibility problem for*** $R_\nu, \boldsymbol{\phi}$
      $\left(R_\nu, R_\nu\tan(\phi_{\nu,\mu_1}), \ldots, R_\nu\tan(\phi_{\nu,\mu_K})\right) \in \mathcal{C}_{\text{MAC}}$,
      $\left(R_\nu, R_\nu\tan(\phi_{\nu,\mu_1}), \ldots, R_\nu\tan(\phi_{\nu,\mu_K})\right) \in \mathcal{C}_{\text{BC}}$,
      $\sum_{i=1}^K \text{Tr}(\boldsymbol{\Omega}_i) \leq P_{\text{R}}, \boldsymbol{\Omega}_i \succeq 0 \forall i$,
    $\rightarrow$ ***if feasible***
      $R_\nu^{\min} := R_\nu$
    $\rightarrow$ ***else***
      $R_\nu^{\max} := R_\nu$
  until: $\rightarrow R_\nu^{\max} - R_\nu^{\min} < \epsilon$
  $\rightarrow$ ***compute the search direction for*** $\boldsymbol{\phi} : \Delta\boldsymbol{\phi}$
    numerical first derivative computation for sum rate
  $\rightarrow$ ***line search for choosing step size: t***
  $\rightarrow$ ***update:*** $\boldsymbol{\phi} = \boldsymbol{\phi} + t\Delta\boldsymbol{\phi}$
until: $\rightarrow$ no further significant improvement on sum rate.

Since there are $K+1$ rate dimensions and each pair (α, β) out of $K+1$ dimensions can be interpreted through an angle $\phi_{\alpha,\beta}$, it may be conjectured that we need $(K+1)!/(2(K-1)!)$ angles to represent all relations between all dimensions, whereas, in essence, we need only K angles. This statement can be proven by setting one dimension fixed, say the νth one, and expressing all the rest K dimensions in terms of the νth dimension and a corresponding angle. Thus, the rates for all dimensions can be expressed as

$$\left(R_\nu, R_\nu\tan(\phi_{\nu,\mu_1}), \ldots, R_\nu\tan(\phi_{\nu,\mu_K})\right),$$

for any $\nu \in \mathcal{N}$, where $\mu_1, \ldots, \mu_K$ are the members of the set $\mathcal{N}\backslash\{\nu\}$, and $|\mathcal{N}\backslash\{\nu\}| = K$. Having the knowledge of the vector $\boldsymbol{\phi} = [\phi_{\nu,\mu_1}, \ldots, \phi_{\nu,\mu_K}]$, one can derive all other angles through the relation

$$\tan(\phi_{\mu_i,\mu_j}) = \frac{\tan(\phi_{\nu,\mu_i})}{\tan(\phi_{\nu,\mu_j})}, \quad \forall i,j \in \{1, \ldots, K\}, \ i \neq j.$$

Hence, likewise the case in Section IV-B.1, for a given vector $\boldsymbol{\phi}$, the maximum sum rate of the proposed cellular two-way scheme can be obtained through

$$\mathcal{P}_\phi : \begin{cases} \max_{R_\nu, \boldsymbol{\Omega}_1, \ldots, \boldsymbol{\Omega}_K} \ R_\nu \\ \text{subject to} \ \sum_{i=1}^K \text{Tr}(\boldsymbol{\Omega}_i) \leq P_{\text{R}}, \boldsymbol{\Omega}_i \succeq 0 \forall i, \\ \quad \left(R_\nu, R_\nu\tan(\phi_{\nu,\mu_1}), \ldots, R_\nu\tan(\phi_{\nu,\mu_K})\right) \in \mathcal{C}_{\text{MAC}}, \\ \quad \left(R_\nu, R_\nu\tan(\phi_{\nu,\mu_1}), \ldots, R_\nu\tan(\phi_{\nu,\mu_K})\right) \in \mathcal{C}_{\text{BC}}, \end{cases}$$

where $\mathcal{C}_{\text{BC}}$ is the set of all constraints related to the broadcast region in the second phase, i.e.,

$$\mathcal{C}_{\text{BC}} := \mathcal{C}_{\text{BS}} \cup \left\{ R_{\text{BS}} \leq \sum_{i=1}^K I_{\text{R}\to\text{MS}}^{(i)} \right\} \quad \text{from} \quad (11).$$

The $\mathcal{P}_\phi$ can be efficiently solved with a bisection method combined with semidefinite feasibility checks [6], [11].

