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The Cellular Relay Carpet: Distributed Cooperation with Ubiquitous Relaying

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Abstract We consider the up- as well as downlink of a cellular network in which base stations (BSs) with large antenna arrays are supported by a large amount of relays. The relays are spread over the entire area of the network, similar to a carpet. The combination of sophisticated multi-user MIMO transmission with a simple form of BS cooperation and communication in small relay cells allows to reduce the complexity at the terminal nodes and to improve the overall system performance by distributed interference management. We investigate different types of relays as well as different relaying strategies and compare them with respect to complexity, required channel state information (CSI), and performance in the interference-limited environment of dense cellular networks. The robustness of the different schemes with respect to channel estimation errors is studied and we conclude that especially relays of very low complexity are not sensitive to CSI imperfections. The relay carpet proves thereby to be an efficient approach to enhance future generations of cellular networks significantly.

Keywords Cellular networks · cooperation · relaying · multi-user · MIMO · channel state information · imperfections

1 Introduction

The increasing demand for ubiquitous data service sets high expectations on future cellular networks. The next generations of cellular systems should not only provide data rates that are higher by orders of magnitude than today's systems, but also have to guarantee high coverage and reliability [2]. In order to stretch the boundaries of current systems, spectral efficiency has to be increased. This can be achieved by expanding the networks in the spatial domain, i.e. to introduce more antennas to the system, either physically or virtually. In the former case, the base stations (BSs) can be equipped with (many) more antennas, eventually leading to massive MIMO [3]. Such large arrays allow to serve many users at the same time, for instance using multi-user MIMO methods, and to mitigate the interference in adjacent cells.

An alternative is to form virtual arrays that consist of antennas from different nodes distributed in space. An example are heterogeneous networks in which additional antennas are spread in the area [4]. Such an approach can e.g. be followed by installing small cells (femto-cells) that coexist besides larger micro- or macro-cells [5] or by assisting the BSs by remote radio units [6]. It is also foreseen that future networks contain relays that are wirelessly fed by the BSs and assist their communication [7]. Relaying, however, is so far primarily intended for range extension.

Either of these approaches can increase the total throughput of the entire network; individual user rates,

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however, remain limited when the mobile stations (MSs) do not have more antennas. As much of the available resources (e.g. bandwidth) as possible should therefore be allocated to each user, up to a reuse factor of one. Due to the interference-limited nature of cellular networks, this can only be achieved by efficient interference management that mitigates or cancels interference. To this end, current research focuses on BS cooperation/coordinated multipoint (CoMP) transmission that attempts to overcome the impairments of interference that limits the exploitation of the degrees of freedom in the network [8, 9].

However, CoMP suffers from severe challenges and difficulties. BSs that perform joint beamforming require very high backhaul rates, not only to support the data rates of their users, but also to exchange user data and channel state information (CSI) with their cooperation partners. Especially if BSs with large arrays are considered, the number of channel coefficients that need to be estimated grows rapidly with the number of involved antennas. This leads to an increasing overhead, as more pilots have to be included into the signals. Achievable performance gains might therefore stagnate or even decrease [10]. Moreover, even when this overhead can be overcome, the performance of CoMP remains limited and the interference-limited nature of cellular networks cannot be turned into a noise-limited one [11].

1.1 Ubiquitous Relaying

An attempt to combine the advantages of the aforementioned approaches, while avoiding their disadvantages, has been introduced in [1]. In this concept, few BSs that cover large areas are supported by a large amount of relays that are spread across the entire area, like a carpet (see Fig. 1). In this relay carpet, the BSs as well as the MSs see the relays as the nodes they communicate with. If dedicated relays are mounted at fixed positions, fast fading between BSs and relays is virtually eliminated. This allows to equip the BSs with a large number of antennas and to apply sophisticated multi-user MIMO transmission. The relays on the other hand are in close vicinity of the MSs which experience less pathloss and better coverage. This has numerous advantages: The BSs only have to track quasi-static channels which simplifies the estimation of CSI, while for the MSs the network appears like a much simpler network consisting of nodes with only few (effective) antennas. The distributed relays also lead to a more equally distributed signal quality in the area of service. Moreover, the signal processing at the BSs and relays can be performed in a distributed way that can improve the interference situation to a significant extent.

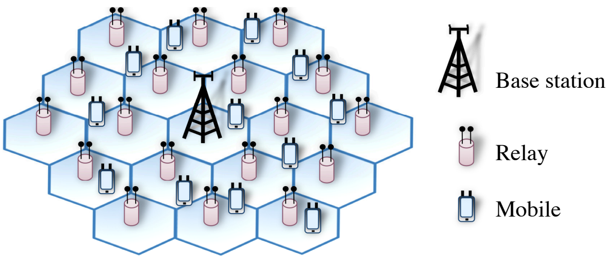

Fig. 1 The relay carpet: a sophisticated BS serves a large amount of MSs in the same physical channel by the help of many distributed relays.

Turning the entire network into a two-hop network is motivated in [1], where an example of a specific two-way relaying protocol with time-division duplex (TDD) relays is applied. The relay carpet concept can however also be seen in a broader context with other relaying schemes. A fundamental advantage of distributing a large number of relays is that they can shape the channel into a beneficial form. Accordingly, the network operators do not have to rely on random channels that can result in deep fades or in users that are shadowed, but can achieve much more homogeneous coverage by the help of the relays. Additionally, when the relays can perform simple signal processing tasks, the complexity at the BSs can be reduced. Thereby, different relay architectures can assist the communication between BSs and MSs in different ways, depending on their available CSI and computational power. The node density and the relay complexity thus lead to a tradeoff in which the overall performance and the total infrastructure costs can be balanced.

1.2 Contribution

In this paper, we extend our study in [1] and discuss different relaying schemes and architectures that can be applied to the relay carpet. We compare different approaches with respect to achievable rates and complexity and propose methods to cope with the interference in such networks, e.g. based on relay filtering and a specific form of BS cooperation, that are of comparably low complexity. Due to this interference mitigation, high performance gains can be achieved. Particularly two-way relaying proves to be very beneficial in contrast to the rather pessimistic results of prior work (cf. e.g. [12]). Furthermore, we investigate the influence of imperfect CSI on these approaches. Finally, we propose a set of transmission schemes that are - according to our simulation results - well suited to achieve interesting performance gains in a scenario as given by our relay carpet setup.

The outline of the paper is as follows: In Section 2 we describe the relay carpet and the different relay architectures considered in this work and formulate the resulting system models. In Section 3, we derive achievable rates for both the up- and downlink that will be used as the performance measure in the remainder of the paper. Transmission schemes for the terminal nodes as well as the relays are developed in Section 4. Aspects of channel estimation at the different nodes are discussed in Section 5 and we also describe their effects on the implementation of the relays and terminal nodes. Extensive simulation results that assess the performance of the relay carpet with and without CSI imperfections are presented in Section 6. Section 7 finally concludes the paper.

Notation: In the following, boldface lowercase and uppercase characters (e.g. $\mathbf{a}$ and $\mathbf{A}$) denote vectors and matrices that generally contain complex values. The operators $(\cdot)^{\mathsf{T}}$ and $(\cdot)^{\mathsf{H}}$ denote transpose and conjugate (Hermitian) transpose, respectively. Expectation, trace, determinant, and null space of a matrix are $\mathsf{E}[\cdot]$, $\mathrm{tr}\{\cdot\}$, $\det\{\cdot\}$, and $\mathrm{null}\,\{\cdot\}$. The $N \times N$ identity matrix is denoted by $\mathbf{I}_N$.

2 The Relay Carpet Network

The basic organization of the network is similar to a conventional cellular network with micro- or macrocells. The area is divided into geographically separated cells, each with one BS that is equipped with a large antenna array and multiple MSs that are served simultaneously. The communication between BS and MSs (downlink) and vice versa (uplink) is assisted by a large amount of relays. We assume that enough relays are placed in each cell such that each active[1] MS is served by at least one relay and that a relay cannot serve more than one MS. The potential disadvantage that moving users might require many hand-overs can be mitigated by assigning more than one relay to serve a specific MS. If GPS information is available, this can be used to predict direction and speed of the users to assign the relays appropriately.

We consider the relays as dedicated infrastructure nodes that are spread in the entire cell. As such, they are intentionally mounted at fixed positions, e.g. on lamp posts, at bus stops, or on the wall of a building, and might therefore have a good connection to the BS. Accordingly, the channels between BS and relays can be assumed to be only slowly fading or even quasi-static; short term fading is virtually eliminated on these links. The MSs, on the other hand, are served by small relay cells and, if sufficiently many relays are deployed, shadowing effects can be avoided to a large extent.

The relays not only improve the connectivity for the MSs, but can also shape the effective channels into a beneficial form. The actual signal processing tasks the relays can perform depend on their architecture and can range from simple active scattering [13] up to sophisticated filtering, interference cancellation [14], or decoding and encoding functionalities [15]. Different relay implementations can thereby affect the signal processing and the complexity at the other nodes. In the following, we describe the different relay architectures that are considered in this paper.

2.1 Relay Architectures

Relays can be classified as full-duplex or half-duplex. While full-duplex relays can simultaneously transmit and receive, half-duplex relays cannot. For instance, half-duplex nodes may operate in TDD mode, i.e. each node transmits and receives in different time slots; in frequency-division duplex (FDD) systems, nodes can transmit and receive at the same time (full-duplex in time) but use different frequency channels. Furthermore, at least two different basic signal processing strategies can be distinguished at the relays: i) The decode-and-forward (DF) strategy involves decoding of the source transmission at the relay, before the re-encoded (and possibly compressed) signal is forwarded to the destination. ii) In case of the compress-and-forward (CF) strategy, the relay does not decode the source transmission but quantizes (and possibly compresses) the signal before forwarding it towards the destination. A special case of the CF strategy is amplify-and-forward (AF) relaying. Here, the signal processing at the relay is purely linear, i.e., transmit antennas at the relay can only forward a linear combination of signals at the receive antennas. In this paper, we consider AF and DF relays and restrict all nodes to perform linear processing. The discussion is limited to a preselection on relaying schemes that seem particularly interesting for the relay carpet scenario. Thereby, no optimality is claimed.

The complexity of relays does not only depend on the relaying strategy but also on further implementation aspects of the relay, as for instance receive and transmit filters. In the following, we consider two different types of relay implementations: In *type A* relays, a very simple implementation is used without a special receive or transmit filter. Before transmitting, these relays only scale the signal with a gain matrix given by a

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[1] Relays can serve more than one MS by sharing the resources with a TDMA or FDMA scheme.

scaled identity matrix. The more complex *type B* relays use a receive and a transmit filter. An especially simple class of relays is given by type A AF relays in an FDD system; such relays can be implemented by a frequency conversion of the received signal.

We apply different bidirectional relaying protocols that do not use a direct link between BSs and MSs. These protocols can be classified into one-way and two-way relaying. In the former case, the up- and downlink are separated in orthogonal resources and the relays either forward only the BS signals to the MSs or only the MS signals to the BSs. In two-way relaying, both directions of communication are combined such that the relays receive the superposition of all BS and MS signals and broadcast a processed version of these signals back to all terminals. With this, the spectral efficiency can be doubled as compared to one-way relaying. An inherent drawback of two-way relaying is that the signal received by a terminal (either BS or MS) also contains the signal that this terminal has previously transmitted and is backscattered by the relays [16]. This so-called *self-interference* needs to be subtracted at the terminal before the signal can be decoded.

