Cooperative Broadcast Performance Prediction Using Inter-Node Distance Distributions

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***Abstract*—In this paper we propose a novel approach for the performance prediction of multistage cooperative broadcast. It is based on inter-node distance distributions, considers each stage independently and employs the expected values of the coverage area and the number of nodes reached. The required distance distributions are derived and a simple but accurate approximation of their expected values is discussed. The proposed approach is then evaluated for a large range of node densities and compared to Monte Carlo simulations. We thereby show that it leads to very accurate performance predictions for the coverage area and the number of nodes reached in each stage, even for low node densities.**

I. Introduction

Multistage cooperative broadcasting has been shown to be a promising concept to efficiently share a message with a large number of nodes in a mobile ad hoc network (MANET), such as a sensor network or a wireless military network [1]. In a first time slot (corresponding to the first stage respectively the first hop of the protocol) the source transmits its message. All nodes which have sufficient signal-to-noise-ratio (SNR) decode the message and retransmit it in the next time slot (i.e. in the next stage respectively hop). This procedure is repeated until all intended nodes could decode the message. In each stage the message is retransmitted either by all nodes which could decode so far (including the source), or only by the nodes which did not retransmit it yet. The retransmission is thereby performed using a transmit diversity scheme among all transmitting nodes (as, e.g., different codebooks for all transmitters or applying phase rolling [2]), such that the individual signal contributions add up in power at the receiving nodes. Hence, large SNRs can be achieved at the receivers and the message spreads quickly through the network. Compared to classical broadcasting large gains can be achieved in throughput as well as in delay.

Cooperative broadcasting has been widely studied in literature. In [3] its performance has been investigated in line networks by deriving an analytical expression for the outage probability. In a similar setup, the authors of [4] determine an upper bound on the maximum achievable gain compared to classical broadcasting. In [5] a two-hop cooperative broadcast protocol is presented and the outage probability and the diversity-multiplexing trade-off are developed. In [1] the transmission dynamics of the multistage cooperative broadcast protocol are derived recursively in a two dimensional network for the continuum limit, i.e. by letting the number of nodes grow to infinity while keeping the total transmit power constant.

Nevertheless, the performance prediction for low node densities is still a problem. While the evaluation with numerical simulations is computationally very costly, [3] and [4] only consider line networks and [5] a two-hop network. Although [1] allows to predict the performance very accurately for high node densities, it is not well suited for low densities.

In this context, we propose a novel performance prediction of multistage cooperative broadcast based on inter-node distance distributions. It allows an accurate performance prediction also for low node densities and has low computational complexity. At the same time, the derived distance distributions are well suited to predict the performance of classical broadcasting as well as single user multi hop transmission. However, these two applications are not covered in this paper.

Various papers have recently considered distance distributions in MANETs. An introduction to the mathematical tools of stochastic geometry and random geometric graphs is given in [6]. The authors of [7] derive the probability density functions (PDFs) of the distance from a source located in the center of a regular polygon to N other nodes uniformly distributed in the polygon. Therewith, they study wireless network characteristics. In [8], the authors derive the complementary cumulative distribution function of Euclidean distances between randomly selected nodes uniformly distributed on a disk and present tight lower bounds on the outage performance. In [9], a stochastic geometric approach is used to characterize the signal-to-interference ratio distribution of a millimeter wave ad hoc network with directional antennas. However, none of the derived distance distributions is suitable for the performance prediction of cooperative broadcast.

In this paper, we derive the necessary distance distributions and discuss simple but accurate approximations of their expected values (which can not be found analytically). Using these distances, we then propose a performance prediction for cooperative broadcast which considers each hop separately and employs the expected values of the coverage range (i.e. the range in which the message could be decoded) and the number of nodes reached per hop. The performance of the proposed approach is evaluated for various node densities and compared to Monte Carlo simulations. We thereby show that, despite the low computational complexity, the proposed approximations are legitimate and the performance prediction is very accurate, even for low node densities.

