Group Decoders for Correlated Massive MIMO Systems: The Use of Random Matrix Theory

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***Abstract*—Future MIMO terminals are expected to be equipped with a higher number of antennas. A possible intermediary solution between using the optimal but complex SIC decoder and the simple but poor performing MMSE decoder, is to consider different group decoders, namely, group SIC (GSIC) and group parallel decoding (GPD). The performances of such decoders strongly depend on the grouping strategy. In this paper, we introduce a unified framework to handle different group decoders. We use tools from random matrix theory to present a tight approximation of the average sum rate achieved in the case of adopting *any* kind of these decoders using only statistical CSI. For large number of data streams and fast changing channels, finding the optimal grouping for each channel realization is very complex. We formulate an optimization problem in which we use our developed approximations to find a *static* grouping which is shown to lead to a performance near to the optimal SIC and much better than the MMSE, especially for high transmit correlation. We also show how the performance depends on different system parameters, such as correlation strength and number of groups.**

I. Introduction

For future systems deploying a massive number of antennas [1], the trade off between the performance and the complexity of the receiver is an active research area. One of the most simple receivers is the linear minimum mean square error (MMSE) receiver, in which the receiver applies a linear Wiener filter proceeded by a stream wise decoder. We denote this receiver by the *MMSE decoder*. The MMSE decoder performs poorly in channels with high correlation. From information theoretic point of view, the optimum decoder that achieves the maximum possible sum rate is the successive interference cancellation (SIC) decoder [2]. For a large number of data streams, SIC has very high complexity and latency. An intermediate strategy between SIC and MMSE decoders would be the so called group decoding.

The concept of group decoding was introduced originally for CDMA systems [3], in which the data streams are divided into groups of smaller sizes and then they can be processed using one of two ways. The first is group SIC (GSIC), in which streams in a certain group are decoded simultaneously using an MMSE decoder with the interference from *previously decoded groups already canceled*. The second is group parallel decoding (GPD), in which streams within the same group are decoded using SIC, with the streams in *all* other groups as interference. The concept of GPD for MIMO was discussed in [4], however the detection strategy within each group was not SIC, but was the maximum likelihood detection (MLD). The algorithm in [4] suffered from noise enhancement as it did not consider the noise in the subspace projection, i.e. it was similar to a block zero forcing step. Modifications for the partitioning were done in [5], where the noise was taken into account in the subspace projection. The GSIC for MIMO was discussed briefly in [6]. In [7] the GSIC was also studied, however the authors were concerned with the outage performance of the system. In [8] a low complexity iterative soft input soft output group decoder similar to the GSIC is introduced. However, it uses an approximation of the MLD within each group. *The sum rate performances of both decoders strongly depend on the number of groups and which streams are grouped together*.

On the other hand, analysis tools based on random matrix theory (RMT) have gained massive attention recently. They are based on the fact that in the limit of large number of antennas, different performance metrics tend to be deterministic and can be approximated using the statistical channel state information (CSI). In [9] a tight approximation of the maximum achievable sum rate of the MIMO system was given. In [10] an approximation of the sum rate of the MMSE decoder was provided, which was based on an approximation of the mean signal to interference and noise ratio (SINR).

In this paper, we start by presenting a unified framework for analyzing the sum rate of different group decoders. Based on RMT, we supply tight approximations of the ergodic sum rates resulting from the use of *any* of the group decoders using only statistical CSI. Since finding the optimal grouping is of high complexity in case of fast changing channel and large number of data streams, we use our developed formulas to suggest a static grouping for the different data streams based only on statistical CSI. We formulate the grouping problem as a constrained optimization problem, which we solve using genetic algorithm optimization [11]. Especially in the case of high number of correlated streams, our results show that the use of GSIC with the suggested static grouping could be a substitute of both the poor performing simple MMSE decoder and the optimal but highly complex and latent SIC decoder. The developed approximations help to gain insights in the performance of the two group decoders. We also reveal how sensitive the performance becomes to the grouping strategies as well as to the number of groups, specifically for the case of transmit correlation.