2.2 System Model

The network under consideration consists of C cells, each containing a single BS and multiple MSs. For notational simplicity, we assume that all cells have the same number M of active MSs and that all nodes of the same kind have the same number of antennas, although an extension to a more general case is straightforward. The number of antennas at the BSs is denoted by N_B, the one of the MSs by N_M. The considered communication is bidirectional, i.e. BS $c \in \{1, \ldots, C\}$ wants to transmit $d_\mathrm{s} \leq N_\mathrm{M}$ data streams to MS (c, k) (the kth MS in cell c) in the downlink and, in turn, each MS wishes to send d_s data streams to its BS in the uplink.

As each BS simultaneously serves multiple MSs located in its corresponding cell, we assume $N_\mathrm{B} \geq M \cdot N_\mathrm{M}$ and write the transmit signal of BS c in the downlink as

$$\mathbf{x}_c^{(\mathrm{B})} = \sum_{k=1}^{M} \mathbf{Q}_{c,k}^{(\mathrm{B})} \cdot \mathbf{s}_{c,k}^{(\mathrm{B})}, \tag{1}$$

where $\mathbf{s}_{c,k}^{(\mathrm{B})} \in \mathbb{C}^{d_\mathrm{s}}$ is the transmit symbol vector from BS c intended for MS (c, k) and $\mathbf{Q}_{c,k}^{(\mathrm{B})} \in \mathbb{C}^{N_\mathrm{B} \times d_\mathrm{s}}$ is the corresponding precoding matrix. In the uplink, the MSs transmit

$$\mathbf{x}_{c,k}^{(\mathrm{M})} = \mathbf{Q}_{c,k}^{(\mathrm{M})} \cdot \mathbf{s}_{c,k}^{(\mathrm{M})}, \tag{2}$$

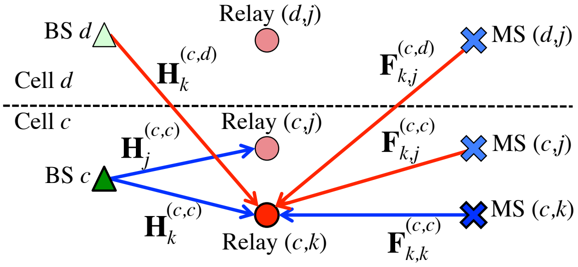

Fig. 2 Network model. The communication between BSs and MSs is assisted by relays.

with $\mathbf{s}_{c,k}^{(\mathrm{M})} \in \mathbb{C}^{d_\mathrm{s}}$ and $\mathbf{Q}_{c,k}^{(\mathrm{M})} \in \mathbb{C}^{N_\mathrm{M} \times d_\mathrm{s}}$ being the transmit symbol vector and the precoding matrix of the signal from MS (c, k) intended for BS c.

The bidirectional communication between BSs and MSs is assisted by $K \geq M$ relays. The relays are equipped with N_R antennas, where $N_\mathrm{B} \geq N_\mathrm{R} \geq N_\mathrm{M}$. A sketch of the network can be seen in Fig. 2. The channel from BS d to relay (c, k) is denoted by $\mathbf{H}_k^{(c,d)} \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{B}}$ and the reverse channel from relay (c, k) to BS d by $\overline{\mathbf{H}}_k^{(d,c)} \in \mathbb{C}^{N_\mathrm{B} \times N_\mathrm{R}}$. The channels from MS (d, j) to relay (c, k) and vice versa are denoted by $\mathbf{F}_{k,j}^{(c,d)} \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{M}}$ and $\overline{\mathbf{F}}_{j,k}^{(d,c)} \in \mathbb{C}^{N_\mathrm{M} \times N_\mathrm{R}}$, respectively. When a TDD protocol is applied, the channels are assumed to be reciprocal, i.e. $\overline{\mathbf{H}}_k^{(d,c)} = \mathbf{H}_k^{(c,d)\mathsf{T}}$ and $\overline{\mathbf{F}}_{j,k}^{(d,c)} = \mathbf{F}_{k,j}^{(c,d)\mathsf{T}}$. If the system is operated in the FDD mode, the channels on the different directions are assumed to be independent as different frequency bands are used. In the following, we describe the end-to-end relations of the system model when different relaying strategies are applied.

2.3 AF One-Way Relaying

In one-way relaying, the up- and downlink are separated, either by different time slots (TDD) or orthogonal frequency bands (FDD). Considering the downlink, the BSs simultaneously transmit their signal (1) and relay (c, k) receives

$$\overrightarrow{\mathbf{r}}_{c,k} = \sum_{d=1}^{C} \mathbf{H}_k^{(c,d)} \cdot \sum_{j=1}^{M} \mathbf{Q}_{d,j}^{(\mathrm{B})} \cdot \mathbf{s}_{d,j}^{(\mathrm{B})} + \overrightarrow{\mathbf{n}}_{c,k}, \tag{3}$$

where $\overrightarrow{\mathbf{n}}_{c,k}$ is the noise induced in the relay. Assuming AF relaying, the relays multiply their receive signals (3) with a gain matrix $\mathbf{G}_{c,k} \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{R}}$ and, after a possible frequency conversion in FDD, retransmit $\overrightarrow{\mathbf{t}}_{c,k} = \mathbf{G}_{c,k} \cdot \overrightarrow{\mathbf{r}}_{c,k}$ to the MSs. The receive signal of MS (c, k) is then

$$\overrightarrow{\mathbf{y}}_{c,k} = \sum_{d=1}^{C} \sum_{j=1}^{K} \overline{\mathbf{F}}_{k,j}^{(c,d)} \cdot \mathbf{G}_{d,j} \cdot \overrightarrow{\mathbf{r}}_{d,j} + \mathbf{w}_{c,k}^{(\mathrm{M})}, \tag{4}$$

with $\mathbf{w}_{c,k}^{(\mathrm{M})}$ being the noise in the MS.

In the uplink, the MSs transmit their signals (2) and the receive signal at the relays can be written as

$$\overleftarrow{\mathbf{r}}_{c,k} = \sum_{d=1}^{C}\sum_{j=1}^{M} \mathbf{F}_{k,j}^{(c,d)} \cdot \mathbf{Q}_{d,j}^{(\mathrm{M})} \cdot \mathbf{s}_{d,j}^{(\mathrm{M})} + \overleftarrow{\mathbf{n}}_{c,k}. \tag{5}$$

After multiplication of $\overleftarrow{\mathbf{r}}_{c,k}$ with $\mathbf{G}_{c,k}$ and forwarding the resulting signal $\overleftarrow{\mathbf{t}}_{c,k} = \mathbf{G}_{c,k} \cdot \overleftarrow{\mathbf{r}}_{c,k}$, BS c receives

$$\overleftarrow{\mathbf{y}}_{c} = \sum_{d=1}^{C}\sum_{j=1}^{K} \overline{\mathbf{H}}_{j}^{(c,d)} \cdot \mathbf{G}_{d,j} \cdot \overleftarrow{\mathbf{r}}_{d,j} + \mathbf{w}_{c}^{(\mathrm{B})}, \tag{6}$$

where $\mathbf{w}_{c}^{(\mathrm{B})}$ is the BS noise.

When half-duplex relays are used, i.e. relays that cannot simultaneously transmit and receive in the same resource block, four orthogonal dimensions are required for a transmission in both directions: the two-hop communication requires two resources on each direction. As an alternative, two-way relaying is an efficient approach to save resources and to improve the spectral efficiency.

2.4 AF Two-Way Relaying

In two-way relaying, both directions of communication, i.e. the down- and the uplink, are combined into the same physical channel [17]. To this end, all BSs and MSs transmit their signals (1) and (2) simultaneously and the relays receive the superposition of all these signals

$$\mathbf{r}_{c,k} = \sum_{d=1}^{C}\sum_{j=1}^{M}\Big(\mathbf{H}_{k}^{(c,d)}\mathbf{Q}_{d,j}^{(\mathrm{B})}\mathbf{s}_{d,j}^{(\mathrm{B})} + \mathbf{F}_{k,j}^{(c,d)}\mathbf{Q}_{d,j}^{(\mathrm{M})}\mathbf{s}_{d,j}^{(\mathrm{M})}\Big) + \mathbf{n}_{c,k}. \tag{7}$$

As before, the AF relays multiply their receive signal vector with a gain matrix $\mathbf{G}_{c,k}$ and broadcast the resulting signal back to all terminal nodes. The resulting signals received by BS c and MS (c,k) are thus given by (8) and (9). These signals not only include the desired signal but also contain what the corresponding node has transmitted itself (self-interference) as well as additional interference from the other nodes of the same kind.

2.5 DF One-Way Relaying

In contrast to the AF case, DF relays completely decode the signals they receive before they forward them. The receive signal of relay (c,k) in the downlink is the same as in (3). This signal can then be filtered by a receive combining matrix $\mathbf{G}_{c,k}^{(\mathrm{Rx})\mathsf{H}}$, which leads to

$$\overrightarrow{\tilde{\mathbf{r}}}_{c,k} = \mathbf{G}_{c,k}^{(\mathrm{Rx})\mathsf{H}} \cdot \Big(\mathbf{H}_{k}^{(c,c)}\mathbf{Q}_{c,k}^{(\mathrm{B})}\mathbf{s}_{c,k}^{(\mathrm{B})} + \overrightarrow{\mathbf{x}}_{c,k}^{(\mathrm{R,i+n})}\Big), \tag{10}$$

where $\overrightarrow{\mathbf{x}}_{c,k}^{(\mathrm{R,i+n})}$ contains all interference and noise terms. The symbol vector $\mathbf{s}_{c,k}^{(\mathrm{B})}$ is decoded, while $\overrightarrow{\mathbf{x}}_{c,k}^{(\mathrm{R,i+n})}$ is considered as noise. After that, the relays newly encode the data symbols, possibly with a different code book. Finally, the resulting symbols $\tilde{\mathbf{s}}_{c,k}^{(\mathrm{B})}$ are premultiplied by a transmit filter matrix $\mathbf{G}_{c,k}^{(\mathrm{Tx})}$ and forwarded to the MSs. The receive signal of MS (c,k) follows as

$$\overrightarrow{\mathbf{y}}_{c,k} = \sum_{d=1}^{C}\sum_{j=1}^{K} \overline{\mathbf{F}}_{k,j}^{(c,d)} \cdot \mathbf{G}_{d,j}^{(\mathrm{Tx})} \cdot \tilde{\mathbf{s}}_{d,j}^{(\mathrm{B})} + \mathbf{w}_{c,k}^{(\mathrm{M})}. \tag{11}$$

In the uplink, the relays receive the signals from the MSs. The receive signal at relay (c,k), after applying the receive filter, is

$$\overleftarrow{\tilde{\mathbf{r}}}_{c,k} = \mathbf{G}_{c,k}^{(\mathrm{Rx})\mathsf{H}} \cdot \Big(\mathbf{F}_{k,k}^{(c,c)}\mathbf{Q}_{c,k}^{(\mathrm{M})}\mathbf{s}_{c,k}^{(\mathrm{M})} + \overleftarrow{\mathbf{x}}_{c,k}^{(\mathrm{R,i+n})}\Big), \tag{12}$$

where $\overleftarrow{\mathbf{x}}_{c,k}^{(\mathrm{R,i+n})}$ contains the relay noise and all MS interference terms. The relay decodes the corresponding MS symbol vector $\mathbf{s}_{c,k}^{(\mathrm{M})}$, encodes it to $\tilde{\mathbf{s}}_{c,k}^{(\mathrm{M})}$, and multiplies it with $\mathbf{G}_{c,k}^{(\mathrm{Tx})}$. After retransmission, BS c receives

$$\overleftarrow{\mathbf{y}}_{c} = \sum_{d=1}^{C}\sum_{j=1}^{K} \overline{\mathbf{H}}_{j}^{(c,d)} \mathbf{G}_{d,j}^{(\mathrm{Tx})} \tilde{\mathbf{s}}_{d,j}^{(\mathrm{M})} + \mathbf{w}_{c}^{(\mathrm{B})}. \tag{13}$$

