Rate Maximization in Coupled MIMO Systems: A Generic Algorithm For Designing Single-Port Matching Networks

Yahia Hassan and Armin Wittneben
Communication Technology Laboratory, ETH Zurich, 8092 Zurich, Switzerland
{hassan, wittneben}@nari.ee.ethz.ch

***Abstract*—Compact arrays are known to be associated with antenna coupling and noise correlation. The noise can be either antenna noise, LNA noise or downstream noise. Due to these effects, it was shown that the matching network affects the performance of MIMO systems with coupled receiver antennas. Since the optimal multiport matching network is of very high complexity as well as very narrow operation bandwidth, development of single-port (SP) matching networks that boost the performance became inevitable. In this paper we develop a gradient-search algorithm to design the matching network for achievable rate maximization of multi user MIMO systems. For *any* combination of noise sources, we rigorously derive the exact gradient of the achievable rate with respect to the components of the matching network. We assume either full knowledge of the spatial channel or knowing its statistical properties. In the later case we optimize the matching network to maximize the Jensen's bound. Substantial performance enhancement is shown when our algorithms are used. Significant reduction in the array area is gained in comparison to the often used $\lambda/2$ antenna spacing without taking coupling into account. This can be vital for future wireless systems adopting massive MIMO arrays. Via eigenvalues distribution simulations at different SNR regimes, we show an intuitive link to the communication theory.**

I. Introduction

Future wireless networks are facing several challenges, one of which is supporting extremely high spectral efficiency. With the scarcity of frequency resources, the use of the spatial domain to increase data rates is the main way to reach the desired performance. In MIMO communication increasing the number of antennas boosts the performance. As a result, massive MIMO arrays have recently attracted considerable attention [1]. However, the (massive) antenna arrays are desired to be compact. Low transmit frequencies (e.g. like in TV white space [2]) as well as antenna form factor constraints require an antenna spacing of only a fraction of a wavelength (λ).

Compact MIMO systems are associated with different physical effects. The most interesting of such are the spatial correlation and antenna coupling. Due to coupling, the additive noise at the receiver that comes from external sources [3], from the ohmic losses in the antennas and from the low noise amplifier (LNA) also become spatially correlated. However, *noise* from different sources has different correlation properties, as studied in [4], [5]. As it was shown in different papers like [1], neglecting antenna coupling in studying compact MIMO systems leads to misleading results. The reason behind that is the fact that antenna coupling introduces impedance mismatch that leads to less signal power absorbed by the receiver, and more noise power generated by the LNAs, thus causing performance degradation. The optimal solution of such problems was discussed in [6], [7] and was shown to be the multiport matching networks. Such a network is very hard to implement and suffers from very narrow operation bandwidth [8]. This led to the research topic of single-port (SP) matching network design for coupled MIMO receivers. In SP matching, each antenna has its own *separate* matching network.

Optimizing the SP matching network for performance enhancement of the compact MIMO systems was studied in various publications. In [9] an algorithm for SP matching network optimization is given for a 2×2 system. The matching networks per-antenna are assumed to be identical. This algorithm is based on the knowledge of the statistical properties of the channel. The work of [9] is extended to non-identical per-antenna matching networks in [10]. However, the noise is assumed to be spatially uncorrelated in both papers. Another approach to minimize the noise generated by the LNA is presented in [11]. An additional algorithm is introduced for maximizing the maximum power transferred to the receiver in [12]. For MIMO systems maximizing signal power transferred to the receiver or minimizing noise power generated by the LNAs are not the only metric to maximize the rate. Recently, a random search algorithm is introduced in [13] to maximize the rate given the instantaneous channel but assuming only uncorrelated noise. Up to our knowledge, designing SP matching networks for rate maximization was not studied in the presence of LNA noise.