Since the $K+1$ dimensional achievable rate region is convex by definition, we can search over $\boldsymbol{\phi}$, whose elements $\phi_i \in [0, \pi/2]$, $i = 1, \ldots, K$, using an unconstrained minimization method. The overall sum rate maximization algorithm is summarized in Table I. While implementing the bisection part of the proposed algorithm in Table I, $R_\nu^{\min}$ is set to 0, $R_\nu^{\max}$ is chosen large enough according to the operation mean SNR value, and ϵ is a small positive number indicating the precision of the bisection algorithm. Moreover, the search direction for ϕ is found through a numerical first derivative computation, i.e., $(f(x+\varepsilon)-f(x))/\varepsilon$. The iterations for the descent algorithm continues until the difference at the sum rate becomes negligible for a new iteration. The convergence and the optimality of the proposed algorithm are ensured through the related conditions of the used methods and convexity of the problem [11].

V. Simulations

In this section we present Monte Carlo simulation results. It is assumed that all elements of $\{\mathbf{H}, \mathbf{G}_i, \forall i\}$ are independently and identically distributed with $\mathcal{CN}(0,\sigma_H^2)$ and $\mathcal{CN}(0,\sigma_G^2)$. The channel matrices are assumed to stay constant over the two phases. All nodes use the same transmit power, i.e., $P = P_{\mathrm{MS}} = P_{\mathrm{BS}} = P_{\mathrm{R}}$, and have the same noise variance, i.e., $\sigma^2 = \sigma_{MS}^2 = \sigma_{BS}^2 = \sigma_R^2$. Hence, the average signal-to-noise ratios for the BS link and the MS links are defined as $\mathrm{SNR}_{\mathrm{BS}} = \sigma_H^2 P/\sigma_n^2$ and $\mathrm{SNR}_{\mathrm{MS}} = \sigma_G^2 P/\sigma_n^2$, respectively.

As a reference system, we assume a scheme, where the relay (hence, the BS) serves MSs one by one, i.e., total transmission lasts for $2K$. In order to be fair to the reference system, it is also assumed that the reference system employs multiuser diversity, i.e., depending on the instantaneous channel conditions, the relay chooses the best user to serve. Note that the sum rate for the reference system can also be computed through the algorithm presented in Table I by setting $K = 1$ and using the corresponding uplink and downlink rate constraints. Throughout the simulations, we use the MATLAB based semidefinite tool *Yalmip* [12] to solve the designed semidefinite problems.

The average sum rates of the proposed and the reference scheme with multiuser diversity is shown in Fig. 4, where $N_T = 4$, $N_R = 2$ and $M = 8$, i.e., sufficiently enough antennas for efficient decoding at all nodes. We investigate the impact of unbalanced link quality, i.e., $\mathrm{SNR}_{\mathrm{BS}} = 10$dB and $\mathrm{SNR}_{\mathrm{MS}} \in [0, 20]$dB. The proposed multiuser two-way relaying scheme with both ZFBF and ZFDPC in the downlink, proposes substantial improvement over the reference. Especially for the symmetric link quality case, i.e., $\mathrm{SNR}_{\mathrm{BS}} = \mathrm{SNR}_{\mathrm{MS}} = 10$dB, up to 4 bps/Hz is gained over the reference for given operation parameters. As $\mathrm{SNR}_{\mathrm{MS}}$ increases, the BS link turns out to be the bottleneck for the sum rate, which suggests that increased number of users can not be supported. This fact is also observed in the figure such that the proposed and the reference schemes approach each other for high $\mathrm{SNR}_{\mathrm{MS}}$ values. Moreover, as expected, the ZFDPC based scheme performs always better than ZFBF based one. As shown in the figure, the MAC sum mutual information $I_{\{\mathrm{BS},\mathrm{MS}_1,\ldots,\mathrm{MS}_K\}}$ stands as an upper bound to achievable sum rate, which depicts that the first phase is the ultimate bottleneck. Although not investigated in this paper, it should be also noted that the proposed scheme can also benefit from user selection for increased number of MSs, i.e., multiuser diversity.

