Coherent Multiuser Relaying with Partial Relay Cooperation

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***Abstract*- We consider a heterogeneous wireless ad hoc network. All nodes are within radio range and may employ an arbitrary number of antennas. Our goal is to achieve a distributed spatial multiplexing gain in this scenario without introducing additional delay. To this end we let 2N of the nodes form N source/destination pairs, that concurrently communicate on the same physical channel. Other nodes in the network act as coherent amplify&forward relays. We use relaying to minimize the cochannel interference between different links (coherent multiuser relaying) and to achieve diversity. Some of the relays may be able to exchange received signal information e.g. by utilizing a suitable short range wireless technology such as Bluetooth (partial cooperation). The main contributions of this paper are (i) a unified derivation of coherent multiuser relaying with an arbitrary number of relays and an arbitrary cooperation pattern and (ii) a novel block zero forcing relay gain allocation for this setting. The derivation naturally includes any combination of multi-antenna (MIMO) nodes. The gain allocation is based on a subspace approach and the complexity is essentially independent of the number of relays. Performance results show, that it achieves the full spatial multiplexing gain N. If the number of relays and/or cooperations exceeds a minimum value, we additionally obtain a distributed diversity gain and array gain for each source/destination link.**

I. Introduction

One of the most challenging tasks in the design of wireless networks is to accommodate a large number of user nodes (i.e. a large sum rate) in a confined bandwidth. Both ad hoc networks and cellular networks (with a fixed infrastructure) traditionally rely on variants of distributed spatial multiplexing to achieve this goal: in large ad hoc networks we resort to multihop links to improve the sum rate. Due to the reduced radio range per hop the transmit power is reduced and the same physical channel may be reused at some spatial distance in the network. Under a point-to-point coding model this leads to a sum rate of the ad hoc network, that scales with the square root of the user node density [1]. A major drawback of multihop is the delay involved in the decode&forward operation at each intermediate node. In cellular networks a fixed basestation infrastructure with high speed backbone network reduces the required radio range of the mobile nodes. Here distributed spatial multiplexing is achieved by frequency reuse in different cells. Due to the backbone network the delay is almost independent of the cell size, but the required infrastructure is expensive. As is well known, with multi-antenna nodes we can extent these classical approaches to realize a spatial multiplexing gain on the link level (MIMO wireless, e.g. [2]).

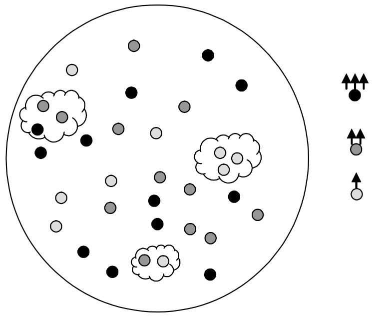

Fig. 1. Heterogeneous wireless ad hoc network. A cloud indicates an underlying wireless personal area network.

In this paper we consider a heterogeneous wireless ad hoc network (Fig. 1). All nodes are within radio range and may employ an arbitrary number of antennas. Our goal is to achieve a distributed spatial multiplexing gain in this scenario. To this end we let 2N of the nodes form N source/destination pairs, that concurrently communicate on the same physical channel. Other nodes in the network act as coherent amplify&forward relays. We use relaying to minimize the cochannel interference between different links (coherent multiuser relaying) and to achieve diversity. Some of the relays may be able to exchange received signal information e.g. by utilizing a suitable short range wireless technology such as Bluetooth (partial cooperation). A major benefit in comparison to multihop signaling is low delay, as all nodes are in radio range. On the other hand we do not save transmit power however. Some of the nodes are in close proximity (e.g. less than 10m) and use a wireless personal area network (WPAN) such as Bluetooth to locally exchange information about their respective received signals. A unidirectional cooperation between nodes i and j implies, that node i communicates its received signal to node j. Full cooperation in a cluster of nodes realizes a virtual antenna array, as each nodes knows the received signals of all other nodes in the cluster. We refer to an arbitrary set of unidirectional cooperations as partial cooperation.