II. SYSTEM MODEL

We consider an ad hoc network where the locations of the nodes are points of a homogeneous Poisson point process (PPP) Φ with intensity δ in $\mathbb{R}^2$. That is, for any subset of the plane of area A, the mean of the number of nodes in this subset is given by $\delta \cdot A$. The channel between any two nodes is determined by a distance dependent path loss with path loss coefficient γ and a random phase shift. Considering Gaussian transmit symbols $s \sim \mathcal{CN}(0,1)$, transmit power P_{Tx} and additive white Gaussian noise $w \sim \mathcal{CN}(0, \sigma_w^2)$, the received signal of a node at distance d from the source can be written as

$$y = \sqrt{P_{\text{Tx}}} \cdot d^{-\gamma/2} \cdot h \cdot s + w, \tag{1}$$

with $h = e^{j\theta},\ \theta \sim \mathcal{U}(0, 2\pi)$. Without loss of generality, the source is considered to be located at the origin.

For our performance evaluation, we consider time slots of fixed length and a fixed transmission rate per hop of $R_{\min}$ in bits per channel use (bpcu). Whenever a node can decode the message (i.e. when it achieves a decoding rate of at least $R_{\min}$), it supports the source in all consecutive time slots by retransmitting the message using a different codebook, until the message has reached all intended nodes. Due to the different codebooks, the signal contributions add up in power at the receiving nodes, leading to a fast spreading of the message.

Note: With only small effort, the introduced approach can be adapted to variable transmission rates and flexible time slot durations, e.g., considering a fixed transmission range per hop or a fixed number of nodes to reach per hop. Furthermore, the distance distributions required for the performance evaluation for the case where each node retransmits the message only once (e.g., as in [1]) can be derived similarly to the distributions shown in the following. Due to space limitations, these related derivations and results are omitted in this paper.

III. INTER-NODE DISTANCES

In the following, we consider a binomial point process (BPP) [7] consisting of N nodes uniformly distributed on a disk of radius ρ. We derive the distributions of the distances between these nodes to a node of interest which is located outside of the disk at a distance D from the center (Fig. 1). These distance distributions and their expected values will be used to predict the performance of multistage cooperative broadcast in Sec. IV.

A. Inter-Node Distance Distributions

In [7] a general method is presented to derive the distribution of the distance between the n-th closest node and an arbitrary node of interest for a general BPP. In the sequel we apply this approach to our specific geometry (Fig. 1). To this end, we first determine the probability of a node being located in a circle of radius r around the node of interest. Therewith, we conclude on the cumulative distribution function (CDF) of the n-th closest node and finally determine the corresponding probability density function (PDF).

The probability that a node is located in a circle of radius r around the node of interest is given as the area of the intersection of this circle with the disk as sketched in Fig. 1, divided by the total area of the disk. The area A of the intersection can be found as the sum of the area of two segments

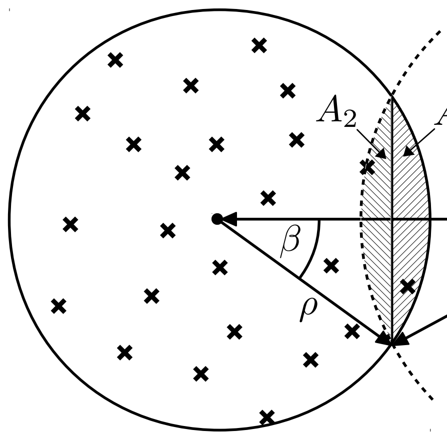

Fig. 1. A typical setup for one hop of the cooperative broadcast: N nodes distributed on a disk of radius ρ jointly transmitting to a node at distance D.