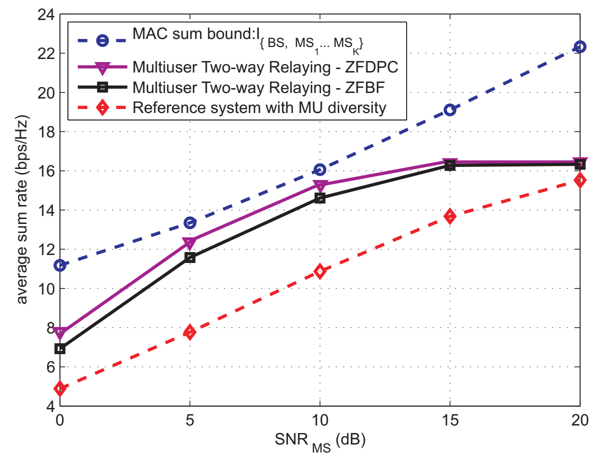

Fig. 4. Average sum rate versus $\mathrm{SNR}_{\mathrm{MS}}$ for $K = 2$, $N_T = 4$, $N_R = 2$, $M = 8$ and $\mathrm{SNR}_{\mathrm{BS}} = 10$dB.

VI. Conclusions

We extended the MIMO two-way relaying to the multiuser case for XOR precoding. The separation of $2K$ messages to the BS and K MSs is accomplished in two level: the interference of exchanged messages between the BS and each MS is cancelled by the usage of XOR precoding, and different MSs are separated spatially. A novel iterative algorithm has been proposed for sum rate maximization, which can be employed in some other similar problems consisting rate region intersections. The proposed scheme is shown to provide substantial sum rate gains over the conventional system with multiuser diversity, which serves a single MS per relaying cycle.

References

[1] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex relay channels", *IEEE Jour. on Selec. Areas in Commun.*, vol. 525, no. 2, pp. 379-389, Feb. 2007.

[2] N. J. Laneman, D. N. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior", *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062-3080, Dec. 2004.

[3] Y. Wu, P. Chou, and S.-Y. Kung, "Information exchange in wireless networks with network coding and physical-layer broadcast", in *Proc. of 39th Annual Conf. on Information Sciences and Systems*, Mar. 2005.

[4] T. J. Oechtering, C. Schnurr, I. Bjelakovic, and H. Boche "Broadcast capacity region of two-phase bidirectional relaying", *IEEE Trans. Inform. Theory*, vol. 54, no. 1, pp. 454-458, Jan. 2008.

[5] T. J. Oechtering and H. Boche "Relay Selection in bidirectional relay communication", in *Proc. IEEE 8th Workshop on of Signal Processing Advances in Wireless Communications*, pp. 1-5, 2007.

[6] I. Hammerström, M. Kuhn, C. Eşli, J. Zhao, A. Wittneben, and G. Bauch, "MIMO two-way relaying with transmit CSI at the relay," in *Proc. IEEE SPAWC 2007, Helsinki, Finland*, June 2007, pp. 5.

[7] C. Chae, T. Tang, R. W. Heath, and S. Cho, "MIMO Relaying With Linear Processing for Multiuser Transmission in Fixed Relay Networks, " *IEEE Tran. on Sig. Proc.*, vol. 56, no. 2, pp. 727-78, Frb. 2008.

[8] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. on Sig. Proc.*, vol. 52, no. 2, pp. 461-471, Feb. 2004.

[9] A. D. Dabbagh and D. J. Love, "Precoding for multiple antenna gaussian broadcast channels with succesive zero-forcing, " *IEEE Trans. on Sig. Proc.*, vol. 55, no. 7, pp. 3837-3850, July 2007.

[10] W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi, "Iterative waterfilling for Gaussian vector multiple access channels, " *IEEE Trans. on Info. Theo.*, vol. 50, pp. 145-152, Jan. 2004.

[11] S. Boyd and L. Vandenberghe, "Convex Optimization, " Cambridge University Press, 2004.

[12] J. Lofberg, "YALMIP : A Toolbox for Modeling and Optimization in MATLAB," in *Proc. of the CACSD Conference, Taipei, Taiwan*, 2004.