To proceed we first review the related state of the art in cooperative signaling. In coded cooperation [3] multiple user nodes cooperate to jointly transmit their own coded information and (partial) coded information of other users. In cooperative diversity schemes [4] multiple nodes support the communication of a source/destination pair to improve diversity. Relaying schemes typically involve a 2-hop traffic pattern [5], where the relaying nodes forward the (processed) received signal in the second time slot to the destination(s). Upper and lower bounds on the capacity of wireless networks with a relay traffic pattern have been determined in [5]. The system model consists of one source/destination pair, while all other nodes operate as relays in order to assist this transmission. In [6] the analysis of [5] is extended and upper and lower bounds on the capacity of MIMO wireless networks are given.

Coherent (synchronous) relaying scheme explicitly or implicitly require a phase synchronization between the relaying nodes. This makes distributed beamforming possible, as the relay gains may be chosen such, that the relayed signals add up coherently at the destination. In multiuser relaying the relay nodes jointly process the signals of multiple source/destination pairs. In [6] is was observed, that the data streams of different users are orthogonalized, if the number of relays is large and the relay gains are appropriately matched to the channel coefficients. In [6,7] relaying scheme for multi-antenna nodes are suggested. For stream orthogonalization with a finite number of relays these schemes require however, that each relay itself can separate the different source streams (for a single source/destination pair this implies, that each relay has at least as many antennas as the source and destination).

In [8] we have introduced a coherent multiuser relaying scheme, which orthogonalizes the streams of different users in a homogeneous network with *single antenna nodes* and amplify&forward relays. In contrast to previous work we do neither require multiple antennas at the relays nor a very large number of relays. Let N be the number of single antenna source/destination pairs and N_r the number of single antenna relays. In [8] we have shown, that for $N_r > N \cdot (N-1)$ it is possible to choose the relay gains such, that the interference between different source/destination links is nulled (multiuser zero forcing relaying). Note, that this does not require any cooperation between the sources and the destinations respectively. In [9] we have shown on the basis of the average sum rate, that multiuser zero forcing relaying achieves the full spatial multiplexing gain N. As a result the sum rate in a dense ad hoc network scales with the square root of the number of nodes. This is the same behavior as in a multihop network. Due to the 2-hop traffic pattern however the delay is independent of the number of nodes. On the other hand the required transmit power per node does not drop with increasing user node density. In [10] the approach is verified on the basis of measured matrix channel impulse responses. In [11] the impact of noisy channel state information on the performance of the minimum relay configuration is analyzed.

In this paper we present a unified formulation of coherent multiuser relaying with an arbitrary number of amplify&forward relays (excess relay case) and an arbitrary cooperation between the relays (Section III). This naturally includes any combination of multi-antenna (MIMO) relays. Our original proposal [8] with single antenna nodes is included as a special case. In Section IV we suggest a novel block zero forcing gain allocation for this setting. It is based on a subspace approach and the complexity is essentially independent of the number of relays. The performance results in Section V show, that the novel gain allocation achieves the full spatial multiplexing gain and realizes a distributed diversity gain and a distributed array gain in the excess relay case.

II. Signal Model

We consider a $\left(N_1 \times \left(N_{r1} \times N_{r2}\right) \times N_2\right)$ network with a total number N_1 of antennas at the N sources (single and multi-antenna sources) and a total number N_2 of antennas at the N destinations. The relay tier includes a total number N_{r1} of receive antennas at all relays and a total number N_{r2} of transmit antennas. We assume a 2-hop relay traffic pattern (Fig. 2), i.e. there is no direct path between the sources and the destinations.

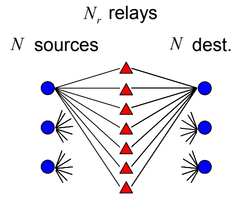

Fig. 2. Multiuser relaying

The amplify&forward (nonregenerative) half duplex relays are coherent in the sense that their local oscillators are phase synchronized to a global phase reference.