$$\begin{aligned} A &= A_1 + A_2 \\ &= \frac{1}{2}\rho^2 \left(2\beta - \sin(2\beta)\right) + \frac{1}{2}r^2 \left(2\alpha - \sin(2\alpha)\right) \end{aligned} \tag{2}$$

with

$$\beta = \arccos\left(\frac{\rho^2 + D^2 - r^2}{2\rho D}\right) \tag{3}$$

and

$$\alpha = \arccos\left(\frac{r^2 + D^2 - \rho^2}{2rD}\right) \tag{4}$$

for $r \in [D - \rho, D + \rho]$, the interval of interest. Hence, the probability that a node is located in a circle of radius $r \in [D - \rho, D + \rho]$ around the node of interest is given as

$$p = \frac{\rho^2 \left(2\beta - \sin(2\beta)\right) + r^2 \left(2\alpha - \sin(2\alpha)\right)}{2\pi\rho^2}. \tag{5}$$

From the binomial distribution, the probability that less than n of the N nodes on the disk are within the circle of radius r is then given by

$$\bar{F}_n(r) = \sum_{k=0}^{n-1} \binom{N}{k} p^k (1-p)^{N-k}. \tag{6}$$

At the same time, $\bar{F}_n(r)$ corresponds to the probability that the n-th closest node is at distance r or more. Hence, the CDF of the distance of the n-th closest node is given by $F_n(r) = 1 - \bar{F}_n(r)$. The corresponding PDF can then be found by taking the derivative of $F_n(r)$ with respect to r as given in [7] as

$$f_n(r) = \frac{-d\bar{F}_n(r)}{dr} = \frac{dp}{dr} \frac{(1-p)^{N-n} p^{n-1}}{B(N-n+1, n)} \tag{7}$$

where

$$B(a,b) = \int_0^1 t^{(a-1)} (1-t)^{(b-1)} dt \tag{8}$$

is the beta function [10]. The derivative of p can be found as

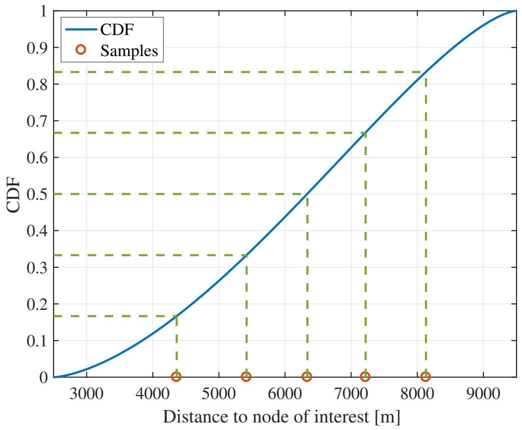

Fig. 2. CDF of the distance of an arbitrary node on the disk to the node of interest for $\rho = 3500$ m, $D = 6000$ m and the approximations of the expected values of the ordered distances for $N = 5$ (red circles).

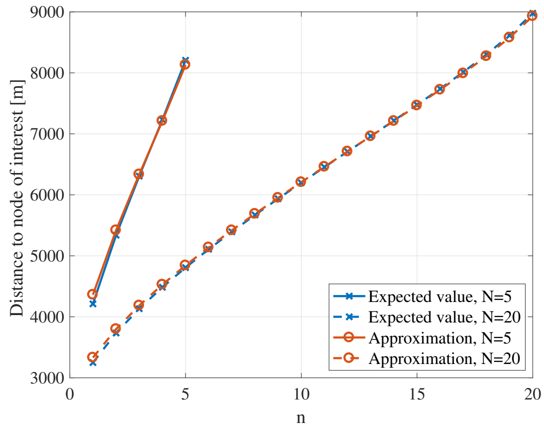

Fig. 3. Expected values of the distance of the n-th closest node to the node of interest and its approximations for $\rho = 3500$ m, $D = 6000$ m and $N \in \{5, 20\}$.