II. System Model and Different Decoders

We consider an uplink of a multi user (MU) MIMO system, with L single antenna transmitters that can be spatially correlated and a receiver that has M antennas. We model the correlation at the transmitter and at the receiver sides using the Kronecker model. Thus, the $M \times L$ channel matrix is

$$\mathbf{H} = \mathbf{R}^{\frac{1}{2}} \mathbf{H}_{\text{IID}} \mathbf{T}^{\frac{1}{2}}, \tag{1}$$

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where $\mathbf{R}$ and $\mathbf{T}$ are the correlation matrices at the receiver and at the transmitters, respectively. The entries of the matrix $\mathbf{H}_{\text{IID}}$ are i.i.d. complex Gaussian random variables with zero mean and *variance* 1.

A. System Equation

This system at hand is described by the classical equation

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{2}$$

where the vectors $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{n}$ denote the transmitted vector, the received vector and the noise vectors, respectively. The elements of the vector of the transmitted data streams $\mathbf{x}$ and the noise vector $\mathbf{n}$ are all assumed to be i.i.d Gaussian distributed random variables, with variances σ_x^2 and σ_n^2, respectively. This means that we assume *no cooperation among the transmitters that all use the same power* and that we have in total L data streams to be transmitted. We assume that all of them use the same power $\sigma_x^2 = 1$, as well as full channel knowledge at the receiver.

The maximum achievable rate of the MIMO system described by (2) in nats per channel use [npcu] is

$$R_M = \log\left(\det\left(\mathbf{I}_M + \frac{1}{\sigma_n^2}\mathbf{H}\mathbf{H}^H\right)\right) \text{[npcu]}, \tag{3}$$

where the $\mathbf{I}_M$ is the $M \times M$ identity matrix. This rate can be achieved by using the optimal SIC. We denote the average of R_M by $\bar{R}_M$.

B. Group Decoders

We address two types of group decoders, namely GSIC and GPD. For both decoders, the first step is always to partition the L users' streams into q groups. Each group j has t_j users' streams. We denote the set of data streams that belong to the j^{th} group by S_j. The whole grouping process can be encapsulated in one $1 \times L$ vector $\mathbf{\Gamma}$, that states which group is assigned to which user stream. The entries of $\mathbf{\Gamma}$ can take values between 1 and q. The following is an example of the vector $\mathbf{\Gamma}$ for a system with $L = 6$, $q = 3$ and $t_1 = t_2 = t_3 = 2$

$$\mathbf{\Gamma} = [1, 3, 1, 2, 3, 2] \tag{4}$$

In this example the $S_1 = \{1, 3\}$, $S_2 = \{4, 6\}$ and $S_3 = \{2, 5\}$.

In the following lines we are going to summarize how both algorithms work. The first group decoder we consider is the GSIC. In GSIC, an MMSE decoding takes place within the group after the removal of the interference from the previously decoded groups. We refer to the set having all the previously decoded streams by $\tilde{S}_j$. For decoding a certain stream i in the group j, we first define the set $\tilde{S}_j^i$ as the union of set $\tilde{S}_j$ and stream i. This means that the stream i in group j would have the following rate

$$r_{\text{GS},i}(\mathbf{\Gamma}) = \log\left(1 + \frac{1}{\sigma_n^2}\mathbf{h}_i^H\left(\frac{1}{\sigma_n^2}\mathbf{H}_{(\tilde{S}_j^i)}\mathbf{H}_{(\tilde{S}_j^i)}^H + \mathbf{I}_M\right)^{-1}\mathbf{h}_i\right), \tag{5}$$

where $\mathbf{h}_i$ is the column vector of $\mathbf{H}$ corresponding to user i and $\mathbf{H}_{(\tilde{S}_j^i)}$ is the matrix $\mathbf{H}$ with *removing* all the columns corresponding to the data streams in $\tilde{S}_j^i$. The rate in (5) can be described as a difference between two rates as follows

$$\begin{aligned} r_{\text{GS},i}(\mathbf{\Gamma}) = \log\left(\det\left(\mathbf{I}_M + \frac{1}{\sigma_n^2}\mathbf{H}_{(\tilde{S}_j)}\mathbf{H}_{(\tilde{S}_j)}^H\right)\right) - \\ \log\left(\det\left(\mathbf{I}_M + \frac{1}{\sigma_n^2}\mathbf{H}_{(\tilde{S}_j^i)}\mathbf{H}_{(\tilde{S}_j^i)}^H\right)\right). \end{aligned} \tag{6}$$

The sum rate when the GSIC decoder is used becomes

$$R_{\text{GS}}(\mathbf{\Gamma}) = \sum_{i=1}^{L} r_{\text{GS},i}(\mathbf{\Gamma}). \tag{7}$$