2.6 DF Two-Way Relaying

In the case of two-way relaying, the BSs and MSs transmit simultaneously and relay (c,k) receives

$$\tilde{\mathbf{r}}_{c,k} = \mathbf{G}_{c,k}^{(\mathrm{Rx})\mathsf{H}}\Big(\mathbf{H}_{k}^{(c,c)}\mathbf{Q}_{c,k}^{(\mathrm{B})}\mathbf{s}_{c,k}^{(\mathrm{B})} + \mathbf{F}_{k,k}^{(c,c)}\mathbf{Q}_{c,k}^{(\mathrm{M})}\mathbf{s}_{c,k}^{(\mathrm{M})} + \mathbf{x}_{c,k}^{(\mathrm{R,i+n})}\Big). \tag{14}$$

$$\mathbf{y}_{c}^{(\mathrm{B})} = \sum_{d=1}^{C}\sum_{j=1}^{K}\sum_{b=1}^{C}\sum_{i=1}^{M}\Big(\overline{\mathbf{H}}_{j}^{(c,d)}\mathbf{G}_{d,j}\mathbf{H}_{j}^{(d,b)}\mathbf{Q}_{b,i}^{(\mathrm{B})}\mathbf{s}_{b,i}^{(\mathrm{B})} + \overline{\mathbf{H}}_{j}^{(c,d)}\mathbf{G}_{d,j}\mathbf{F}_{j,i}^{(d,b)}\mathbf{Q}_{b,i}^{(\mathrm{M})}\mathbf{s}_{b,i}^{(\mathrm{M})}\Big) + \sum_{d=1}^{C}\sum_{j=1}^{K}\overline{\mathbf{H}}_{j}^{(c,d)}\mathbf{G}_{d,j}\mathbf{n}_{d,j} + \mathbf{w}_{c}^{(\mathrm{B})} \tag{8}$$

$$\mathbf{y}_{c,k}^{(\mathrm{M})} = \sum_{d=1}^{C}\sum_{j=1}^{K}\sum_{b=1}^{C}\sum_{i=1}^{M}\Big(\overline{\mathbf{F}}_{k,j}^{(c,d)}\mathbf{G}_{d,j}\mathbf{H}_{j}^{(d,b)}\mathbf{Q}_{b,i}^{(\mathrm{B})}\mathbf{s}_{b,i}^{(\mathrm{B})} + \overline{\mathbf{F}}_{k,j}^{(c,d)}\mathbf{G}_{d,j}\mathbf{F}_{j,i}^{(d,b)}\mathbf{Q}_{b,i}^{(\mathrm{M})}\mathbf{s}_{b,i}^{(\mathrm{M})}\Big) + \sum_{d=1}^{C}\sum_{j=1}^{K}\overline{\mathbf{F}}_{k,j}^{(c,d)}\mathbf{G}_{d,j}\mathbf{n}_{d,j} + \mathbf{w}_{c,k}^{(\mathrm{M})} \tag{9}$$

Now both data symbol vectors $\mathbf{s}_{c,k}^{(\mathrm{B})}$ and $\mathbf{s}_{c,k}^{(\mathrm{M})}$ are desired. These are decoded by successive interference cancellation (SIC) [18]. The relay can then combine the decoded data streams by an XOR operation with zero padding [19]. The combined data symbol vector $\tilde{\mathbf{s}}_{c,k}^{(\mathrm{R})}$ is precoded by $\mathbf{G}_{c,k}^{(\mathrm{Tx})}$ and the resulting signal is broadcasted. BS c and MS (c,k) then receive this signal under interference from the other relays

$$\mathbf{y}_c^{(\mathrm{B})} = \sum_{d=1}^{C}\sum_{j=1}^{K} \overline{\mathbf{H}}_j^{c,d} \cdot \mathbf{G}_{d,j}^{(\mathrm{Tx})} \cdot \tilde{\mathbf{s}}_{d,j}^{(\mathrm{R})} + \mathbf{w}_c^{(\mathrm{B})} \tag{15}$$

$$\mathbf{y}_{c,k}^{(\mathrm{M})} = \sum_{d=1}^{C}\sum_{j=1}^{K} \overline{\mathbf{F}}_{k,j}^{c,d} \cdot \mathbf{G}_{d,j}^{(\mathrm{Tx})} \cdot \tilde{\mathbf{s}}_{d,j}^{(\mathrm{R})} + \mathbf{w}_{c,k}^{(\mathrm{M})}. \tag{16}$$

When the relay signal is decoded, the terminals can apply another XOR operation with the data symbols they have previously transmitted. With this form of self-interference cancellation, the desired signal can be reconstructed at the terminals. In order to decode all signals from relays in their own cell, the BSs can again apply SIC.

3 Achievable Rates

Once precoding and relay gain matrices are chosen, achievable rates for one-way and two-way relaying can be formulated for both directions of communication. It is thereby assumed that the data symbols in the vectors $\mathbf{s}_{c,k}^{(\mathrm{B})}$ and $\mathbf{s}_{c,k}^{(\mathrm{M})}$ are independent and identically distributed (i.i.d) according to $\mathcal{CN}(0,1)$. The elements of the noise terms in the relays and terminals, $\mathbf{n}_{c,k}$, $\mathbf{w}_c^{(\mathrm{B})}$, and $\mathbf{w}_{c,k}^{(\mathrm{M})}$, are assumed to be i.i.d. $\mathcal{CN}(0,\sigma_n^2)$ and $\mathcal{CN}(0,\sigma_w^2)$, respectively.

3.1 AF Relaying

In one-way relaying, the achievable rate of the downlink transmission from BS c to MS (c,k) is calculated by

$$\overrightarrow{R}_{c,k} = \log_2 \det\left\{\mathbf{I}_{N_\mathrm{M}} + \left(\overrightarrow{\mathbf{K}}_{c,k}^{(\mathrm{i+n})}\right)^{-1} \cdot \overrightarrow{\mathbf{K}}_{c,k}^{(\mathrm{sig})}\right\}, \tag{17}$$

where $\overrightarrow{\mathbf{K}}_{c,k}^{(\mathrm{sig})}$ and $\overrightarrow{\mathbf{K}}_{c,k}^{(\mathrm{i+n})}$ are covariance matrices of the desired signal and interference plus noise, which are given in Appendix A.

In the uplink, we assume that the BSs try to jointly decode all signals from the MSs within their corresponding cell. The achievable sum-rate of the uplink at BS c is thus

$$\overleftarrow{R}_c = \log_2 \det\left\{\mathbf{I}_{N_\mathrm{B}} + \left(\overleftarrow{\mathbf{K}}_c^{(\mathrm{i+n})}\right)^{-1} \cdot \overleftarrow{\mathbf{K}}_c^{(\mathrm{sig})}\right\}, \tag{18}$$

where $\overleftarrow{\mathbf{K}}_c^{(\mathrm{sig})}$ is the covariance matrix of the desired signal at BS c that now contains the signals from all MSs in cell c. Accordingly, $\overleftarrow{\mathbf{K}}_c^{(\mathrm{i+n})}$ contains the noise as well as the signals originated from all other MSs. These matrices are also derived in Appendix A.

In the two-way case, where all up- and downlink signals are combined, the receive signals at the terminals additionally contain the signals these nodes have injected into the network themselves as well as the signals from the other nodes of the same kind. The achievable rate of the downlink

$$R_{c,k}^{(\mathrm{DL})} = \log_2 \det\left\{\mathbf{I}_{N_\mathrm{M}} + \left(\mathbf{K}_{\mathrm{M},c,k}^{(\mathrm{i+n})} + \mathbf{K}_{\mathrm{M},c,k}^{(\mathrm{self})}\right)^{-1} \cdot \mathbf{K}_{\mathrm{M},c,k}^{(\mathrm{sig})}\right\} \tag{19}$$

thus additionally contains the covariance matrix of the self-interference $\mathbf{K}_{\mathrm{M},c,k}^{(\mathrm{self})}$. For the uplink, we distinguish between interference that is caused by the BSs (including self-interference) and remaining interference from the MSs. The achievable sum rate at BS c thus results in

$$R_c^{(\mathrm{UL})} = \log_2 \det\left\{\mathbf{I}_{N_\mathrm{B}} + \left(\mathbf{K}_{\mathrm{B},c}^{(\mathrm{i+n})} + \mathbf{K}_{\mathrm{B},c}^{(\mathrm{BS\ int})}\right)^{-1} \cdot \mathbf{K}_{\mathrm{B},c}^{(\mathrm{sig})}\right\}. \tag{20}$$

Again, the interested reader is referred to the appendix for the exact form of the covariance matrices.

3.2 DF Relaying

Achievable rates are also derived for the case when DF relays are used. The one-way case is considered first. When the BSs have transmitted their signals in the downlink and relay (c,k) has applied its receive filter, it decodes the symbol vector $\mathbf{s}_{c,k}^{(\mathrm{B})}$ that is contained in the receive signal (10). To this end, the interference in $\overrightarrow{\mathbf{x}}_{c,k}^{(\mathrm{R,i+n})}$ is treated as noise and the resulting rate on the BS-to-relay link can be given as

$$\overrightarrow{R}_{c,k}^{(\mathrm{BR})} = \log_2 \det\left\{\mathbf{I}_{N_\mathrm{R}} + \left(\overrightarrow{\mathbf{K}}_{\mathrm{R},c,k}^{(\mathrm{i+n})}\right)^{-1} \cdot \overrightarrow{\mathbf{K}}_{\mathrm{R},c,k}^{(\mathrm{sig})}\right\} \tag{21}$$

with

$$\overrightarrow{\mathbf{K}}_{\mathrm{R},c,k}^{(\mathrm{sig})} = \mathbf{G}_{c,k}^{(\mathrm{Rx})\mathsf{H}} \cdot \mathbf{H}_k^{(c,c)} \mathbf{Q}_{c,k}^{(\mathrm{B})} \mathbf{Q}_{c,k}^{(\mathrm{B})\mathsf{H}} \mathbf{H}_k^{(c,c)\mathsf{H}} \mathbf{G}_{c,k}^{(\mathrm{Rx})} \tag{22}$$

and

$$\overrightarrow{\mathbf{K}}_{\mathrm{R},c,k}^{(\mathrm{i+n})} = \mathsf{E}\left[\overrightarrow{\mathbf{x}}_{c,k}^{(\mathrm{R,i+n})} \cdot \overrightarrow{\mathbf{x}}_{c,k}^{(\mathrm{R,i+n})\mathsf{H}}\right]. \tag{23}$$

The newly encoded data symbols $\tilde{\mathbf{s}}_{c,k}^{(\mathrm{B})}$ are multiplied with $\mathbf{G}_{c,k}^{(\mathrm{Tx})}$ and forwarded to the MSs. The achievable rate on the second hop can similarly be calculated and results in $\overrightarrow{R}_{c,k}^{(\mathrm{RM})}$. The final rate of the BS-to-MS link follows then as

$$\overrightarrow{R}_{c,k} = \min\left\{\overrightarrow{R}_{c,k}^{(\mathrm{BR})}, \overrightarrow{R}_{c,k}^{(\mathrm{RM})}\right\}. \tag{24}$$

The end-to-end rate of the uplink can be obtained in a similar way. When the rate of the transmission from MS (c,k) to relay (c,k) is $\overleftarrow{R}_{c,k}^{(\mathrm{MR})}$ and the one of the link from this relay to BS c is $\overleftarrow{R}_{c,k}^{(\mathrm{RB})}$, the resulting sum rate of the uplink to BS c is

$$\overleftarrow{R}_{c} = \sum_{k=1}^{M} \min\left\{\overleftarrow{R}_{c,k}^{(\mathrm{MR})}, \overleftarrow{R}_{c,k}^{(\mathrm{RB})}\right\}. \tag{25}$$

Note that the achievable rates follow from the assumption that equally long time slots and the same bandwidth is used for both hops. The end-to-end rate could be improved by optimizing the time and frequency allocation of the two individual links. This, however, is unpractical in the cellular context as the different links of the up- and downlink in adjacent cells would not necessarily be separated anymore.