$$\frac{dp}{dr} = \frac{2r}{\pi\rho^2}\left(\frac{\rho}{D}\sqrt{1-b^2} - \frac{r}{D}\sqrt{1-a^2} + \arccos(a)\right), \quad (9)$$

where

$$a = \frac{r^2 + D^2 - \rho^2}{2rD}, \quad b = \frac{\rho^2 + D^2 - r^2}{2\rho D}. \quad (10)$$

B. Expected Values of Inter-Node Distances

For the performance prediction of the cooperative broadcast, which will be presented in Sec. IV, only the expected values of the distances are required. Unfortunately, the expected values of the PDFs derived above can not be found analytically. Thus, they have to be evaluated numerically. As this requires high computational effort, we discuss an approximation in the following which allows to determine the expected values of the distances much faster at high accuracy.

To this end, we consider the CDF of the distance of a single node, i.e. the probability that a uniformly distributed node on the disk is at distance r or less from the node of interest. This CDF is given by the normalized covered area A (2) as

$$C(r) = \frac{\rho^2(2\beta - \sin(2\beta)) + r^2(2\alpha - \sin(2\alpha))}{2\pi\rho^2} \quad (11)$$

with $r \in [D-\rho, D+\rho]$. It is shown in blue in Fig. 2 for $\rho = 3500$ m and $D = 6000$ m. Having N nodes independently and identically distributed (i.i.d.) on the disk, the distance of each node is distributed according to this CDF. The corresponding CDF values are samples from the uniform distribution $\mathcal{U}(0,1)$ (as any CDF value). However, regarding the ordered distances of these nodes to the node of interest, $d_1 \leq d_2 \leq, \ldots, d_N$, the corresponding ordered $U_n = C(d_n)$, $n \in \{1, \ldots, N\}$, are distributed according to the beta distribution $\mathrm{Beta}(n, N+1-n)$ with mean $\mu_n = n/(N+1)$ and variance $\sigma_n^2 = \left(n^2 + n(N+1)\right) / \left((N+1)^2(N+2)\right)$ [10]. That is, the expected CDF value of the n-th closest node is given by $n/(N+1)$.

Following [10] by considering $d_n = Q(U_n)$, i.e. $Q = C^{-1}$, expanding $Q(U_n)$ in a Taylor series around μ_n and taking the expectation of it, we get

$$\begin{aligned}\mathsf{E}[d_n] =& Q(\mu_n) + \mathsf{E}[(U_n - \mu_n)]Q'(\mu_n) \\ &+ \frac{1}{2}\mathsf{E}[(U_n - \mu_n)^2]Q''(\mu_n) + \ldots \\ =& Q(\mu_n) + \frac{n^2 + n(N+1)}{2(N+1)^2(N+2)}Q''(\mu_n) + \ldots .\end{aligned} \quad (12)$$

Hence, as the term of $Q'(\mu_n)$ evaluates to zero and by neglecting the higher order derivatives, we end up with

$$\mathsf{E}[d_n] \approx Q(\mu_n) = C^{-1}\left(\frac{n}{N+1}\right) \quad (13)$$

as a straight forward approximation of the expected values of the ordered distances. This procedure of inversely sampling the CDF is illustrated in green with the resulting samples in red for $N = 5$ in Fig. 2.

The accuracy of this approximation improves with increasing N, as the higher order derivatives in (12) are scaled with factors in the order of N^{-2} or less. However, as shown in Fig. 3 for $\rho = 3500$ m and $D = 6000$ m, a value of $N = 5$ (very low node density) already yields high accuracy. Only the approximation of the distance to the closest node is slightly off the real value. For $N = 20$, the approximation is very accurate for all nodes.

Unfortunately, the inverse of (11), C^{-1}, can not be found analytically (note: α and β are also functions of r). Hence, the values of r corresponding to $C(r) = \frac{1}{N+1}, \frac{2}{N+1}, \ldots, \frac{N}{N+1}$ have to be found numerically. However, this can be done very efficiently by a line search on $r \in [D-\rho, D+\rho]$.