In this paper we design the SP matching network to maximize the achievable rate of a MIMO system with a compact receive antenna array. The novelty of our work comes from the fact that we derive the exact analytical gradient of the achievable rate with respect to different components of the matching network. We hinge our idea on the fact that the matching network affects the signal and noise from different components differently. Hence, our gradient is generic for *any noise contribution* from any source. We start with a complete system model that maps circuit theory to the classical MIMO model. Next, we derive the gradient of the achievable rate of the system with respect to the components of the matching network. We discuss the cases of having either full or statistical knowledge of the spatial channel. In the later, we design the matching network that maximizes Jensen's bound of the achievable rate [14]. Afterward, we discuss our simulation results. Comparing the performance of using our designed matching networks to using the conventional one that ignores coupling shows that using our algorithms can substantially reduce the area size. Finally, we show the effect of the operation SNR on the behavior of the algorithm. Such results give deeper insight on how the algorithm works from

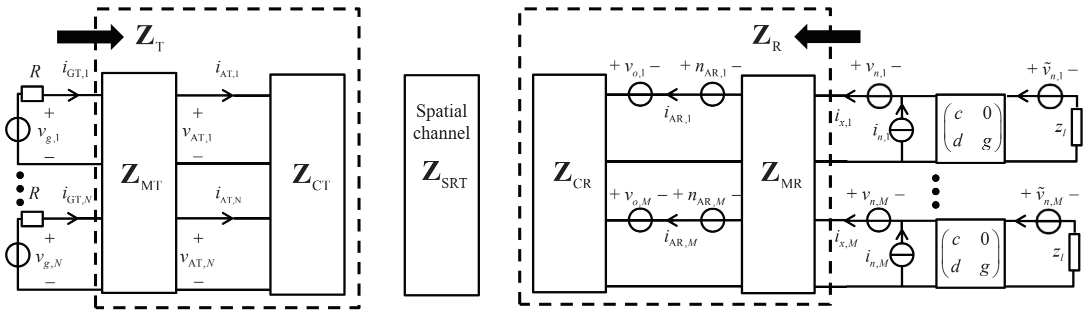


Fig. 1: Circuit model for MIMO communications. The transmitter is on the left and the receiver is on the right.

a communication theoretic point of view.

II. MIMO System Model

In communication theory, a MIMO system with N transmit antennas and M receive antennas, is often described by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{H}$ is the channel matrix, and $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{n}$ are the vectors of the transmit signals, received signals and noise, respectively. In the sequel, we summarize the equivalent relation between the transmitter generator voltages $\mathbf{v}_g$, and the receiver load voltages $\mathbf{v}_l$ and noise $\mathbf{u}_n$ at the impedance z_l after the LNA. Note that z_l models all the circuitry that are after the LNA. We extend the model presented in [5] in order to capture the effects of post LNA circuitry.

A. System Components

The circuit models of the transmitter and the receiver are shown in Fig.1. The matrices $\mathbf{Z}_{\text{MT}}$ and $\mathbf{Z}_{\text{MR}}$ denote the *lossless* matching networks at the transmitter side and at the receiver side. The receiver matching network has the form

$$\mathbf{Z}_{\text{MR}} = \begin{bmatrix} j\mathbf{Z}_{\text{MR},11} & j\mathbf{Z}_{\text{MR},12} \\ j\mathbf{Z}_{\text{MR},21} & j\mathbf{Z}_{\text{MR},22} \end{bmatrix}. \tag{2}$$

For a matching network to be lossless it should be pure imaginary and symmetric [5]. This means that $\mathbf{Z}_{\text{MR},11} = \mathbf{Z}^T_{\text{MR},11}$, $\mathbf{Z}_{\text{MR},22} = \mathbf{Z}^T_{\text{MR},22}$and $\mathbf{Z}_{\text{MR},12} = \mathbf{Z}^T_{\text{MR},21}$. Note that SP matching means that the submatrices of the matching network in (2) are *diagonal*.