The second group decoder we consider is the GPD. In GPD, the data streams within each group are decoded using SIC while having data streams in other groups as interference. The rate a certain stream i in a group j gets depends on its sequence of decoding within its group. However, the *sum rate of the streams in the group j, i.e. elements of S_j*, $r_{\text{GP},j}$ is constant and equal to

$$\begin{aligned} r_{\text{GP},j}(\mathbf{\Gamma}) = \log\left(\det\left(\mathbf{I}_M + \frac{1}{\sigma_n^2}\mathbf{H}\mathbf{H}^H\right)\right) - \\ \log\left(\det\left(\mathbf{I}_M + \frac{1}{\sigma_n^2}\mathbf{H}_{(S_j)}\mathbf{H}_{(S_j)}^H\right)\right). \end{aligned} \tag{8}$$

The sum rate when the GPD is used becomes

$$R_{\text{GP}}(\mathbf{\Gamma}) = \sum_{i=1}^{q} r_{\text{GP},i}(\mathbf{\Gamma}). \tag{9}$$

From equations (7) and (9), we can see that the sum rates of both decoders depend on the grouping strategy adopted. There exist $N_{\text{GS,c}}$ and $N_{\text{GP,c}}$ possible grouping combinations for GSIC and GPD, respectively, which are defined as follows:

$$\begin{aligned} N_{\text{GS,c}} &= \prod_{i=1}^{q}\binom{L - \sum_{j=0}^{i-1} t_j}{t_i}, \text{ where } t_0 = 0. \\ N_{\text{GP,c}} &= \frac{N_{\text{GS,c}}}{q!}. \end{aligned} \tag{10}$$

The division by $q!$ in (10) comes from the fact that the groups are processed in parallel not sequentially. Note that both the SIC and MMSE decoders are special cases of either group algorithms. For example the GSIC with L groups each of size 1 is the classical SIC. While GSIC with only one group of size L is the MMSE decoding.

III. Sum Rate Approximations for Group Decoders

A consequence of considering MIMO systems with a large number of antennas, is that the mean sum rate tends to be deterministic and a function of the statistical CSI. This helps to efficiently simplify the group decoders design.

A. Approximation of the Expectation of the Achievable Rate

A very tight approximation of $\bar{R}_M$ was presented in [9], which was developed using the relation between the Stieltjes transform and the Shannon transform. Theorem 2 in [9] can be summarized as: *In the limit of large L and M, but with* *maintaining the ratio $c = \frac{M}{L}$ constant, the ergodic rate $\bar{R}_M$ would almost surely be:*

$$\tilde{R}_M = \log\left(\det\left(\mathbf{I}_L + c\alpha\mathbf{T}\right)\right) + \log\left(\det\left(\mathbf{I}_M + \beta\mathbf{R}\right)\right) - \sigma_n^2 M\alpha\beta, \quad (11)$$

where

$$\alpha = \frac{1}{M}\mathrm{tr}\left(\mathbf{R}\left(\sigma_n^2\left[\mathbf{I}_M + \beta\mathbf{R}\right]\right)^{-1}\right) \quad (12)$$

$$\beta = \frac{1}{L}\mathrm{tr}\left(\mathbf{T}\left(\sigma_n^2\left[\mathbf{I}_L + c\alpha_i\mathbf{T}\right]\right)^{-1}\right), \quad (13)$$

where (12) and (13) are fixed point equations that are solved iteratively until convergence. More details about the derivation of equations (11),(12) and (13) and the properties of the correlation matrices $\mathbf{R}$ and $\mathbf{T}$, defined in (1), can be found in [9].

B. Ergodic Sum Rate Approximation of Group Decoders

We are now ready to provide an approximation of the sum rates of the different group decoders. We are going to start with the rates of the GSIC decoder. As it can be seen from (6), the rate for the stream i is being presented as difference between two rates. Hence, its mean $\bar{r}_{\text{GS},i}(\mathbf{\Gamma})$ is

$$\bar{r}_{\text{GS},i}(\mathbf{\Gamma}) = \mathrm{E}\left[\log\left(\det\left(\mathbf{I}_M + \frac{1}{\sigma_n^2}\mathbf{H}_{(\tilde{S}_j)}\mathbf{H}_{(\tilde{S}_j)}^H\right)\right)\right] - \mathrm{E}\left[\log\left(\det\left(\mathbf{I}_M + \frac{1}{\sigma_n^2}\mathbf{H}_{(\tilde{S}_j^i)}\mathbf{H}_{(\tilde{S}_j^i)}^H\right)\right)\right]. \quad (14)$$