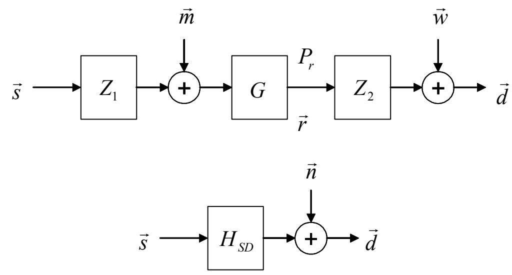

Fig. 3. Signal model

Fig. 3 depicts the signal model. The transmit symbols of the source antennas are stacked in the transmit symbol vector $\vec{s}$. The source sum transmit power is given by the expectation
$P_s = E_{\vec{s}}\left[\vec{s}^H \cdot \vec{s}\right]$

The matrix Z_1 denotes the channel matrix from the source antennas to the relay receive antennas and Z_2 the channel matrix from the relay transmit antennas to the destination antennas. The received signal at the relays includes the additive white Gaussian noise component $\vec{m} = CN\left(0, I \cdot \sigma_m^2\right)$. The relays multiply the received signal vector with the *gain matrix* G to obtain the relay transmit signal $\vec{r}$. The relay sum transmit power follows as

$$P_r = E_{\vec{s},\vec{m}}\left[\vec{r}^H \cdot \vec{r}\right] \tag{1}$$

Note that the columns of the gain matrix G correspond to the receive antennas and the rows to the transmit antennas of the relay tier. The vector $\vec{w} = CN\left(0, I \cdot \sigma_w^2\right)$ denotes the local noise contribution at the destinations. The decision vector $\vec{d}$ comprises the received signals at all destination antennas. It is given by

$$\vec{d} = Z_2 \cdot G \cdot Z_1 \cdot \vec{s} + Z_2 \cdot G \cdot \vec{m} + \vec{w} \equiv H_{SD} \cdot \vec{s} + \vec{n} \tag{2}$$

H_{SD} is the *equivalent channel matrix* and $\vec{n}$ the equivalent destination noise. It has the correlation matrix

$$\Lambda_{nn} = \left(Z_2 \cdot G \cdot G^H \cdot Z_2^H\right) \cdot \sigma_m^2 + I \cdot \sigma_w^2 \tag{3}$$

Due to the relay noise contribution, Λ_{nn} is in general not diagonal.

A simple and efficient approach to optimize the gain matrix G is the block zero forcing (ZF) criterion. For block ZF we choose the gain matrix subject to a relay sum power constraint (1) such, that there is no interference between different source/destination links. In other words block ZF nulls all elements of the equivalent channel matrix H_{SD}, which correspond to transmit/receive antenna pairs at non-associated sources and destinations. Besides simplicity an additional advantage of block ZF is the transparency to the source power allocation (near-far problem).

In the special case of single antenna sources and destinations the equivalent channel matrix H_{SD} is diagonal. To enhance the clarity of the exposition we will constrain our attention to this case throughout the paper. Furthermore we assume, that

- no channel state information is available at the sources. Consequently we use i.i.d. complex normal transmit symbols $\vec{s} = CN\left(0, I \cdot \sigma_s^2\right)$),
- the relay tier has the same number of receive and transmit antennas, i.e. $N_{r1} = N_{r2} = N_r$.

A configuration with N single antenna source, N associated single antenna destinations and a relay tier with N_r antennas will be denoted as $\left(N \times N_r \times N\right)$. For a given channel matrix Z_1 and i.i.d. source symbols the relay transmit signal has the sum power

$$P_r = \left\|G \cdot Z_1\right\|_2^2 \cdot \sigma_s^2 + \left\|G\right\|_2^2 \cdot \sigma_m^2 \tag{4}$$

The operator $\| \; \|_2^2$ denotes the squared Froebenius norm of a matrix. The signal to interference plus noise ratio $\mathrm{SINR}^{(k)}$ at destination (k) follows readily

$$\mathrm{SINR}^{(k)} = \frac{\sigma_s^2 \cdot \left|H_{SD}(k,k)\right|^2}{\sigma_s^2 \cdot \sum_{m \neq k}\left|H_{SD}(k,m)\right|^2 + \Lambda_{nn}(k,k)} \tag{5}$$

Let some of the N_r relays cooperate by exchanging their received signals. A unidirectional cooperation between relay i and j implies, that relay i communicates its received signal to relay j (e.g. by using a short range WPAN). As a result the element $G[j,i]$ of the gain matrix may be nonzero. To visualize the cooperation pattern, we introduce the cooperation matrix R_{coop}. A uni-directional cooperation i-j is identified by $R_{coop}[j,i] = 1$. All other elements are zero.

$$\begin{bmatrix}1&1&0\\1&1&0\\0&0&1\end{bmatrix} \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} \begin{bmatrix}1&1&0\\0&0&1\end{bmatrix}$$

(a) (b) (c) (d)

Fig. 4. Cooperation matrix for (a): one (2x2) MIMO relay and one single antenna relay, (b) three single antenna relays, (c) one (3x3) MIMO relay, (d) one relay with 2 receive and 1 transmit antenna and one single antenna relay.