In the case of two-way relaying, the BSs and MSs transmit simultaneously in the multiple access (MAC) phase. The receive signal of relay (c,k) is given by (14). Both data symbol vectors $\mathbf{s}_{c,k}^{(\mathrm{B})}$ and $\mathbf{s}_{c,k}^{(\mathrm{M})}$ are decoded using SIC, which leads to a pair of resulting achievable rates $R_{c,k}^{(\mathrm{BR})}$ and $R_{c,k}^{(\mathrm{MR})}$, one for the signal from BS c intended for MS (c,k) and vice versa. The combined and newly encoded relay symbol vector $\tilde{\mathbf{s}}_{c,k}^{(\mathrm{R})} \sim \mathcal{CN}(\mathbf{O}, \mathbf{I})$ is precoded by the relay transmit filter and forwarded to the terminals in the broadcast (BC) phase. BS c and MS (c,k) then receive this signal under interference from the other relays. With self-interference cancellation, the desired signal can be reconstructed at the terminals, where the BSs again apply SIC to decode all signals from their relays. When the achievable rate pairs of the broadcast phase with respect to each relay/user are given by $R_{c,k}^{(\mathrm{RB})}$ and $R_{c,k}^{(\mathrm{RM})}$, the resulting rates achievable on the two-hop up- and downlink are finally

$$R_{c,k}^{(\mathrm{DL})} = \min\left\{R_{c,k}^{(\mathrm{BR})}, R_{c,k}^{(\mathrm{RM})}\right\} \tag{26}$$

$$R_{c}^{(\mathrm{UL})} = \sum_{k=1}^{M} \min\left\{R_{c,k}^{(\mathrm{MR})}, R_{c,k}^{(\mathrm{RB})}\right\}. \tag{27}$$

Choosing the rates like this ensures that they lie inside the achievable rate region. However, no optimality is claimed.

3.3 Prelog Factor

Note that we have dropped the prelog factors in the achievable rates. Depending on the relaying strategy as well as the considered duplex mode, different factors (1, $\frac{1}{2}$, or $\frac{1}{4}$) would apply. Also in conventional cellular networks where no relays are present, the up- and downlink have to be separated by orthogonal resource blocks that can have different sizes for both directions. For the sake of comparability and to avoid discussions on how the resource blocks are divided for the up- and downlink, we omit theses factors.

Additionally, also the use of full-duplex relays is possible when the transmit and receive chains are sufficiently separated. No prelog loss would occur in this case. Alternatively, the relays can convert the BS signals to a frequency band that is currently not used (cf. cognitive radio [20]). In this way, the relay-to-MS link operates in a secondary band that is free at the moment of usage. The additional costs of such bands thus do not have to be included into the spectral efficiency.

4 Transmission Schemes

In order to gain more understanding in what the limiting factors of the considered network are, we analyze the individual terms of the receive signals at the terminals. To this end, we apply spatially white signaling at all involved nodes and a scaled identity matrix at the relays. In this way, no interference is mitigated and the whole network is flooded with signals. This allows to measure the individual signal contributions for both the up- and downlink and to identify the strongest interference sources. Based on this analysis, we can then design precoding and relay gain matrices that mitigate the most severe interference terms.

We apply a per node transmit power of $P_{\mathrm{B}} = 40\,\mathrm{W}$ at the BSs and $P_{\mathrm{R}} = 6\,\mathrm{W}$ and $P_{\mathrm{M}} = 0.2\,\mathrm{W}$ at the relays and MSs. The precoding and relay gain matrices are accordingly

$$\mathbf{Q}_{c,k}^{(\mathrm{B})} = \sqrt{P_{\mathrm{B}}/N_{\mathrm{B}}} \cdot \mathbf{I}_{N_{\mathrm{B}}}, \quad \mathbf{Q}_{c,k}^{(\mathrm{M})} = \sqrt{P_{\mathrm{M}}/N_{\mathrm{M}}} \cdot \mathbf{I}_{N_{\mathrm{M}}},$$

and

$$\mathbf{G}_{c,k} = \sqrt{P_{\mathrm{R}}/\mathrm{tr}\left\{\mathsf{E}\left[\mathbf{r}_{c,k} \cdot \mathbf{r}_{c,k}^{\mathsf{H}}\right]\right\}} \cdot \mathbf{I}_{N_{\mathrm{R}}}.$$

The resulting receive signal powers of the BSs and MSs, for one-way as well as two-way relaying are shown in Fig. 3. The network consists of $C = 19$ cells, each containing $M = K = 6$ MSs/relays. In the figure, we distinguish which relays have forwarded the different signal contributions (own relay, other in-cell relays, or relays

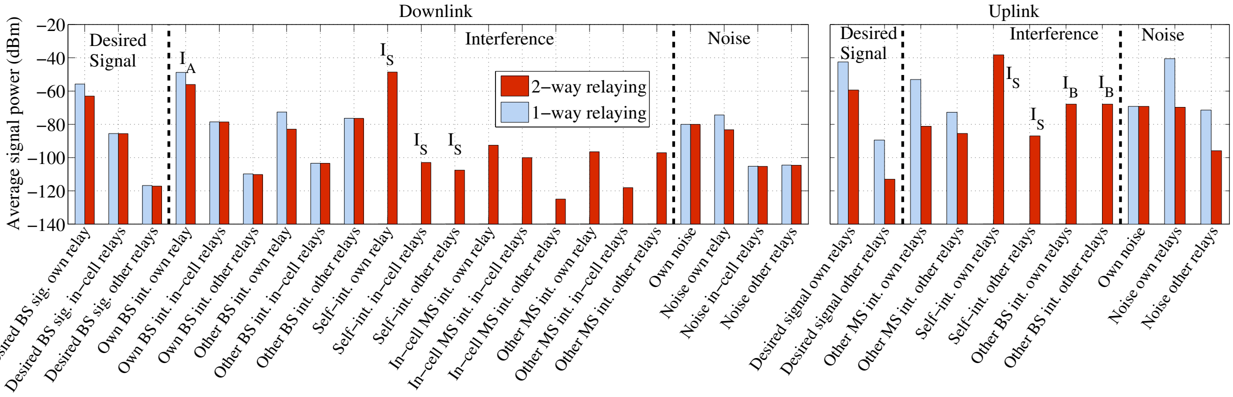

Fig. 3 Receive signal powers distinguished by their sources (one-way and two-way AF relaying protocol).

from other cells in the downlink and own relays and relays from other cells in the uplink). More details on the simulation parameters are given in Section 6.

From the figure, we can conclude where the different interference contributions come from. In contrast to one-way relaying, additional interference terms appear in two-way relaying: the signals transmitted by the other terminals of the same kind, including self-interference. These signals are not present in one-way relaying because the different directions of communication are separated by orthogonal resources. The total signals received by the relays are thus of less power and one-way relays can apply a higher gain factor in order to meet the transmit power. Consequently, the (existing) signal contributions in one-way relaying are of higher power than in the two-way case. The dominant interference terms can be classified into: i) BS signals intended for other MSs in the same cell ($\mathrm{I_A}$ in the figure), in the case of two-way relaying ii) self-interference ($\mathrm{I_S}$), iii) interference from other BSs in the uplink ($\mathrm{I_B}$), and iv) remaining interference. The terms $\mathrm{I_S}$ and $\mathrm{I_B}$ do not exist in one-way relaying.

In the following, we construct precoding and relaying schemes that attempt to mitigate the interference seen by the terminals. In order to develop schemes that are relevant for practical implementation, we restrict ourselves to simple linear precoding techniques that can be computed in a non-iterative fashion. Consequently, we do not claim any optimality of the proposed schemes, but rather understand them as example implementations for the relay carpet that are, due to the low complexity, of high practical relevance. However, the precoding and relay gain matrices are designed in a distributed way, i.e., the global task of improving the network performance is shared among the different nodes according to their complexity and abilities. Each node computes its precoding or gain matrix based on *locally* available CSI. Moreover, the transmissions on the BS-relay links should be independent of the ones on the relay-MS links. This has the advantage that the precoding at the BSs has not to be updated as often as the precoding on the relay-MS links. This is because the channels between BSs and fixed relays presumably have a much longer coherence time than the channels between the relays and the (possibly moving) MSs.

In order to mitigate the interference present in the network and to improve the achievable rates, we propose the following transmission schemes for the BSs and relays. The signaling of the users is spatially white such that the MSs do not require transmit CSI.

4.1 Block Zero-Forcing at the BSs

A strong interference source that mitigates the performance in the downlink is the BS signal intended for other MSs ($\mathrm{I_A}$ in Fig. 3). In order to cancel it, we apply block zero-forcing at the BSs to the in-cell relays. The transmit signal of BS c is

$$\mathbf{x}_c^{(\mathrm{B})} = \sum_{k=1}^{M} \mathbf{Q}_{c,k}^{(\mathrm{B})} \cdot \mathbf{s}_{c,k}^{(\mathrm{B})} = \sum_{k=1}^{M} \mathbf{Z}_{c,k} \cdot \tilde{\mathbf{V}}_{c,k} \cdot \mathbf{P}_{c,k} \cdot \mathbf{s}_{c,k}^{(\mathrm{B})}, \quad (28)$$

where

$$\mathbf{Z}_{c,k} = \mathrm{null}\left\{ \left[\mathbf{H}_1^{(c,c)\mathsf{T}}, \ldots, \mathbf{H}_{k-1}^{(c,c)\mathsf{T}}, \mathbf{H}_{k+1}^{(c,c)\mathsf{T}}, \ldots, \mathbf{H}_M^{(c,c)\mathsf{T}} \right]^{\mathsf{T}} \right\}$$

ensures that the signal intended for MS (c,k) is nulled at the other relays in this cell and $\tilde{\mathbf{V}}_{c,k}$ are the right hand singular vectors of the virtual channel $\tilde{\mathbf{H}}_k^{(c,c)} = \mathbf{H}_k^{(c,c)} \cdot \mathbf{Z}_{c,k}$. The diagonal power loading matrix $\mathbf{P}_{c,k}$ weights each stream according to the waterfilling solution as in [21].

4.2 AF relay gain matrices

In its simplest form, AF relaying is performed with a scaled identity matrix

$$\mathbf{G}_{c,k} = \sqrt{P_\mathrm{R}/\mathrm{tr}\left\{\mathsf{E}\left[\mathbf{r}_{c,k}\cdot\mathbf{r}_{c,k}^\mathsf{H}\right]\right\}}\cdot\mathbf{I}_{N_\mathrm{R}}. \tag{29}$$

These type A relays forward their receive signal scaled according to the power constraint, without modifying it. This form of AF relaying does not require any CSI at the relays.

More sophisticated type B relays that have access to local CSI can form linear combinations of all input streams to a beneficial output signal vector. The relay can e.g. design the relay gain matrix such that undesired signals are minimized while the desired signal components should remain at a good quality. To this end, the relay gain matrices are factorized into

$$\mathbf{G}_{c,k} = \sqrt{\alpha_{c,k}}\cdot\mathbf{G}_{c,k}^{(\mathrm{Tx})}\cdot\mathbf{G}_{c,k}^{(\mathrm{Rx})\mathsf{H}}, \tag{30}$$

where $\mathbf{G}_{c,k}^{(\mathrm{Rx})}$ is a receive filter, $\mathbf{G}_{c,k}^{(\mathrm{Tx})}$ a transmit filter, and $\alpha_{c,k}$ a scaling factor to adjust the transmit power.