In the limit of large L and M, the mean rate of stream i, $\bar{r}_{\text{GS},i}$ would almost surely be

$$\tilde{r}_{\text{GS},i}(\mathbf{\Gamma}) = \tilde{R}_M(\mathbf{T}_{(\tilde{S}_j,\tilde{S}_j)}) - \tilde{R}_M(\mathbf{T}_{(\tilde{S}_j^i,\tilde{S}_j^i)}), \quad (15)$$

where the matrix $\mathbf{T}_{(\tilde{S}_j,\tilde{S}_j)}$ is the matrix $\mathbf{T}$ with the *rows and columns* corresponding to the data streams in $\tilde{S}_j$ removed. The matrix $\mathbf{T}_{(\tilde{S}_j^i,\tilde{S}_j^i)}$ is defined similarly. By $\tilde{R}_M(\mathbf{T}_{(\tilde{S}_j,\tilde{S}_j)})$ we mean that we calculate the approximation of the rate in (11), (12) and (13) using the new transmit correlation matrix $\mathbf{T}_{(\tilde{S}_j,\tilde{S}_j)}$. Using (15) the mean sum rate of the GSIC $\bar{R}_{\text{GS}}(\mathbf{\Gamma})$ can be approximated by

$$\tilde{R}_{\text{GS}}(\mathbf{\Gamma}) = \sum_{i=1}^{L}\tilde{r}_{\text{GS},i}(\mathbf{\Gamma}). \quad (16)$$

By inspection of the group rate of the GPD in (8), we can directly deduce that the approximation of the mean of $r_{\text{GP},j}$ would directly be

$$\tilde{r}_{\text{GP},j}(\mathbf{\Gamma}) = \tilde{R}_M(\mathbf{T}) - \tilde{R}_M(\mathbf{T}_{(S_j,S_j)}). \quad (17)$$

Hence, the mean of the sum rate for the GPD $\bar{R}_{\text{GP}}(\mathbf{\Gamma})$ can be approximated by

$$\tilde{R}_{\text{GP}}(\mathbf{\Gamma}) = \sum_{j=1}^{q}\tilde{r}_{\text{GP},j} = q\tilde{R}_M(\mathbf{T}) - \sum_{j=1}^{q}\tilde{R}_M(\mathbf{T}_{(S_j,S_j)}). \quad (18)$$

Note that S_j is different from $\tilde{S}_j$. While the former is the set of streams to be decoded in group j, the later is the set of all the streams decoded before processing group j. Thus, S_j is used to calculate $\tilde{R}_{\text{GP}}$, while $\tilde{S}_j$ is used to calculate $\tilde{R}_{\text{GS}}$. *For the rest of the paper we are going to assume that the users are ordered on a line with inter-antenna spacing d_t and the receive antennas are organized similarly with inter-antenna spacing d_r.*

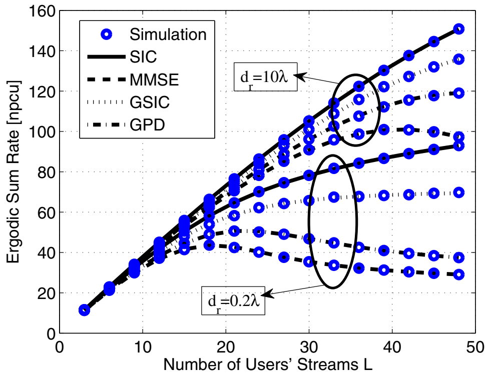

Fig. 1. Sum rate performance of GSIC and GPD for $\sigma_n^2 = 0$ dB, $M = 48$ with antenna separation distance $d_r = 0.2\lambda, 0.5\lambda$, $L = 3, 6 \ldots, 48$ uncorrelated streams and $t_1 = t_2 = t_3 = L/3$

We are now going to test how close our approximations are to the simulation results. For a system with $M = 48$ antennas, we vary the number of transmit antennas L from 3 to 48 with a step size of 3. The transmit antennas are assumed to be widely spaced and completely uncorrelated. For this case, all the possible groupings give the same mean rate, since all the streams have the same statistical properties. The scattering profile at the receiver is assumed to be wide-sense stationary uncorrelated scattering in 2D. As discussed in [12], the entries of the matrix $\mathbf{R}$ are given as

$$\mathbf{R}_{i,j} = J_0\left(\frac{2\pi d_{i,j}}{\lambda}\right), \quad (19)$$

where $d_{i,j}$ is the separation between the i^{th} and j^{th} antenna in terms of the wavelength λ and J_0 is the first kind Bessel function of order 0. We set σ_n^2 to 0 dB. Due to space limitation, we drop the effect of antenna coupling, which would only slightly affect our results quantitatively not qualitatively.