C. Approximation of Inter-Node Distances

For a large number of nodes N on the disk, their actual distances d_n to the node of interest can be closely approximated by the expected value in (13). This can intuitively be seen by considering the empirical CDF of the d_n. The higher N, the better this empirical CDF approximates the actual CDF (11).

Hence, for a large number of nodes inversely sampling the CDF leads to accurate approximations of the real distances.

IV. Performance Prediction of Multistage Cooperative Broadcast

For the first hop of multistage cooperative broadcast it is simple to find the communication range and the PDF of the number of covered nodes analytically. However, it becomes very hard from the second hop on, as the distance which can be overcome in one hop depends on the number of nodes reached in the previous hop (Poisson distributed random variable) as well as on their spatial distribution. In this context, we present a low complexity performance prediction of the multistage cooperative broadcast in the following.

Analogously to Monte Carlo evaluations, we consider each hop separately and determine in every direction the coverage range and the nodes reached. Based on these results, the next hop is evaluated likewise and so on until the required area is covered respectively all required nodes are reached. However, instead of averaging over many realizations as in the Monte Carlo evaluations, we employ the expected values of the coverage distance and the number of nodes reached in each hop (requiring only low computational complexity). To this end, we make the following assumptions for each hop:

- The coverage area contains the expected number of nodes.
- These nodes are distributed according to a BPP.
- The message spreads circularly.

Hence, the expected coverage range can be determined from the distance distributions respectively its expected values derived in Sec. III. Due to the circular spreading, it is the same in all directions.

Intuitively, these steps can be repeated iteratively in each hop. Thereby, the assumptions and simplifications made are motivated as follows. If the locations of the nodes form a PPP in $\mathbb{R}^2$, the nodes contained in any subset of the plane are uniformly distributed in this subset. Hence, for a sufficiently large node density the message spreads circularly as from all directions the distribution of the distances from a node of interest outside of the disk to the nodes inside look alike. Therefore, also the communication range in all directions is approximately the same. For a high node density, the circular spreading of a message in multistage cooperative broadcast was also shown in [1]. Furthermore, for a large number of nodes in the disk, the distances to these nodes are well approximated by the expected values of the distances as discussed in Sec. III-C. If the coverage distance is accurately approximated also the covered number of nodes is accurately approximated by its expected value. Hence, for high node densities, each hop can be considered separately, based on the coverage area of the previous hop.

The performance prediction of the cooperative broadcast is summarized in the following and its performance evaluated. We thereby show that the stated assumptions are also valid for low node densities and allow to accurately predict the average performance of the multistage cooperative broadcast.

A. First hop

As we consider a fixed transmission rate $R_{\min}$, the distance which can be overcome in the first hop, $d_{\max,1}$, is also fixed and can be found from the achievable rate

$$R_{\min} = \log_2\left(1 + \frac{P_{\mathrm{Tx}} \cdot |h|^2 \cdot d_{\max,1}^{-\gamma}}{\sigma_w^2}\right) \tag{14}$$

as

$$d_{\max,1} = \left(\frac{P_{\mathrm{Tx}}}{(2^{R_{\min}} - 1)\cdot \sigma_w^2}\right)^{1/\gamma}, \tag{15}$$

as $|h|^2 = 1$.

B. Further hops

From the second hop on, we consider that the number of nodes reached so far (denoted by N_{k-1}) is exactly given by its expected value. Furthermore, we assume that these nodes are uniformly distributed on a disk of radius $d_{\max,k-1}$ centered around the origin, where k denotes the hop index.