For the design of the receive filter, we distinguish between one-way and two-way relaying. In the one-way case, the receive filter $\mathbf{G}_{c,k}^{(\mathrm{Rx})}$ is chosen to suppress the interference coming from the BSs of adjacent cells. Such a filter can be obtained by $\mathbf{G}_{c,k}^{(\mathrm{Rx})} = \left[\mathbf{v}_1^{(c,k)},\ldots,\mathbf{v}_{d_\mathrm{s}}^{(c,k)}\right]$ [22]. Therein, $\mathbf{v}_i^{(c,k)}$ is the eigenvector corresponding to the ith smallest eigenvalue of the covariance matrix of the relay's interference channel

$$\boldsymbol{\Gamma}_{c,k} = \sum_{\substack{d=1\\d\neq c}}^{C}\mathbf{H}_k^{(c,d)}\cdot\mathbf{H}_k^{(c,d)\mathsf{H}}. \tag{31}$$

With this, $\mathbf{G}_{c,k}^{(\mathrm{Rx})}$ is independent of the actual BS signaling and has thus not to be updated when a BS changes its precoding. Moreover, when the relay position is fixed, this covariance matrix is mainly static and simple to estimate.

In two-way relaying, we can additionally enhance the uplink performance by choosing a receive filter that does not only reduce the interference from adjacent BSs but tries also to keep the signal from its MS at a good quality. To this end, $\mathbf{G}_{c,k}^{(\mathrm{Rx})}$ can be chosen as the solution of the optimization problem

$$\begin{aligned}\mathbf{G}_{c,k}^{(\mathrm{Rx})} &= \arg\min\mathrm{tr}\left\{\mathbf{G}_{c,k}^{(\mathrm{Rx})\mathsf{H}}\left(\boldsymbol{\Gamma}_{c,k}+\sigma_n^2\mathbf{I}_{N_\mathrm{R}}\right)\mathbf{G}_{c,k}^{(\mathrm{Rx})}\right\}\\ &\qquad\text{such that } \mathbf{G}_{c,k}^{(\mathrm{Rx})\mathsf{H}}\mathbf{F}_{k,k}^{(c,c)} = \mathbf{I}_{N_\mathrm{M}},\end{aligned} \tag{32}$$

which is given by

$$\begin{aligned}\mathbf{G}_{c,k}^{(\mathrm{Rx})} &= \left(\boldsymbol{\Gamma}_{c,k}+\sigma_n^2\mathbf{I}_{N_\mathrm{R}}\right)^{-1}\cdot\mathbf{F}_{k,k}^{(c,c)}\cdot\\ &\quad\cdot\left(\mathbf{F}_{k,k}^{(c,c)\mathsf{H}}\left(\boldsymbol{\Gamma}_{c,k}+\sigma_n^2\mathbf{I}_{N_\mathrm{R}}\right)^{-1}\mathbf{F}_{k,k}^{(c,c)}\right)^{-1}.\end{aligned} \tag{33}$$

This approach is a MIMO extension of the minimum variance distortionless response (MVDR) filter [23].

The transmit filter of the relay is chosen as a transmit matched filter (MF)

$$\mathbf{G}_{c,k}^{(\mathrm{Tx})} = \overline{\mathbf{F}}_{k,k}^{(c,c)\mathsf{H}} \tag{34}$$

with respect to the channel to the corresponding MS. The combined relay gain matrix is then scaled with

$$\alpha_{c,k} = \frac{P_\mathrm{R}}{\mathrm{tr}\left\{\mathbf{G}_{c,k}^{(\mathrm{Tx})}\mathbf{G}_{c,k}^{(\mathrm{Rx})\mathsf{H}}\mathsf{E}\left[\mathbf{r}_{c,k}\mathbf{r}_{c,k}^\mathsf{H}\right]\mathbf{G}_{c,k}^{(\mathrm{Rx})}\mathbf{G}_{c,k}^{(\mathrm{Tx})\mathsf{H}}\right\}}.$$

Note that the gain matrices of these type B relays are chosen such that the relays mainly improve the links to the MSs, because the BS-relay links are presumably already strong due to the high transmit power and the zero-forcing at the BSs. Also note that the receive filters at the relays depend only on the covariance matrix of the BS-relay interference links. The individual channel coefficients need not to be known. Moreover, the relay receive filters do not have to be updated very often, since these channels change only slowly when the relays are at fixed positions. Additionally, the precoding at the BSs can be done with respect to the effective channel that includes the specific relay receive filters, i.e. the block zero-forcing and waterfilling is given as a function of the effective channel $\mathbf{G}_{c,k}^{(\mathrm{Rx})\mathsf{H}}\cdot\mathbf{H}_k^{(c,c)}$ instead of $\mathbf{H}_k^{(c,c)}$ only. This further improves the overall performance.

4.3 DF Relay Filter Design

The same filter techniques can also be applied to DF relays. When type A DF relays are considered, the relay filter matrices are

$$\mathbf{G}_{c,k}^{(\mathrm{Rx})\mathsf{H}} = \mathbf{I}_{N_\mathrm{R}} \tag{35}$$

$$\mathbf{G}_{c,k}^{(\mathrm{Tx})} = \sqrt{P_\mathrm{R}/N_\mathrm{M}}\cdot\mathbf{I}_{N_\mathrm{R}}. \tag{36}$$

For the more sophisticated type B relays, the filters from the AF case can be adopted. In this case, the receive filter in the one-way protocol contains, as for AF relaying, the eigenvectors corresponding to the d_s smallest eigenvalues $\mathbf{G}_{c,k}^{(\mathrm{Rx})} = \left[\mathbf{v}_1^{(c,k)},\ldots,\mathbf{v}_{d_\mathrm{s}}^{(c,k)}\right]$. The transmit filter is a scaled transmit MF

$$\mathbf{G}_{c,k}^{(\mathrm{Tx})} = \sqrt{\frac{P_\mathrm{R}}{\mathrm{tr}\left\{\overline{\mathbf{F}}_{k,k}^{(c,c)\mathsf{H}}\cdot\overline{\mathbf{F}}_{k,k}^{(c,c)}\right\}}}\,\overline{\mathbf{F}}_{k,k}^{(c,c)\mathsf{H}}, \tag{37}$$

such that it meets the relay transmit power constraint.

For two-way relaying, the receive filter can be replaced by the MVDR solution as in (33).

4.4 Self- and BS-Interference Cancellation

In two-way relaying, both directions of communication are combined into the same physical channel. A strong contribution of interference in this case is thus the self-interference that propagates back from the relays ($\mathrm{I_S}$ in Fig. 3). This interference, however, can be canceled at each node when the effective channel from itself via the relays back to it is known. At the MSs, this effective channel is described by an $N_\mathrm{M} \times N_\mathrm{M}$ matrix which can be estimated with pilot symbols that are included in the MS signal. Alternatively or in addition, the self-interference can be used to obtain CSI estimates [16]. When the self-interference is completely cancelled, the covariance matrix $\mathbf{K}_{\mathrm{M},c,k}^{(\mathrm{self})}$ in (19) disappears and the resulting rate is significantly improved.

Self-interference can also be canceled at the BSs in the same way. However, this might not be sufficient to achieve high uplink rates, as the sum of signals from all other BSs $d \neq c$ is a strong contribution of the interference at BS c ($\mathrm{I_B}$ in Fig. 3). Therefore, we propose that (at least close) BSs cooperate with each other in a way that they share their transmit symbols. In this way, the BSs can not only cancel their self-interference, but can also reconstruct and cancel the interference caused by neighboring BSs. The known data symbols or pilot/ training sequences included in the signals can be used to estimate the effective channels via the relays and no CSI needs to be shared. As a result, the covariance matrix $\mathbf{K}_{\mathrm{B},c}^{(\mathrm{BS\,int})}$ disappears in (20) completely. This form of cooperation improves the uplink rates of two-way relaying drastically.

4.5 Performance Evaluation

In Fig. 4, achievable rates of the aforementioned transmission schemes are compared with the rates of a cellular network in which the BSs serve the MSs directly by block zero-forcing and no relays are used. The BSs, relays, and MSs transmit with a fixed transmit power of $P_\mathrm{B} = 40\,\mathrm{W}$, $P_\mathrm{R} = 6\,\mathrm{W}$, and $P_\mathrm{M} = 0.2\,\mathrm{W}$, respectively. It can be seen that significant gains can be achieved with the relay carpet, even with the simple type A relays. The performance of TDD and FDD relaying is comparable. The reciprocal channels in the TDD case thus do not have a significant impact. Therefore, the simulations described in Section 6 are limited to the TDD case. For FDD systems, very similar results can be expected. In the following, we take a closer look at the relay and BS signal processing and describe how the required CSI can be estimated and how imperfections affect the performance at the different nodes.

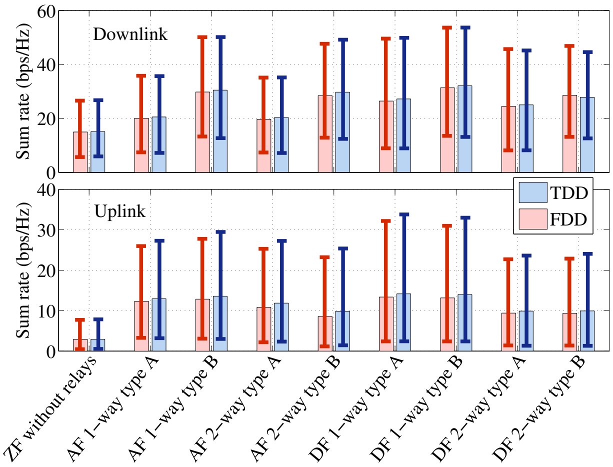

Fig. 4 Achievable sum rates of the different relaying schemes compared to a conventional network in which the BS without relays serves the MSs directly with zero-forcing. Shown are the mean rates (bars) as well as the 5% and 95% percentiles. The prelog factors are not considered.

5 Aspects of Channel Estimation

For the transmission schemes introduced in the previous section, CSI is necessary at the BSs, relays, and MSs in different forms. We distinguish between CSI at the receiver (CSIR) and at the transmitter (CSIT). Usually, acquiring CSIR (e.g. based on a training sequence included in the received transmission) is considered not as difficult as obtaining CSIT. In TDD systems assuming channel reciprocity, CSIT can be determined from CSIR estimated to decode a previous transmission. In case of FDD this is not possible due to the different frequencies. One way of acquiring CSIT nevertheless is using a feedback channel: the receiver is feeding its CSIR (possibly quantized and compressed) back to the sender. However, CSIT may then be outdated or noisy (e.g. due to quantization). Another way to acquire CSIT in a FDD system would be that the receiver transmits a training sequence on the transmit frequency of the transmitter in a separate time slot. The transmitter then estimates the CSIR and determines the CSIT assuming channel reciprocity. In the following, we discuss which nodes need which form of CSI, how they can acquire it, and what impact this has on the node complexity. Furthermore, we introduce error models for CSI imperfections in order to determine the robustness of the transmission schemes.

5.1 Acquisition of CSI

At the BSs, channel estimation is necessary for different tasks: CSIR to decode the uplink signals, CSIR to cancel self- and BS-interference and CSIT for the calculation of the beamforming matrices. Whereas CSIR can be acquired at the BS based on training sequences as described above, CSIT needs to be estimated at the relay and fed back to the BS, or the relay can transmit a training sequence on the transmit frequency of the BS on demand.

At the MSs, no CSIT is required by the schemes presented in this paper, only CSIR for decoding the downlink signal, and in case of two-way relaying, for canceling the self-interference.