Fig. 1 shows the comparison between the approximations derived and the results from simulations for SIC, MMSE, GSIC and GPD. For the two group detectors we have $q = 3$ and $t_1 = t_2 = t_3 = L/3$. We plot the performance comparison for the two cases of $d_r = 0.2\lambda$ and $d_r = 10\lambda$. Even for not so large number of antennas, the approximations derived are very tight. In the case of small number of streams L, the difference between the achieved sum rate for different decoders is minor. This is a consequence of having channels with low eigenvalue spread, which means that the channels of different users have very weak correlation. However, the performance gap between different decoders becomes more evident as the number of streams L increases. Note that the performances of the receivers degrade strongly at $d_r = 0.2\lambda$ as the channel becomes more correlated. The MMSE decoder suffers the most as the number of users increase, especially in the case of high correlation. Even in the case of approximately uncorrelated channel, i.e. $d_r = 10\lambda$, dividing the decoding steps in GSIC to $q = 3$ leads to a loss much smaller than doing the decoding completely in one step (MMSE). *Note that we have large sum rates, as we have large L and M and each user transmits with power σ_x^2.*

IV. Grouping Based on Statistical CSI

In this section we study how the grouping affects the performance of the different decoders, and how to use the approximations derived before to suggest a grouping based on the statistical CSI. In general, we have a decoder with specific properties, such as decoder type, number of groups $q = \breve{q}$ and the number of streams in each group $t_i = \breve{t}_i \forall i \in \{1, .., q\}$. Finding the vector $\mathbf{\Gamma}_{\text{E,max}}$ that maximizes the sum rate $R_{\text{GE}}(\mathbf{\Gamma})$, where E can be either S for GSIC or P for GPD, for an instantaneous channel $\mathbf{H}$ can be formulated as the following constrained optimization problem

$$\begin{aligned} \underset{\mathbf{\Gamma}}{\text{maximize}} \quad & R_{\text{GE}}(\mathbf{\Gamma}) \\ \text{subject to} \quad & q = \breve{q}, t_i = \breve{t}_i \forall i \in \{1, .., \breve{q}\}. \end{aligned} \tag{20}$$

In words, the optimization problem in (20) is an L dimensional problem with integer values that are restricted from 1 to q with the constraint that the number of elements having the value i should be strictly equal to t_i. Such a problem is NP hard. For large L and for fast changing channel, solving (20) for every channel realization becomes unrealistic. Hence we suggest to use the vector $\tilde{\mathbf{\Gamma}}_{\text{E,max}}$ that maximizes the average sum rate approximation $\tilde{R}_{\text{GE}}$, i.e. the output of the following optimization problem

$$\begin{aligned} \underset{\mathbf{\Gamma}}{\text{maximize}} \quad & \tilde{R}_{\text{GE}}(\mathbf{\Gamma}) \\ \text{subject to} \quad & q = \breve{q}, t_i = \breve{t}_i \forall i \in \{1, .., \breve{q}\}. \end{aligned} \tag{21}$$

For a small number of users L finding the global maximum of (21) can be done using a brute force exhaustive search over all possible combinations. However, for larger L the complexity becomes a burden. A class of low complexity algorithms which is known to perform well for this kind of problems is the class of genetic algorithms (GA) [11], which we choose to solve the optimization problem at hand. For the upcoming results we consider a system with $L = M = 48$ and uncorrelated receiver, i.e. $\mathbf{R} = \mathbf{I}_M$.

A. Effect of Number of Groups

To consider the effect of increasing the number of groups q, while maintaining an equal number of users per group, i.e. $t_i = L/q \forall i = 1..q$, we plot the mean sum rate scaling against the inverse of the noise power. In Fig. 2, we plot the performance for the strong correlation case, i.e. $d_t = 0.2\lambda$. The performance for the case of moderate correlation of $d_t = 0.4\lambda$ is plotted in Fig. 3. The curves are annotated in the legend as (decoder type/Gr q), e.g. GSIC/ Gr 8 means the use of GSIC with $q = 8$.