The expected number of covered nodes (including the source) is given by

$$\mathsf{E}[N_{k-1}] \geq \pi \cdot \mathsf{E}[d_{\max,k-1}]^2 \cdot \delta \tag{16}$$

where the lower bound follows from Jensen's inequality (for $k = 2$ it is fulfilled with equality as $d_{\max,1}$ is deterministic). As an integer number of nodes is required to determine the coverage range of the next hop, we consider the ceiling of this value as the number of covered nodes (due to the $\geq$ in (16)), i.e.

$$N_{k-1} = \lceil \pi \cdot \mathsf{E}[d_{\max,k-1}]^2 \cdot \delta \rceil. \tag{17}$$

If all these nodes are simultaneously transmitting the same message with a different codebook, the expected achievable rate of a node of interest outside of this disk at distance D from the origin is given by Jensen's inequality as

$$\mathsf{E}[R] \leq \log_2\left(1 + \frac{P_{\mathrm{Tx}} \sum_{n=1}^{N_{k-1}} \mathsf{E}[d_n^{-\gamma}]}{\sigma_w^2}\right), \tag{18}$$

where d_n are the distances to the N_{k-1} nodes inside the disk. The minimum required received signal power to achieve $R_{\min}$ can then be approximated by

$$P_{\min} = P_{\mathrm{Tx}} \cdot \sum_{n=1}^{N_{k-1}} \mathsf{E}[d_n]^{-\gamma} \approx (2^{R_{\min}} - 1) \cdot \sigma_w^2. \tag{19}$$

The approximation $\approx$ comes from the fact that from Jensen's inequality $\mathsf{E}[d_n^{-\gamma}] \geq \mathsf{E}[d_n]^{-\gamma}$. Thus, we can't conclude on a bound based on the expected values of d_n.

Eq. (19) is key to estimate the distance $d_{\max,k}$ which can be overcome in this hop. In Sec. III we have determined the expected values of the sorted distances $\mathsf{E}[d_n]$ of all N_{k-1} nodes within a disk to a node of interest outside this disk. These expectations are a function of the disk radius ρ (Fig. 1) and the distance D between the node of interest and the center of the disk. Thus we can estimate the maximum range achieved in this hop as the value of D for which Eq. (19) is just fulfilled. As it is impossible to find this distance analytically, it has to be determined by a simple line search.

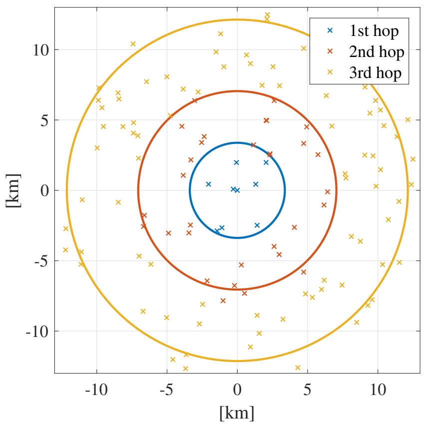

Fig. 4. Covered nodes per hop of one initialization of the Monte Carlo simulations for $\delta = 0.2$ nodes/km^2 (crosses) and the prediction of the coverage area (solid circles).

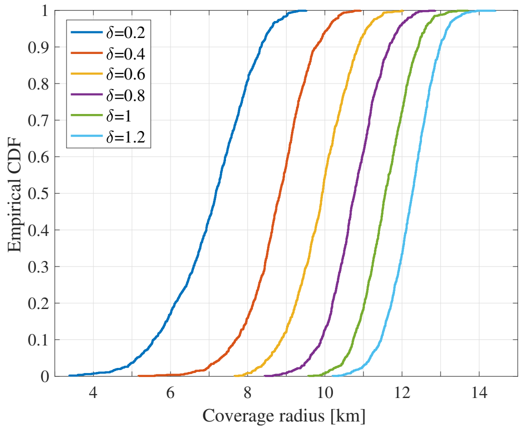

Fig. 6. Empirical CDF of the distance reached in the second hop of the Monte Carlo simulations for the cooperative broadcast.