At the relays, the necessity of CSIR and CSIT depends on the type of the relay and the signal processing. Whereas a type A AF relay does not need any CSI at all, a type B AF relay needs to know the relay-MS channel $\mathbf{F}_{k,k}^{(c,c)}$ as well as the BS interference covariance matrix $\boldsymbol{\Gamma}_{c,k}$ for the computation of the transmit and receive filter. For TDD relays, the CSIT can be acquired via the CSIR. For FDD, either a feedback from the BS/MS is necessary or the transmission of training sequences by the BS and MS on the transmit frequency of the relay. As only the channel covariance matrix from the BSs is required, the estimation is much simpler than for the full channel. The dimensions of the channel covariance matrix is only $N_\mathrm{R} \times N_\mathrm{R}$ and can be acquired by averaging the received signal over time. For DF relays, CSIR is always necessary for the decoding, also for type A relays. When type B DF relays are used, the CSIT can be obtained from CSIR when the relays operate in a TDD mode. In the FDD case, this is not possible and the acquisition of CSIT by feedback or pilot transmission comes on top.

5.2 Node Functionality

The simplest form of relays are type A AF relays in FDD mode. In this case, the relays can be seen as simple frequency converters that amplify their input signal without the requirement of any CSI. In order to allow its BS to estimate the BS-relay channel, these relays have to be able to transmit a training sequence on demand. This kind of relay can be referred to as a “drilled” relay, as it only responds to requests of the BS. Apart from some synchronization mechanisms, such relays do not need any additional functionalities. If the relays operate in TDD mode, an additional buffer to store the received signal before it can be retransmitted is required.

The more sophisticated type B AF relays additionally need to acquire CSI such that they can calculate their receive and transmit filters. To this end, the relays need either to be able to estimate the required channels themselves or to receive the CSI that is delivered from their BS and/or MS. As a result, such relays require a decoding functionality that does not differ much from the one in DF relays.

DF relays are the most complex relays considered in this paper. Additional to the CSIR necessary for the decoding, the signals need to be re-encoded. For type B DF relays, also CSIT is required that can be obtained as in the AF case.

While the relaying protocol (whether one-way or two-way) does not matter for the relay complexity in AF relays, it influences the tasks of the terminal nodes. For one-way relaying, the terminal nodes just need to evaluate the training sequences and decode the signal. For two-way relaying instead, they additionally need to estimate and subtract the self interference (and the interference of the other BSs). Especially for the BSs, that cancel the other BS interference, two-way relaying thus adds some complexity to the terminals. However, when the relays are static, the CSI for interference cancellation needs to be tracked with a comparably low frequency. If DF relays are used, the task of self-interference cancellation is simpler. Only self-interference has to be compensated, which can be done in the digital domain by an XOR operation.

5.3 Estimation Error Models

As the positions of BSs and relays are fixed, we consider the channel between a BS and a relay as quasi-static. Acquiring CSIT of a certain quality for this link seems possible and of less complexity than for the link between a relay and a possibly moving MS. These considerations motivate the chosen transmission schemes, where the schemes suffering stronger from imperfect CSIT are only used on the channel between BS and relay, while the more robust schemes are used on the link between relay and MS. In the following, we investigate the robustness of the considered schemes regarding imperfect CSI. Depending on the link, we use different estimation error models to simulate these imperfections.

Complete Channel Matrix: For the BS beamforming and the relay filters, the actual channel matrices $\mathbf{H}_k^{(c,c)}$ and $\mathbf{F}_{k,k}^{(c,c)}$ need to be known at the respective nodes. Imperfections on this type of CSI is modeled as

$$\hat{\mathbf{H}}_k^{(c,c)} = \sqrt{L_\mathrm{p}} \left(\sqrt{1-\vartheta^2}\mathbf{H}_k^{(c,c)} + \vartheta \mathbf{W}_k^{(c,c)} \right), \tag{38}$$

where L_p denotes the pathloss, $\mathbf{H}_k^{(c,c)}$ the true small scale fading channel, $\mathbf{W}_k^{(c,c)}$ the estimation error matrix and $\vartheta^2 \in [0,1]$ the CSI noise scaling factor. The elements of $\mathbf{W}_k^{(c,c)}$ are $\mathcal{CN}(0,1)$. The pathloss L_p is assumed to be known perfectly (averaged over time). Only the small scale fading is assumed to change. This model captures effects as outdated CSI, noisy estimation, or quantization. We define the estimation signal to noise ratio (SNR) as $\mathrm{SNR}_{\mathbf{H}} = \frac{1-\vartheta^2}{\vartheta^2}$ as a measure for the quality of the CSI. As the channels between the BSs and the relays are considered quasi-static, high SNRs can be expected.

For the estimation of $\mathbf{F}_{k,k}^{(c,c)}$, the same model is used. Thereby, the estimation SNR given by $\mathrm{SNR}_{\mathbf{F}} = \frac{1-\vartheta^2}{\vartheta^2}$ can differ from the one at the BS, as this channel cannot be assumed to be quasi-static.

Channel Covariance Matrix: For the error of the estimation of the channel covariance matrix $\boldsymbol{\Gamma}_{c,k}$, required for the calculation of $\mathbf{G}_{c,k}^{(\mathrm{Rx})}$, we use the model

$$\hat{\boldsymbol{\Gamma}}_{c,k} = \boldsymbol{\Gamma}_{c,k} + \sigma_\Gamma^2 \mathbf{W}_{c,k}\mathbf{W}_{c,k}^\mathsf{H}, \tag{39}$$

where $\mathbf{W}_{c,k}$ is again the estimation error matrix with its elements chosen as above and $\sigma_\Gamma^2 \in [0,\infty)$ is the noise scaling factor. The instantaneous estimation SNR of this model is defined as $\mathrm{SNR}_\Gamma = \frac{\mathrm{tr}\{\boldsymbol{\Gamma}_{c,k}\}}{N_\mathrm{R}\sigma_\Gamma^2}$. With this definition, we can relate the estimation noise power to the actual signal power. The estimation error is assumed to be small, as the channel covariance matrices can be averaged over time.

CSI for Interference Cancellation: For the cancellation of the self-interference at the BSs and the MSs, we consider the compound channels

$$\mathbf{H}_{c,k}^{(\mathrm{comp})} = \sum_{d=1}^{C}\sum_{j=1}^{K} \overline{\mathbf{H}}_j^{(c,d)} \mathbf{G}_{d,j} \mathbf{H}_j^{(d,c)} \tag{40}$$

$$\mathbf{F}_{c,k}^{(\mathrm{comp})} = \sum_{d=1}^{C}\sum_{j=1}^{K} \overline{\mathbf{F}}_{k,j}^{(c,d)} \mathbf{G}_{d,j} \mathbf{F}_{j,k}^{(d,c)} \tag{41}$$

from the BS/MS to the relay and back which can be estimated e.g. with the training sequence contained in the signal. The estimation error for this compound channel is modeled as

$$\hat{\mathbf{H}}_{c,k}^{(\mathrm{comp})} = \mathbf{H}_{c,k}^{(\mathrm{comp})} + \sigma \mathbf{W}_{c,k}, \tag{42}$$

with $\mathbf{W}_{c,k}$ the estimation error matrix as above and $\sigma \in [0,\infty)$ the CSI noise scaling factor. For the cancellation of the self-interference, the BS/MS subtracts the estimated self-interference. For the computation of the achievable rate, only the remainder of the self-interference covariance matrix is of importance

$$\hat{\mathbf{K}}_{\mathrm{M},c,k}^{(\mathrm{self})} = \sigma^2 \mathbf{W}_{c,k}\mathbf{W}_{c,k}^\mathsf{H}. \tag{43}$$

To relate the estimation noise power to the actual self-interference power we define the instantaneous estimation SNR of this error model as $\mathrm{SNR}_\mathrm{self} = \frac{\mathrm{tr}\left\{\mathbf{K}_{c,k}^{(\mathrm{self})}\right\}}{N_\mathrm{i}\sigma^2}$, where $\mathbf{K}_{c,k}^{(\mathrm{self})}$ is the self interference covariance matrix and $i \in \{\mathrm{B},\mathrm{M}\}$.

The same model is used for the cancellation of the interference from other BSs. The remainder of the other BS signal covariance matrix is modeled as

$$\hat{\mathbf{K}}_{\mathrm{B},c}^{(\mathrm{BSint})} = \sigma^2 \mathbf{W}_{c,k}\mathbf{W}_{c,k}^\mathsf{H}, \tag{44}$$

with all parameters as above. As these channels are assumed to be quasi-static and all data is expected to be known at the receiver, high SNRs can be expected.

6 Simulation Results

We study the performance of the described relay carpet approach by means of computer simulations in a realistic setup. We focus on the sum rate that is achievable in a cell of interest and compare the performance to a non-cooperative reference scenario, that is a cellular network without relays in which the BSs serve multiple MSs by zero-forcing and waterfilling on the direct BS-MS channels.

Simulation setup: The network consists of $C = 19$ hexagonal cells, where 18 cells are arranged in two circles around a middle cell that is the cell of interest. The distance between adjacent BSs is 1000 m. In each cell, there are $M = K$ MSs and relays with $N_\mathrm{M} = 2$ and $N_\mathrm{R} = 4$ antennas. The BS antenna arrays have $N_\mathrm{B} = M \cdot N_\mathrm{R}$ antennas. All antennas are omnidirectional and we apply the WINNER II channel model as in [24] to get a realistic network model. The channels are drawn according to the WINNER II scenario C2 with line-of-sight condition on the channels between each BS and its corresponding relays and non-line-of-sight on all others. If not stated otherwise, the chosen transmit powers at the BSs, relays, and MSs are $P_\mathrm{B} = 40\,\mathrm{W}$, $P_\mathrm{R} = 6\,\mathrm{W}$, and $P_\mathrm{M} = 0.2\,\mathrm{W}$, and the noise variances are $\sigma_n^2 = \sigma_w^2 = 5 \cdot 10^{-12}\,\mathrm{W}$. The underlying assumption thereby is that the systems are operated in a total bandwidth of 100 MHz.

Perfect CSI at all nodes: Empirical cumulative distribution functions (CDFs) of achievable sum rates in the down- as well as uplink are shown in Fig. 5, where $K = M = 6$ relays are placed at a distance of $d_{\mathrm{BR}} = 350\,\mathrm{m}$ in a circle around each BS. The MSs are uniformly distributed in the small relay cells such that each MS is served by one relay. A "deadzone" with a radius of $\frac{2}{3}d_{\mathrm{BR}}$ is applied around each BS. In this zone, no MSs are placed. This assumption is intuitive, as it does not really make sense that MSs that are very close to a BS are served by relays which are further away. MSs in this zone can be served directly from their BS, as in a conventional system. To this end, the deadzone radius is chosen such that the average receive power on the deadzone border is the same for a signal from the BS as from the relays. The CDFs show that, compared to the non-cooperative reference scenario, very high gains can be achieved by the relay carpet approach. As stated in Section 3.3, prelog factors are not considered in the presented achievable rates. Hence, in case all resources have to be counted and the relays are half-duplex, the rates of all considered variants of one-way relaying must be scaled with 1/2 before comparing them to the reference scenario or the two-way relaying schemes. Then, two-way relaying outperforms one-way relaying in all investigated schemes. If the resources of the second hop do not have to be accounted for or full-duplex relays can be used, the one-way curves show their potential gains compared to the two-way approach.