Fig. 4 illustrates the cooperation matrix for some typical configurations. Note, that each off-diagonal one represents either a co-located antenna pairs or a unidirectional cooperation. Let $N_{r,c}$ denote the total number of cooperations between the relays (including "self cooperations" i-i). In the *pure relaying case* $N_{r,c} = N_r$ (Fig. 4, (a)) and the gain matrix is diagonal. In the *distributed antenna case* (full cooperation) $N_{r,c} = N_r^2$ (Fig. 4, (c)) and all elements of the gain matrix may be nonzero.

III. Zero Forcing Gain Matrix With Partial Relay Cooperation

In this section we present a new unified formulation for the ZF gain vector in the presence of a arbitrary relay cooperation pattern. Our exposition is based on the simple equality [12]

$$vec(ABC) = \left(C^T \otimes A\right) \cdot vec(B) \tag{6}$$

The operation $\vec{a} = vec(A)$ converts the matrix A into a vector $\vec{a}$ by stacking all columns. The operator $\otimes$ indicates the Kronecker product. For a (2×2) matrix A we obtain e.g.

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{bmatrix}$$

Let the equivalent channel vector be defined as $\vec{h}_{SD} = vec\left(H_{SD}\right)$ and the compound gain vector as $\vec{g}_0 = vec(G)$. With (2) and (6) we obtain

$$\vec{h}_{SD} = \left(Z_1^T \otimes Z_2\right) \cdot \vec{g}_0 \equiv A_0 \cdot \vec{g}_0 \tag{7}$$

We refer to A_0 as the compound channel matrix. For a given relay cooperation pattern we may drop the zero elements of the compound gain vector: $\vec{g}_0 \Rightarrow \vec{g} = \left(N_{r,c} \times 1\right)$ and the corresponding columns of the compound channel matrix: $A_0 \Rightarrow A = \left(N^2 \times N_{r,c}\right)$ without loss of generality. Thus the equivalent channel vector is given by

$$\vec{h}_{SD} = A \cdot \vec{g} \tag{8}$$

The equivalent channel matrix H_{SD} has "signal elements", which contribute signal power at the respective destination, and "interference elements", which generate interstream interference. In the present case of noncooperating destinations, the signal elements are the N diagonal elements of H_{SD} and the interference elements the $N(N-1)$ off-diagonal elements. We will constrain our attention to this case in the sequel. Let $\vec{h}_{SD,s}$ be the vector of signal elements of $\vec{h}_{SD}$ and $\vec{h}_{SD,I}$ the vector of interference elements. We define the compound signal matrix $A_S = \left(N \times N_{r,c}\right)$ and the compound interference matrix $A_{ZF} = \left(N(N-1) \times N_{r,c}\right)$ such, that

$$\begin{aligned} \vec{h}_{SD,s} &= A_S \cdot \vec{g} \\ \vec{h}_{SD,I} &= A_{ZF} \cdot \vec{g} \end{aligned} \tag{9}$$

Multiuser ZF relaying requires $\vec{h}_{SD,I} = 0$. Let $N_{ZF,0} = null\left(A_{ZF}\right)$ be a nullspace of A_{ZF}, i.e. $A_{ZF} \cdot N_{ZF,0} = 0$ and $N_{ZF,0}^H \cdot N_{ZF,0} = I$. Clearly any ZF gain vector lies in this nullspace, i.e. for any vector $\vec{y}$ we obtain a ZF gain vector $\vec{g}_{ZF}$ as

$$\vec{g}_{ZF} = N_{ZF,0} \cdot \vec{y} \tag{10}$$

We will refer to $\vec{y}$ as the nullspace gain vector in the sequel. A sufficient, but not necessary condition for a nonempty nullspace is given by $N_{rc} > N(N-1)$. We refer to the case