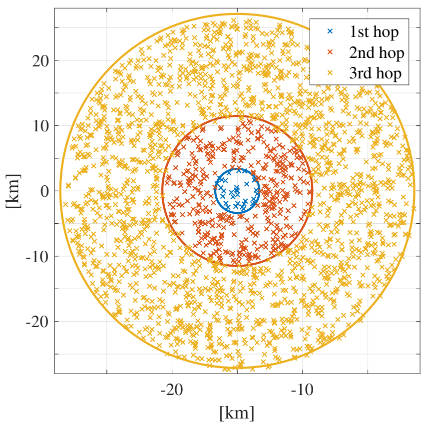

Fig. 5. Covered nodes per hop of one initialization of the Monte Carlo simulations for $\delta = 1$ nodes/km^2 (crosses) and the prediction of the coverage area (solid circles).

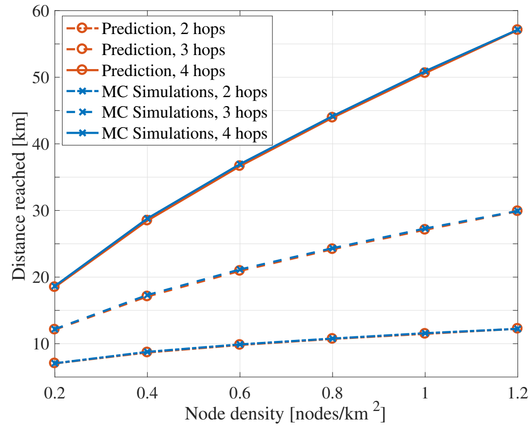

Fig. 7. Average distance reached per hop in multistage cooperative broadcast: prediction compared to extensive Monte Carlo simulations.

C. Performance Evaluation

The performance of the multistage cooperative broadcast can be predicted for large number of nodes and hops by iteratively applying the steps described above until the required average number of nodes is reached or the required average distance is overcome. The following results show the predicted performance compared to Monte Carlo simulations over 4 hops. The parameters have been set to $R_{\text{min}} = 0.5$ bpcu, $\sigma_w^2 = 6.25 \cdot 10^{-14}$ W, $P_{\text{Tx}} = 0.001$ W and $\gamma = 3$ with node densities of $\delta \in \{0.2, 0.4, \ldots, 1.2\}$ nodes/km^2. These numbers are motivated by a military MANET with typically large dimensions and low data rates.

The figures Fig. 4 and Fig. 5 show two examples of the prediction of the coverage per hop (solid circles) and the actual nodes reached in each hop for one initialization of the Monte Carlo simulations (crosses). We show the first 3 hops and consider $\delta = 0.2$ nodes/km^2 as well as $\delta = 1$ node/km^2. It can be observed that although the prediction of the coverage area is already quite accurate for $\delta = 0.2$ nodes/km^2 (Fig. 4), there are still nodes significantly outside of the predicted area. The coverage area is only roughly approximated by a disk due to the strong influence of the actual node topology. For $\delta = 1$ node/km^2 (Fig. 5), this effect is already strongly reduced and the coverage area is well approximated by a disk with radius given by the predicted coverage distance.

The strong influence of the random node topology for low node densities can also be seen in Fig. 6, which exemplarily shows the empirical CDFs of the coverage distance in the second hop of the Monte Carlo simulations. We consider the coverage distance for each random node topology of the Monte Carlo simulations as the maximal range achieved in the x-direction[1]. It can be seen, that the variance for low densities is rather high, while for increased density it is reduced. Actually, according to [1], for $\delta \to \infty$, the variance becomes zero and the prediction is perfect.

Despite the strong influence of the random node topology, the average distance reached in each hop is very well predicted by our approach. This is evidenced by Fig. 7 which compares the results for two, three and four hops as a function of the node

[1]For a given node topology the coverage distance depends on the direction considered. For symmetry reasons the average coverage distance is however independent of the direction.