In the following, we only look at the sum of the up- and downlink rates. Fig. 6 shows these sums versus the distance between BS and RS for selected type B relays. The results represented by dashed lines (with deadzones around the BSs) confirm the results in the CDFs of Fig. 5, which were found for $d_{\mathrm{BR}} = 350\,\mathrm{m}$. With increasing d_{BR} the performance of the considered relaying schemes improves up to 40 m; only for $d_{\mathrm{BR}} < 200\,\mathrm{m}$ the reference scenario performs better. If we compare the rates also for the case without deadzones, shown as solid lines in Fig. 6, the gains look less impressive. In the reference scenario, MSs that are very close to a BS achieve very high rates by the direct BS transmission with waterfilling, which favors strong users. However, we regard the scenario in which MSs in close vicinity of a BS are served by relays as not of much relevance as the BSs can serve these MSs directly. In this regard, the two-hop communication can operate in parallel or on top of the conventional direct approach. Moreover, the transmission via the relays has still advantages. The CSI estimation at the BSs is simplified as fast fading is eliminated from the point of view of the BSs, since the relays are, in contrast to the MSs, not moving. Additionally, the relay schemes achieve much higher rates on the cell edge whereas in the case of direct transmission, the high rates that contribute most to the average are for MSs located very close to the BS. The relaying schemes thus lead to a more balanced and fairer rate distribution.

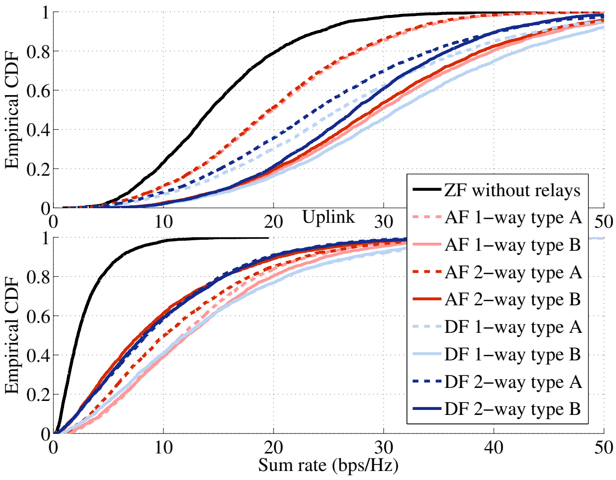

Fig. 5 CDF of achievable sum rates for up- and downlink. The transmit powers are $P_{\mathrm{B}} = 40\,\mathrm{W}$, $P_{\mathrm{R}} = 6\,\mathrm{W}$, $P_{\mathrm{M}} = 0.2\,\mathrm{W}$, and noise variances $\sigma_n^2 = \sigma_w^2 = 5 \cdot 10^{-12}\,\mathrm{W}$ per 100 MHz bandwidth.

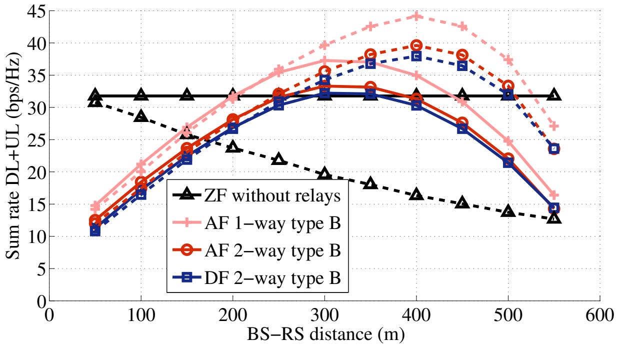

Fig. 6 Average sum rates (up- plus downlink) for different d_{BR}. The solid lines are without deadzone, the dashed lines are with a deadzone of $2d_{\mathrm{BR}}/3$ around the BSs.

Achievable sum rates for varying transmit powers are shown in Fig. 7. Here, the distance between BSs and relays is again $d_{\mathrm{BR}} = 350\,\mathrm{m}$ and a deadzone is applied. The curves show that, while the network is still interference-limited, notably higher slopes can be achieved with the relays. This indicates that in the regime around 20 to 40 dBm, more degrees of freedom can be exploited. Interestingly, AF relaying performs very good, even though this relaying strategy also amplifies noise and interference. Similar performance can only be achieved by DF relaying in the one-way protocol. This type of relaying, however, requires 4 orthogonal resources for one transmission in each direction. It

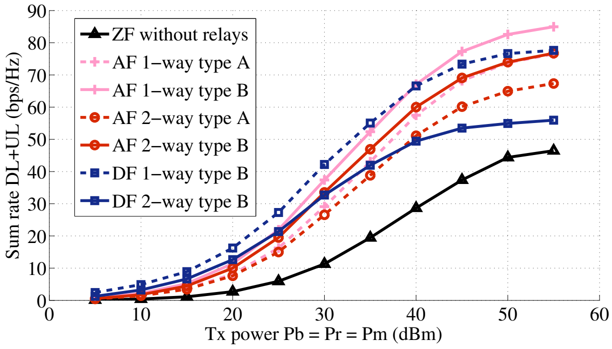

Fig. 7 Achievable sum rates (up- plus downlink) for varying transmit powers. The noise variances are fixed to $\sigma_n^2 = \sigma_w^2 = 5 \cdot 10^{-12}$ W.

is therefore less efficient than two-way relaying which requires only 2 resources.

Imperfect CSI: So far, perfect CSI was assumed for all simulations, i.e. the BS beamforming and relay gain matrices are all computed based on the correct channels. In the following, we study the influence of CSI imperfections as discussed in Section 5. The influence of the CSI noise on the up- and downlink performance is shown in Fig. 8. In the first three columns, only one type of CSI imperfections is considered at one time: i) only at the BSs for the calculation of the ZF beamforming, ii) only at the relays, and iii) only for the interference cancellation in the case of two-way AF relaying. In the column on the right, all nodes are affected by CSI imperfections in the same way, i.e. all estimation SNRs are equal.

i) It can be seen that good CSI is crucial for the beamforming at the BSs. Otherwise, the performance degrades rapidly. This is not surprising, as zero-forcing is known to be sensitive with respect to channel knowledge. Nonetheless, as we consider the channels between BSs and relays as quasi-static, a high SNR can be expected in our setup. In the uplink, only two-way relaying depends on the BS beamforming.

ii) At the relays, CSI imperfections only have an influence on type B relays. The chosen relaying schemes are however quite robust; the interference mitigating receive filter and the transmit matched filter degrade the performance with poor SNRs not significantly.

iii) In the case of AF two-way relaying, the cancellation of self- and BS-interference is crucial. This type of interference is very strong at all nodes and has thus to be known accurately in order to get good end-to-end performance. This form of relaying is thus only beneficial if the terminals can estimate the corresponding channels appropriately, especially at the BSs where the interference from other BSs has also to be cancelled.

Denser cellular network: In the previous simulations, all cells contain $K = M = 6$ relays and MSs. However, the gains achievable with the relay carpet can even be increased with higher numbers. Fig. 9 shows average sum rates for different numbers of users, where $M = K$ and $N_\mathrm{B} = M \cdot N_\mathrm{R}$ grow accordingly. The relays are randomly placed with a uniform distribution in the cell with a deadzone of 300 m around the BSs. The transmit powers are again $P_\mathrm{B} = 40$ W, $P_\mathrm{R} = 6$ W, $P_\mathrm{M} = 0.2$ W. The curves are plotted for the case of perfect CSI at all nodes (solid lines) as well as for the case in which the different nodes are affected by CSI estimation errors (dashed lines). In the latter case, the BS beamforming is based on CSI with an SNR of 20 dB, the relays with an SNR of 10 dB, and an SNR of 30 dB for the self- and BS-interference cancellation.

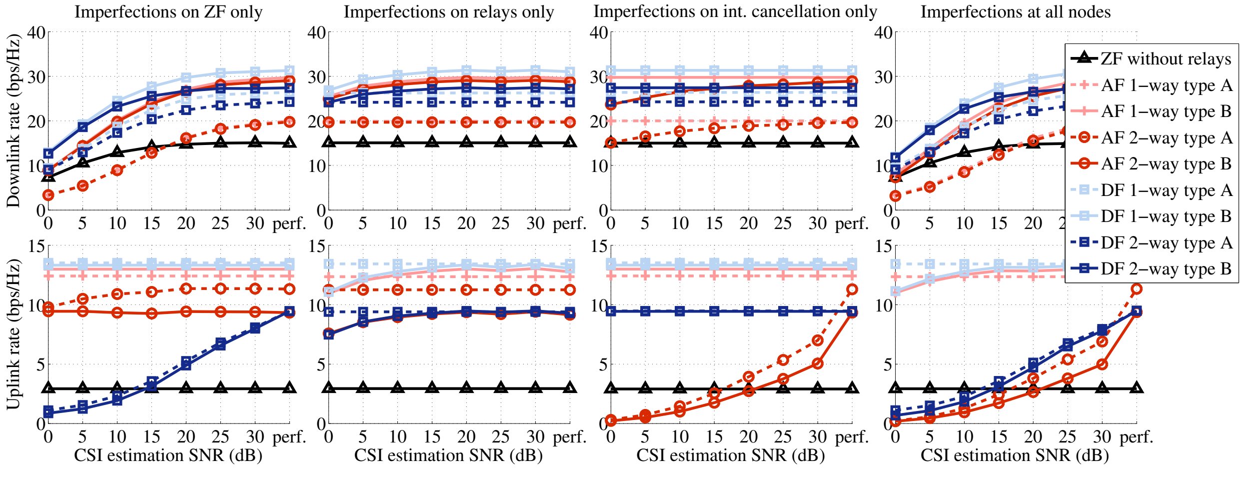

Fig. 8 Influence of imperfect CSI at the different nodes. In the first three columns, only one type of CSI estimation is affected by imperfections, the others are assumed to be perfect. In the column on the right, the estimation SNR is the same at all involved nodes.

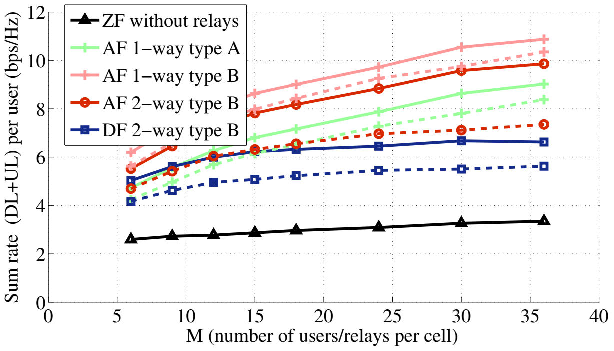

Fig. 9 Increasing number of relays/users with (dashed) and without (solid) imperfections.

It can be seen that the performance of AF relaying improves with the number of relays/MSs. This indicates that the impact of interference is weaker than in the other cases. Interestingly, AF relaying outperforms DF relaying. As more nodes are present in the network, the interference sources become denser and the DF relays have more difficulties to decode their receive signals. The denser network, however, has less impact on AF relaying, which makes it a more suitable choice for the relay carpet. It can also be observed that especially the simple type A AF relays achieve a good performance and the degradation with CSI imperfections are small. As these relays are of very low complexity, more of these relays can be deployed with little costs. The lower rates as compared to the more complex type B relays can thus be recovered by deploying more or them. Also the use of idle MSs as relays can further improve the performance, as a growing network increases the throughput.

7 Conclusions

The relay carpet is a promising approach to drastically enhance the performance of cellular networks. By the use of ubiquitous relaying, interference can be reduced and the network achieves a more homogeneous coverage. Through the distributed form of interference management, the spatial degrees of freedom can be better exploited and the frequency reuse factor can be improved towards one. Turning the cellular network into a two-hop network also simplifies CSI estimation at the terminals and allows for massive MIMO antenna arrays at the BSs. This approach is not only scalable in terms of the number of involved nodes/antennas, but it is also transparent to the implemented communication technology and can be applied on top of other approaches such as CoMP, heterogeneous networks, or others.