$$N_{rc} = N(N-1) + 1 \tag{11}$$

as the *minimum cooperation configuration*. In this case the nullspace gain vector is a scalar. In order to obtain the full spatial multiplexing gain N the equivalent channel matrix H_{SD} should have full rank, i.e. all diagonal elements should be nonzero. As

$$\operatorname{rank}\left(Z_2 \cdot G \cdot Z_1\right) \le \min\left(\operatorname{rank}\left(Z_2\right), \operatorname{rank}\left(G\right), \operatorname{rank}\left(Z_1\right)\right)$$

the gain matrix G should at least have rank N. This imposes some constraints on suitable relay cooperation patterns. In particular any cooperation pattern, which involves a total number of less than N receive and/or transmit antennas in the relay tier, leads to a rank deficient gain matrix (pinhole channel).

IV. Optimization Of The Nullspace Gain Vector

For the minimum cooperation configuration the nullspace gain vector (10) is a scalar: $\vec{y} = y$ and $|y|$ is uniquely determined by the relay sum power constraint (4). If there is an excess number of cooperations, we may use the additional degrees of freedom to optimize the performance. The dimensionality of the nullspace gain vector $\vec{y}$ grows proportionally to the number of cooperations. The gain vector (10) captures the signal contribution (9)

$$\vec{h}_{SD,s} = A_S \cdot N_{ZF,0} \cdot \vec{y} \tag{12}$$

It is convenient to transform the nullspace such, that the elements of $\vec{y}$ have decreasing impact on the vector $\vec{h}_{SD,s}$. With the singular value decomposition

$$U \cdot S \cdot V^H = A_S \cdot N_{ZF,0} \tag{13}$$

we obtain the desired transformation

$$N_{ZF} = N_{ZF,0} \cdot V \tag{14}$$

Note, that only

$$N_{ys} = \min\left(\operatorname{rank}\left(A_S\right), \operatorname{rank}\left(N_{ZF}\right)\right) \tag{15}$$

elements of $\vec{y}$ contribute to $\vec{h}_{SD,s}$. The other elements do not contribute signal energy at the destinations. However they may still have impact on the performance, as they influence the relay noise contribution at the destinations.

We suggest the following heuristic approach to the optimization of the compound gain vector:

1. determine $\vec{y}$ such, that the minimum diagonal element of the equivalent channel matrix H_{SD} is maximized
2. perform this maximization subject to the constraint $|\vec{y}|_2^2 = 1$ (i.e. $\|G\|_2^2 = 1$).

The constraint 2. relates to the average relay sum transmit power $\bar{P}_r$. To illustrate this we consider a channel matrix Z_1 with i.i.d. elements with unit variance and we assume, that the ZF gain matrices are uncorrelated to Z_1. With (4) we obtain

$$\begin{aligned} \bar{P}_r &= E_{Z_1}\left[P_r\right] = E_{Z_1}\left[\left\|G \cdot Z_1\right\|_2^2 \cdot \sigma_s^2\right] + \|G\|_2^2 \cdot \sigma_m^2 \\ &= \left(N \cdot \sigma_s^2 + \sigma_m^2\right) \cdot \|G\|_2^2 \end{aligned} \tag{16}$$

Clearly this assumption does not hold in reality, as the gain matrix is a deterministic function of Z_1 and Z_2. Due to (7) however G is a function of the product of certain elements of these matrices. This reduces the correlation and in reality (16) holds surprisingly well in many cases. Nevertheless in all simulations we have normalized the gain vector such, that the instantaneous sum relay transmit power satisfies $P_r = N \cdot \sigma_s^2$. The max-min approach 1. is motivated by fairness and diversity considerations. As an immediate consequence of the suggested approach we need to consider only those elements of the gain vector, that contribute signal energy at the destinations. Due to (15) these are the first N_{ys} elements of the gain vector.