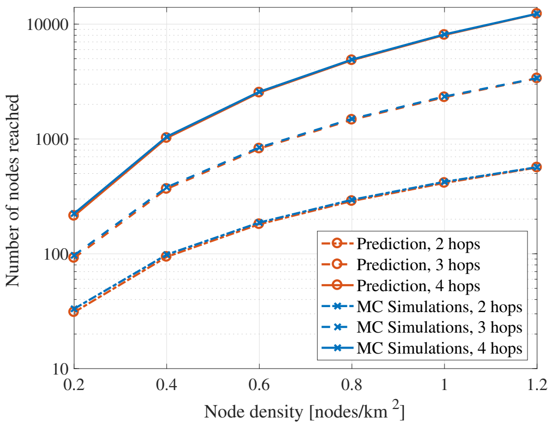

Fig. 8. Average number of nodes reached per hop in the multistage cooperative broadcast: prediction compared to extensive Monte Carlo simulations.

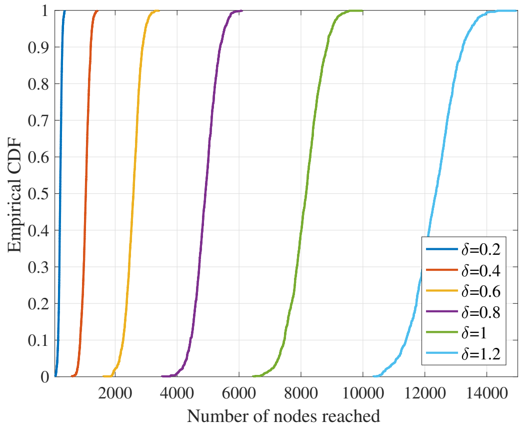

Fig. 9. Empirical CDF of the number of nodes reached in the fourth hop of the Monte Carlo simulations. Parameter is the node density δ [nodes/km^2].

density δ. Even over 4 hops, the accuracy is still very high. That is, the randomness of the first 3 hops does not affect the prediction accuracy of the fourth hop.

As is to be expected by the accurate results on the average coverage distance, we also get very precise results for the number of nodes reached per hop. The corresponding predictions and average values of the Monte Carlo simulations are shown in Fig. 8.

Nevertheless, although the average values are very accurate, the variance is quite high. This can be seen in Fig. 9 which shows the empirical CDF of the number of nodes reached in the fourth hop. Especially for high node densities (achieving large coverage ranges), the variance is very large. This comes from the fact, that the number of nodes covered changes quadratically with the coverage distance. Hence, already small variations in the coverage range lead to large variations in the number of nodes.

D. *Impact of Parameters on Results*

While the presented results are very promising, a reasonable question is how generally valid these results are with changing parameters and increasing number of hops.

As was shown for the setup above, the accuracy with increasing number of hops is not decreasing until 4 hops. This is also expected for higher number of hops (no Monte Carlo simulations available due to computational complexity). As the number of covered nodes increases quadratically with the coverage radius (and thus with the number of hops), the expected values approximate the real distances more accurately at higher hop counts (c.f. Sec. III-C) and the approximation of the expected values (13) is also more precise.

Considering the parameters, the prediction accuracy is expected to decrease when only a very low expected number of new nodes is covered in each hop. This might occur for very low node densities, or also for high $R_{\min}$, a high path loss or low P_{tx} at low node densities. Nevertheless, in these operating regimes the performance of the cooperative broadcast is anyways strongly affected by outages (i.e. the message can not be delivered to all intended nodes), and is therefore not in the focus of our performance prediction.

V. Conclusions

In this paper, we presented a novel performance prediction of the multistage cooperative broadcast based on the expected values of the inter-node distances. To this end, we derived the required distance distributions, discussed an accurate and computationally simple approximation of its expected values and evaluated its performance in comparison to extensive Monte Carlo simulations. It was shown that the proposed approach leads to very accurate results at low computational complexity. Hence, it can be applied to predict the performance for large networks in reasonable time.

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