Several interesting observations can be seen in Fig. 2. For GSIC, having two groups tremendously boosts the performance. The reason for that is the fact that in the case of the MMSE each stream is decoded having $L-1$ other streams as interference, while in the GSIC the streams in the first group have $L-1$ interferers, while the streams in the second group have $L/2-1$ interferers. The higher the number of groups gets, the better the performance achieved. But most importantly is the increase of the slope of the performance with the increase of the number of groups. For GPD, the performance decreases strongly when the number of groups change from 2 to 4. With lower number of groups, less streams suffer from high interference, as we do SIC within each group. This fact also leads to the strong decrease in the slope of the sum rate against the inverse of σ_n^2.

In the case of $d_t = 0.4\lambda$ in Fig. 3, we can see a similar behavior of the curves. However their are several observations to witness. First of all, the rates of all decoders are higher than for the case of $d_t = 0.2\lambda$. Also, the performance gap between the GPD with $q = 2$ and $q = 4$ is smaller, as well as the difference in the slope is obvious. The reason behind those two observations is the fact that at $d_t = 0.4\lambda$, the streams are less correlated, hence, their effect as inteferers to each other decreases strongly. The most important message of the two curves is the fact that we can use the GSIC decoder with our proposed fixed grouping, without having a very large number of groups and still get a performance very close to the optimal, but complex and latent SIC. The difference between the SIC curve and the GSIC curves to get sum rate of 150[npcu] for $q = 8$ and having $d_t = 0.2\lambda$ or $d_t = 0.4\lambda$ is about 1.3dB and 0.5dB, respectively. For the case of having $q = 4$, the performance gap becomes approximately 3.2dB and 1.2dB. These gaps are smaller for a lower target sum rate.

B. Performance Sensitivity to Grouping

We now quantify the performance difference due to different grouping. We define the vector $\tilde{\mathbf{\Gamma}}_{\text{E,min}}$, where E can be either S for GSIC or P for GPD as the output of an optimization problem similar to (21), but with minimization instead of maximization. We plot the rate scaling against the inverse of the noise power in Fig. 4. For GSIC, a large performance drop is found for the case of $q = 2$ when the $\tilde{\mathbf{\Gamma}}_{\text{S,min}}$ is used instead of the $\tilde{\mathbf{\Gamma}}_{\text{S,max}}$ which is shown in Fig. 2. For $q = 8$, there is still a performance drop but not as catastrophic as in the case of $q = 2$. This can be intuitively explained by the fact that an increase in the number of groups results in a decrease in the number of interfering streams for many streams. Hence, the difference between the maximum and the minimum performance decreases. When GPD is used, the performance drop is also huge for the case of $q = 2$, as well as the slope of the curves. This shows that a wrong grouping could lead to very highly correlated interferers, which leads to such a performance drop. A very interesting remark is, that a wrong selection of grouping may lead to a performance very close to the MMSE decoder as in the case of $q = 8$.

We evaluate how the minimum and maximum rates change with the change of the separation distance d_t, which reflects the correlation strength. We are going to use the two group decoders for $t_1 = t_2 = t_3 = 16$ and $\sigma_n^2 = -5$dB. As given by (10), there are $N_{\text{GS,c}} \approx 1.36 \times 10^{21}$ and $N_{\text{GP,c}} \approx 2.26 \times 10^{20}$ possible combinations to group the data streams for GSIC

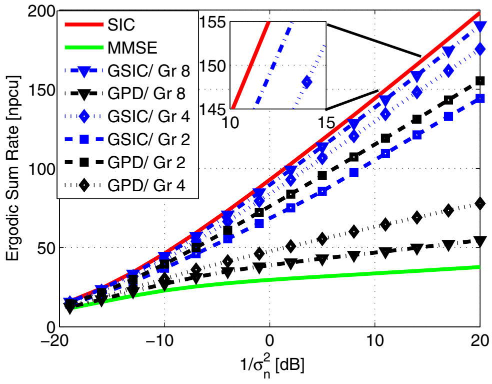

Fig. 2. Rate against $1/\sigma_n^2$ for using $\tilde{\mathbf{\Gamma}}_{\text{E,max}}$ at $d_t = 0.2\lambda$, $L = M = 48$, $q = 2, 4, 8$ with $t_i = L/q \forall i = 1, .., q$.(best grouping)