We define the reduced gain vector $\vec{y}_s \equiv \vec{y}\left[1:N_{ys}\right]$ and let $N_{ZF,s}$ be the corresponding part of the nullspace N_{ZF} from (14). According to our suggested approach we determine the subspace gain vector as follows: with $\vec{h}_{SD,s} \equiv A_S \cdot N_{ZF,s} \cdot \vec{y}_s$ solve

$$\vec{y}_s = \arg\max_{\vec{y}_s}\left(\min\left(\vec{h}_{SD,s} \odot \vec{h}^*_{SD,s}\right)\right) \quad (17)$$

subject to $\left|\vec{y}_s\right| = 1$. The symbol $\odot$ denotes the Hadamard (element-wise) product.

For $N_{ys} = 2$, (17) has an analytical solution. Consider the QR-decomposition $A_S \cdot N_{ZF,s} = R \cdot Q$ such that R is a lower triangular matrix and Q is a unitary matrix. We substitute the gain vector by $\vec{y}_s = Q^H \cdot \vec{z}$ and obtain the triangular set of equations

$$\vec{h}_{SD,s} = \left(A_S N_{ZF,s}\right) \cdot \left(\vec{y}_s\right) = \left(RQ\right) \cdot \left(Q^H \vec{z}\right) = R\vec{z} \quad (18)$$

Without loss of generality we let

$$\vec{z} = \left[\left(1-\rho_2^2\right)^{1/2} \quad \rho_2 \cdot \exp\left(j\varphi_2\right)\right]^T \; \forall 0 \le \rho_2 \le 1 \quad (19)$$

Note, that $\left|\vec{z}\right| = 1$. As R is lower triangular,

$$R = \begin{bmatrix} r_{11} & 0 \\ r_{21} & r_{22} \end{bmatrix}$$

the angle φ_2 affects only the element $\vec{h}_{SD,s}[2]$. Thus for any given ρ_2 the value $\left|\vec{h}_{SD,s}[2]\right|$ is maximized by $\varphi_2 = \measuredangle\left(r_{21} \cdot r^*_{22}\right)$ and (17) is reduced to an optimization of ρ_2. This lead to the following solution:

if $\left|r_{21}\right| \ge \left|r_{11}\right| \Rightarrow \rho_{2,opt} = 0$

else

$$\tilde{\rho}_2 = \frac{\left|r_{11}\right| - \left|r_{21}\right|}{\sqrt{\left|r_{22}\right|^2 + \left(\left|r_{11}\right| - \left|r_{21}\right|\right)^2}}$$
$$\rho_{2,opt} = \min\left(\tilde{\rho}_2, \left|r_{22}\right| / \sqrt{\left|r_{22}\right|^2 + \left|r_{21}\right|^2}\right) \quad (20)$$

At time of writing for $N_{ys} \ge 3$ we resort to numerical optimization.

V. Performance Results

For all simulation results we let the channel matrices have i.i.d. complex normal random elements with unit variance. The source symbols as well are $CN(0,I)$. Thus the source sum transmit power P_s is proportional to the number of sources. Both sources and relays use the same sum transmit power: $P_s = P_r$. Relay and destination noise have the same variance: $\sigma_m^2 = \sigma_w^2$. The reference signal to noise ratio SNR_{ref} determines the noise variance at relays and destinations: $\sigma_m^2 = \sigma_w^2 = 1/SNR_{ref}$. In a (1x1x1) system the reference SNR is equal to (i) the average SNR at the relay and to (ii) the average SNR at the destination, if the relay would be noiseless. The equivalent SINR of the source/destination link (k) is given in (5). The maximum rate supported by this link in bit/complex dimension/source channel use is

$$R^{(k)} = \log_2\left(1 + \mathrm{SINR}^{(k)}\right) \quad (21)$$

Note, that this rate involves one channel use by each the source tier and the relay tier.

A. Figures of Merit

In this section we study the performance of multiuser ZF relaying with partial relay cooperation in terms of:

- *distributed array gain*: the average destination SINR normalized to the average destination SINR of a (1x1x1) system with *noiseless* relay.
- *distributed spatial multiplexing gain*: let $\bar{R}\left(\mathrm{SNR}_{ref}\right)$ be the mean sum rate as a function of the reference SNR. We approximate this function as
$$\bar{R}\left(\mathrm{SNR}_{ref}\right) \approx b \cdot \log_2\left(1 + a \cdot \mathrm{SNR}_{ref}\right) \quad (22)$$
and determine $a = \hat{a}$ and $b = \hat{b}$ such, that the mean error magnitude is minimized. $\hat{b}$ is the estimated spatial multiplexing gain.
- *effective diversity gain@outage probability*: this measure is determined on the basis of (i) the mean destination SINR and (ii) the outage destination SINR at a reference outage probability (here: $P_{out} = 10^{-2}$). The estimated effective diversity gain is one half of the number of degrees of freedom of a chi2-distributed random variable with the same mean and outage value.