With BSs that can cooperate with each other to cancel other BS interference, two-way AF relaying shows large performance gains and can be made very efficient for cellular networks. The limited form of BS cooperation introduces only a small overhead when the channels to the relays have a long coherence time. Moreover, no clustering of BSs is required; any information that helps to reconstruct and cancel interference is beneficial. If the prelog loss due to the use of multiple resources for one transmission is considered, two-way relaying clearly outperforms one-way relaying. On the other hand, the one-way schemes with simple AF relays are very robust with respect to imperfections. The relays can thereby be of very low complexity; especially in FDD, they can be implemented as simple frequency converters. By deploying a large number of them, the throughput of cellular networks can still be enhanced significantly with comparably low costs. If full-duplex relays can be used or when the second hop is for free (e.g. as a secondary link), one-way AF relays can lead to a better performance than two-way relaying.

The sample transmission schemes introduced in this paper already show a significant gain as compared to a conventional multi-user MIMO approach. However, the proposed schemes are not optimal in any way. The performance can be further increased, e.g. when the schemes are combined with power control and/or transmit cooperation at the BSs, not least as some user data and CSI is already available at these nodes. Also the possibility to include user cooperation into the proposed network can be beneficial. We thus consider the relay carpet to be promising option for future cellular networks that can improve their performance by the required factors.

Appendix A: Derivation of Covariance Matrices

The covariance matrices that are used for the rate calculations in Section 3 are derived in the following. For the case of the downlink in AF one-way relaying, the covariance matrix of the desired signal is

$$\overrightarrow{\mathbf{K}}_{c,k}^{(\mathrm{sig})} = \sum_{d=1}^{C}\sum_{j=1}^{K}\sum_{d'=1}^{C}\sum_{j'=1}^{K} \overline{\mathbf{F}}_{k,j}^{(c,d)}\mathbf{G}_{d,j}\mathbf{H}_{j}^{(d,c)}\mathbf{Q}_{c,k}^{(\mathrm{B})}\cdot$$
$$\cdot\mathbf{Q}_{c,k}^{(\mathrm{B})\mathsf{H}}\mathbf{H}_{j'}^{(d',c)\mathsf{H}}\mathbf{G}_{d',j'}^{\mathsf{H}}\overline{\mathbf{F}}_{k,j'}^{(c,d')\mathsf{H}}$$

and the one of the interference and noise follows as

$$\begin{aligned}\overrightarrow{\mathbf{K}}^{(\mathrm{i+n})}_{c,k} &= \mathsf{E}\left[\overrightarrow{\mathbf{y}}_{c,k} \cdot \overrightarrow{\mathbf{y}}^{\mathsf{H}}_{c,k}\right] - \overrightarrow{\mathbf{K}}^{(\mathrm{sig})}_{c,k} \\ &= \sum_{\substack{d=1\\ d\neq c}}^{C} \sum_{j=1}^{M} \sum_{b=1}^{C} \sum_{i=1}^{K} \sum_{b'=1}^{C} \sum_{i'=1}^{K} \overline{\mathbf{F}}^{(c,b)}_{k,i} \mathbf{G}_{b,i} \mathbf{H}^{(b,d)}_{i} \mathbf{Q}^{(\mathrm{B})}_{d,j} \cdot \\ &\qquad \cdot \mathbf{Q}^{(\mathrm{B})\mathsf{H}}_{d,j} \mathbf{H}^{(b',d)\mathsf{H}}_{i'} \mathbf{G}^{\mathsf{H}}_{b',i'} \overline{\mathbf{F}}^{(c,b')\mathsf{H}}_{k,i'} \\ &\quad + \sum_{\substack{j=1\\ j\neq k}}^{M} \sum_{b=1}^{C} \sum_{i=1}^{K} \sum_{b'=1}^{C} \sum_{i'=1}^{K} \overline{\mathbf{F}}^{(c,b)}_{k,i} \mathbf{G}_{b,i} \mathbf{H}^{(b,c)}_{i} \mathbf{Q}^{(\mathrm{B})}_{c,j} \cdot \\ &\qquad \cdot \mathbf{Q}^{(\mathrm{B})\mathsf{H}}_{c,j} \mathbf{H}^{(b',c)\mathsf{H}}_{i'} \mathbf{G}^{\mathsf{H}}_{b',i'} \overline{\mathbf{F}}^{(c,b')\mathsf{H}}_{k,i'} \\ &\quad + \sigma_n^2 \sum_{b=1}^{C} \sum_{i=1}^{K} \overline{\mathbf{F}}^{(c,b)}_{k,i} \mathbf{G}_{b,i} \mathbf{G}^{\mathsf{H}}_{b,i} \overline{\mathbf{F}}^{(c,b)\mathsf{H}}_{k,i} + \sigma_w^2 \mathbf{I}_{N_{\mathrm{M}}}.\end{aligned}$$

In the uplink, they are

$$\begin{aligned}\overleftarrow{\mathbf{K}}^{(\mathrm{sig})}_{c} = \sum_{k=1}^{M} \sum_{d=1}^{C} \sum_{j=1}^{K} \sum_{d'=1}^{C} \sum_{j'=1}^{K} \overline{\mathbf{H}}^{(c,d)}_{j} \mathbf{G}_{d,j} \mathbf{F}^{(d,c)}_{j,k} \mathbf{Q}^{(\mathrm{M})}_{c,k} \cdot \\ \cdot \mathbf{Q}^{(\mathrm{M})\mathsf{H}}_{c,k} \mathbf{F}^{(d',c)\mathsf{H}}_{j',k} \mathbf{G}^{\mathsf{H}}_{d',j'} \overline{\mathbf{H}}^{(c,d')\mathsf{H}}_{j'},\end{aligned}$$

and

$$\begin{aligned}\overleftarrow{\mathbf{K}}^{(\mathrm{i+n})}_{\mathrm{B},c} &= \sum_{\substack{b=1\\ b\neq c}}^{C} \sum_{k=1}^{M} \sum_{d=1}^{C} \sum_{j=1}^{K} \sum_{d'=1}^{C} \sum_{j'=1}^{K} \overline{\mathbf{H}}^{(c,d)}_{j} \mathbf{G}_{d,j} \mathbf{F}^{(d,b)}_{j,k} \mathbf{Q}^{(\mathrm{M})}_{b,k} \cdot \\ &\qquad \cdot \mathbf{Q}^{(\mathrm{M})\mathsf{H}}_{b,k} \mathbf{F}^{(d',b)\mathsf{H}}_{j',k} \mathbf{G}^{\mathsf{H}}_{d',j'} \overline{\mathbf{H}}^{(c,d')\mathsf{H}}_{j'} + \\ &\quad + \sigma_n^2 \sum_{d=1}^{C} \sum_{j=1}^{K} \overline{\mathbf{H}}^{(c,d)}_{j} \mathbf{G}_{d,j} \mathbf{G}^{\mathsf{H}}_{d,j} \overline{\mathbf{H}}^{(c,d)\mathsf{H}}_{j} + \sigma_w^2 \mathbf{I}_{N_{\mathrm{B}}}.\end{aligned}$$

In the AF two-way case, the downlink covariance matrix of the desired signal at MS (c,k) is given by

$$\begin{aligned}\mathbf{K}^{(\mathrm{sig})}_{\mathrm{M},c,k} = \sum_{d=1}^{C} \sum_{j=1}^{K} \sum_{d'=1}^{C} \sum_{j'=1}^{K} \overline{\mathbf{F}}^{(c,d)}_{k,j} \mathbf{G}_{d,j} \mathbf{H}^{(d,c)}_{j} \mathbf{Q}^{(\mathrm{B})}_{c,k} \cdot \\ \cdot \mathbf{Q}^{(\mathrm{B})\mathsf{H}}_{c,k} \mathbf{H}^{(d',c)\mathsf{H}}_{j'} \mathbf{G}^{\mathsf{H}}_{d',j'} \overline{\mathbf{F}}^{(c,d')\mathsf{H}}_{k,j'}.\end{aligned}$$

For the interference, we distinguish the covariance matrix of self-interference and of the remaining interference plus noise:

$$\begin{aligned}\mathbf{K}^{(\mathrm{self})}_{\mathrm{M},c,k} = \sum_{d=1}^{C} \sum_{j=1}^{K} \sum_{d'=1}^{C} \sum_{j'=1}^{K} \overline{\mathbf{F}}^{(c,d)}_{k,j} \mathbf{G}_{d,j} \mathbf{F}^{(d,c)}_{j,k} \mathbf{Q}^{(\mathrm{M})}_{c,k} \cdot \\ \cdot \mathbf{Q}^{(\mathrm{M})\mathsf{H}}_{c,k} \mathbf{F}^{(d',c)\mathsf{H}}_{j',k} \mathbf{G}^{\mathsf{H}}_{d',j'} \overline{\mathbf{F}}^{(c,d')\mathsf{H}}_{k,j'}\end{aligned}$$

$$\mathbf{K}^{(\mathrm{i+n})}_{\mathrm{M},c,k} = \mathsf{E}\left[\mathbf{y}^{(\mathrm{M})}_{c,k} \cdot \mathbf{y}^{(\mathrm{M})\mathsf{H}}_{c,k}\right] - \mathbf{K}^{(\mathrm{sig})}_{\mathrm{M},c,k} - \mathbf{K}^{(\mathrm{self})}_{\mathrm{M},c,k}.$$

For the uplink, covariance matrices of the desired signal (jointly from all corresponding MSs), of the BS-interference (including self-interference), and of the remaining interference plus noise follow similarly as

$$\begin{aligned}\mathbf{K}^{(\mathrm{sig})}_{\mathrm{B},c} = \sum_{k=1}^{M} \sum_{d=1}^{C} \sum_{j=1}^{K} \sum_{d'=1}^{C} \sum_{j'=1}^{K} \overline{\mathbf{H}}^{(c,d)}_{j} \mathbf{G}_{d,j} \mathbf{F}^{(d,c)}_{j,k} \mathbf{Q}^{(\mathrm{M})}_{c,k} \cdot \\ \cdot \mathbf{Q}^{(\mathrm{M})\mathsf{H}}_{c,k} \mathbf{F}^{(d',c)\mathsf{H}}_{j',k} \mathbf{G}^{\mathsf{H}}_{d',j'} \overline{\mathbf{H}}^{(c,d')\mathsf{H}}_{j'},\end{aligned}$$

$$\begin{aligned}\mathbf{K}^{(\mathrm{BS\ int})}_{\mathrm{B},c} = \sum_{\substack{b=1\\ b\neq c}}^{C} \sum_{k=1}^{M} \sum_{d=1}^{C} \sum_{j=1}^{K} \sum_{d'=1}^{C} \sum_{j'=1}^{K} \overline{\mathbf{H}}^{(c,d)}_{j} \mathbf{G}_{d,j} \mathbf{H}^{(d,b)}_{j} \mathbf{Q}^{(\mathrm{B})}_{b,k} \cdot \\ \cdot \mathbf{Q}^{(\mathrm{B})\mathsf{H}}_{b,k} \mathbf{H}^{(d',b)\mathsf{H}}_{j'} \mathbf{G}^{\mathsf{H}}_{d',j'} \overline{\mathbf{H}}^{(c,d')\mathsf{H}}_{j'},\end{aligned}$$

and

$$\mathbf{K}^{(\mathrm{i+n})}_{\mathrm{B},c} = \mathsf{E}\left[\mathbf{y}^{(\mathrm{B})}_{c} \cdot \mathbf{y}^{(\mathrm{B})\mathsf{H}}_{c}\right] - \mathbf{K}^{(\mathrm{sig})}_{\mathrm{B},c} - \mathbf{K}^{(\mathrm{BS\ int})}_{\mathrm{B},c}.$$

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