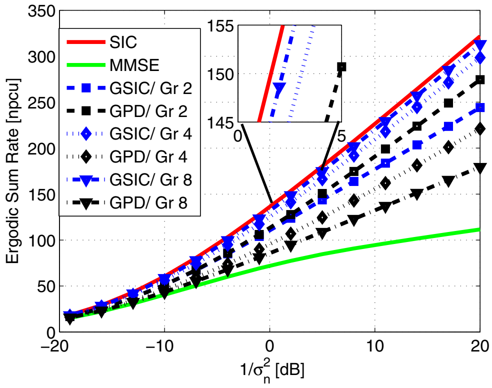

Fig. 3. Rate against $1/\sigma_n^2$ for using $\tilde{\mathbf{\Gamma}}_{\text{E,max}}$ at $d_t = 0.4\lambda$, $L = M = 48$, $q = 2, 4, 8$ with $t_i = L/q \forall i = 1, .., q$.(best grouping)

and GPD, respectively. In Fig. 5 we plot $\tilde{R}_{\text{GS}}(\tilde{\mathbf{\Gamma}}_{\text{S,max}})$ and $\tilde{R}_{\text{GS}}(\tilde{\mathbf{\Gamma}}_{\text{S,min}})$ for the GSIC (blue) and $\tilde{R}_{\text{GP}}(\tilde{\mathbf{\Gamma}}_{\text{P,max}})$, as well as $\tilde{R}_{\text{GP}}(\tilde{\mathbf{\Gamma}}_{\text{P,max}})$ for GPD (black) as a function of the antenna separation distance at the transmitter d_t. In the cases of high correlation, there is a large dependency on the grouping algorithm. This can be witnessed by the huge performance difference between the grouping that leads to the maximum sum rate approximation and the one leading to the minimum rate. However, the difference in the performance becomes very small at high separation (low correlation), as at low correlation all the streams would perform very close statistically.

V. Conclusion

We analyzed the performance of two different group decoding schemes, namely the GSIC and the GPD. Using tools from RMT and given statistical CSI, we developed tight approximations of the ergodic sum rates achieved when such decoders are deployed. For correlated streams, we suggested a grouping strategy which was shown to perform very near to the optimal SIC decoder and much better than the simple MMSE decoder. We also showed the effect of using different number of groups on the performance, as well as the effect of the correlation power between the streams. Our results show that the group decoders are strong and efficient candidates for massive MIMO systems.

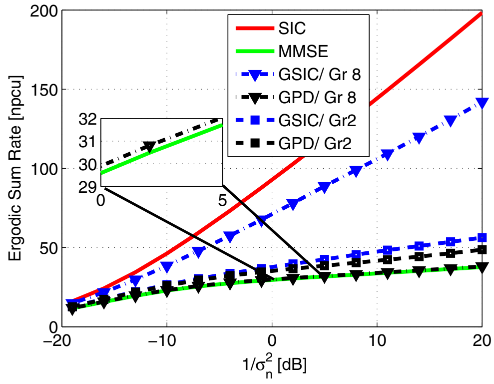

Fig. 4. Rate against $1/\sigma_n^2$ for using $\tilde{\mathbf{\Gamma}}_{\text{E,min}}$ at $d_t = 0.2\lambda$, $L = M = 48$, $q = 2, 8$ with $t_i = L/q \forall i = 1, .., q$. (worst grouping)

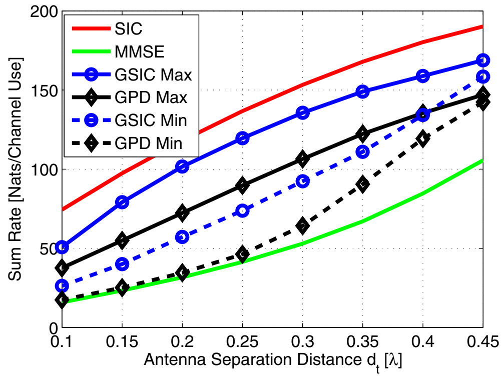

Fig. 5. Maximum and minimum rates for GSIC and GPD Vs. d_t for $L = M = 48$ at $\sigma_n^2 = -5$dB, $q = 3$ and $t_1 = t_2 = t_3 = 16$.

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