The effective diversity gain depends to a certain extend on the reference outage probability. In contrast to the standard asymptotic definition of the diversity gain (infinite SNR) the effective diversity gain more directly reflects the diversity improvement in the operating region of the system under consideration. As an example in Fig. 5 we consider a system with one source/destination pair and $N_r = 1 \cdots 8$ relays. The solid lines indicate the empirical cumulative distribution functions (cdfs) for 10000 channel realizations each and the dashed lines the approximations. In this case there is a very close fit and the reference point has marginal impact on the estimated diversity factor. The approximations capture the slope of the empirical cdfs at the reference point $P_{out} = 10^{-2}$ very well.

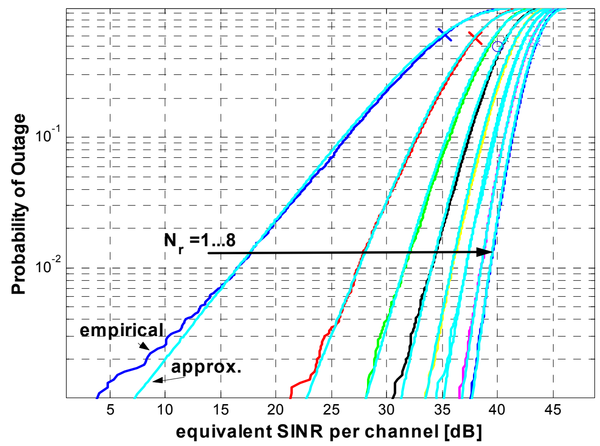

Fig. 5. Empirical cdf of the destination SINR and chi2-approximation for a $(1\times N_r \times 1)$ configuration.

B. Performance Results

In Figs. 6-8 we consider the performance of multiuser ZF relaying with three source/destination pairs. The minimum cooperation configuration (11) requires $N_{r,c,\min}=7$ cooperations in this case. The array gain, diversity gain and mean sum rate are plotted versus the excess number $N_{ex}=N_{r,c}-7$ of relay cooperations. The results are shown for four systems with different cooperation patterns:

System I is pure relaying; i.e. the cooperation matrix (Section II) is diagonal. This situation is typical for an ad hoc network, where some user nodes act as noncooperating relays. System II employs one MIMO relay with three antenna elements and N_{ex} additional noncooperating relays. This is typical for a cellular relaying application, where the cellular operator has installed a dedicated MIMO relay to improve range, coverage and sum rate. The noncooperating relays may be additional user nodes, that volunteer to support the communication.

Both systems III and IV are examples of ad hoc networks with partial relay cooperation. In both systems seven user nodes act as relays. The partial cooperation between the relay nodes could be based on an underlying short range wireless personal area network such as Bluetooth or Ultra Wideband (UWB), which enables cooperation between spatially adjacent nodes. For system III, the excess cooperations are padded along the first minor diagonal of the cooperation matrix. For system IV the excess cooperations are chosen at random. For each channel snapshot a new random cooperation pattern is used. For $N_{ex}=5$ e.g. we have for systems I-III:

$$R_{coop}^{(1)}=I_{12}\quad R_{coop}^{(2)}=\begin{bmatrix}11100000\\11100000\\11100000\\00010000\\00001000\\00000100\\00000010\\00000001\end{bmatrix}\quad R_{coop}^{(3)}=\begin{bmatrix}1100000\\0110000\\0011000\\0001100\\0000110\\0000010\\0000001\end{bmatrix}\tag{23}$$

For reference we also show the performance of a $(1\times N_{ex}+1\times 1)$ system.

The performance of all multiuser systems is essentially determined by the number of excess cooperations. In the minimum cooperation configuration ($N_{ex}=0$) all systems achieve the same array and diversity gain as the single user reference system and the sum rate is tripled. This indicates, that for the given setup the minimum cooperation configuration essentially orthogonalizes the multiuser system to N single user systems, i.e. to $(1\times 1\times 1)$. For all performance measures the systems rank according to their system number. Systems III and IV have very similar performance. Note, that both are employing the same number of relays. This illustrates, that for a given number of relays and excess cooperations the performance is quite insensitive to the actual cooperation pattern.

As shown in Fig. 6, for all systems the proposed gain allocation scheme is able to translate an excess number of cooperations into an distributed array gain. Note that an array gain of 0db implies, that the average SNR at the destination is the same as in a $(1\times 1\times 1)$ system with *noiseless* relay. For this reason the array gain for a small number of excess cooperations is negative. The reference system achieves a larger array gain than the multiuser systems. This may be a result of our optimization criterion, which is targeted at diversity gain rather than maximizing the average SNR.

For the same number of excess cooperations the number of degrees of freedom in the channel matrices increases with the number of relays, thus providing more potential for diversity gain. The diversity ranking of systems I to IV in Fig. 7 follows this intuition. System I essentially achieves the same performance as the reference system. The diversity gains of systems III and IV seem to saturates around 5.

The mean sum rate (Fig. 8) of all multiuser systems is very similar. The minor differences may essentially be attributed to the different array gains. In comparison to the reference system the mean sum rate is almost tripled, as all systems achieve the full spatial multiplexing gain $\hat{b}=3$ (22).

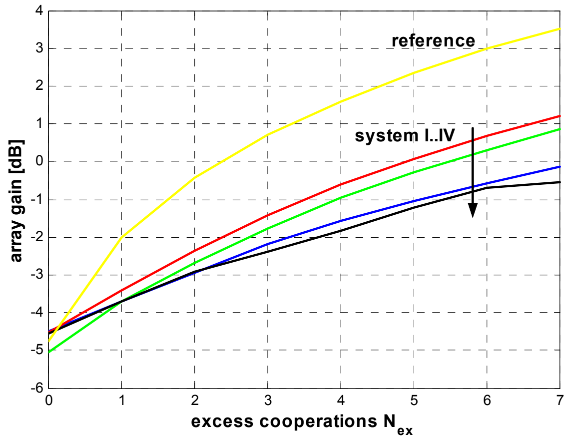

Fig. 6. Array gain versus the excess number of cooperations.

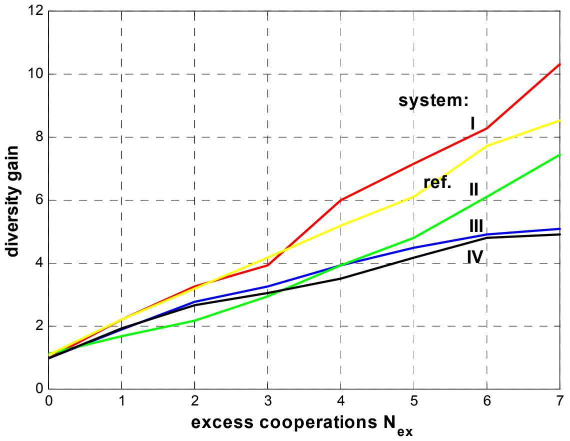

Fig. 7. Effective diversity gain versus the excess number of cooperations.

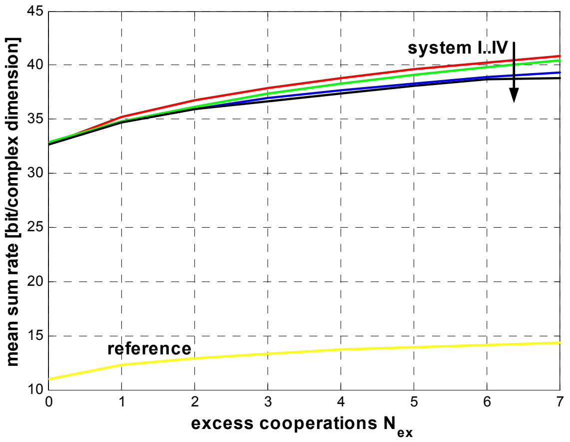

Fig. 8. Mean sum rate versus the excess number of cooperations.

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