The matrices $\mathbf{Z}_{\text{CT}}$ and $\mathbf{Z}_{\text{CR}}$ denote the impedance matrices of the antenna arrays of the transmitter and the receiver, respectively. Each matrix $\mathbf{Z}_{\text{Ci}}$ (where i can either be T to denote the transmitter or R to denote the receiver), is formed from two components. The first is the radiation component $\mathbf{Z}_{\text{Ai}}$ and the second is the real valued, diagonal lossy ohmic component $\mathbf{R}_{\text{Ai}}$

$$\mathbf{Z}_{\text{Ci}} = \mathbf{Z}_{\text{Ai}} + \mathbf{R}_{\text{Ai}}. \tag{3}$$

For arrays of half wavelength dipoles, the matrix $\mathbf{Z}_{\text{Ai}}$ can be calculated using formulas in [15]. Such antennas are called *canonical minimum scattering* antennas. This means that an open circuit antenna does not interact with the electric field around it [5]. Note that the impedance matrix is a *symmetric* matrix, i.e. $\mathbf{Z}_{\text{Ci}} = \mathbf{Z}^T_{\text{Ci}}$. We denote the input impedance matrices looking into the matching networks at the transmitter side and at the receiver side by $\mathbf{Z}_T$ and $\mathbf{Z}_R$, respectively. From circuit theory [5], the impedance matrix $\mathbf{Z}_R$ is related to the matrices $\mathbf{Z}_{\text{MR}}$ and $\mathbf{Z}_{\text{CR}}$ by

$$\begin{aligned} \mathbf{Z}_R &= j\mathbf{Z}_{\text{MR},11} + \mathbf{Z}_{\text{MR},12}(j\mathbf{Z}_{\text{MR},22} + \mathbf{Z}_{\text{CR}})^{-1}\mathbf{Z}^T_{\text{MR},12} \\ &= j\mathbf{Z}_{\text{MR},11} + \mathbf{F}_R\mathbf{Z}^T_{\text{MR},12}. \end{aligned} \tag{4}$$

Similarly, matrix $\mathbf{Z}_T$ is related to the matrices $\mathbf{Z}_{\text{MT}}$ and $\mathbf{Z}_{\text{CT}}$. The terms R and z_l denote the impedance of the source and the post LNA impedance, respectively. On the transmit side, the antennas currents $\mathbf{i}_{\text{AT}}$ are related to the generator voltages $\mathbf{v}_g$ by the relation

$$\mathbf{i}_{\text{AT}} = \mathbf{M}_T\mathbf{v}_g. \tag{5}$$

The matrix $\mathbf{M}_T$ is a function of the antenna coupling matrix and the matching network at the transmitter, and it has the unit Siemens. More details about it can be found in [5]. For notational convenience, we will drop the subindex MR of the matching networks submatrices, for example we denote $\mathbf{Z}_{\text{MR},11}$ by $\mathbf{Z}_{11}$.

We introduce the transimpedance matrix $\mathbf{Z}_{\text{SRT}}$ describing the effect of the physical propagation channel between the transmitter and the receiver, i.e. the spatial channel. It relates the currents flowing in the transmitter antenna array $\mathbf{i}_{\text{AT}}$ to the open circuit voltage at the receiver antenna array $\mathbf{v}_o$

$$\mathbf{v}_o = \mathbf{Z}_{\text{SRT}}\mathbf{i}_{\text{AT}}. \tag{6}$$

The physical spatial channel is modeled using the Kronecker model [16]

$$\mathbf{Z}_{\text{SRT}} = \mathbf{R}^{\frac{1}{2}}_{\text{RX}}\mathbf{Z}_p\mathbf{R}^{\frac{1}{2}}_{\text{TX}}. \tag{7}$$

The entries of the matrix $\mathbf{Z}_P$ are independent identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. The two matrices $\mathbf{R}_{\text{TX}}$ and $\mathbf{R}_{\text{RX}}$ are the spatial correlation matrices at the transmit antenna array and, respectively, the receive antenna array. The elements of the correlation matrices are functions of the scattering profiles around the transmit and receive arrays as well as the antenna elements spacing in terms of wavelength.

B. Noise Sources

There are four main noise sources at the receiver. The antennas are responsible for two of them, namely the external noise $\mathbf{n}_{\text{ext}}$ collected by the radiation component of the antenna array $\mathbf{Z}_{\text{AR}}$ and the noise generated by the losses $\mathbf{R}_{\text{AR}}$ in the antennas $\mathbf{n}_l$. Both $\mathbf{n}_{\text{ext}}$ and $\mathbf{n}_{\text{l}}$ are assumed to be Gaussian and generated from 3D isotropic noise sources with temperatures T_{AE} and T_{AL}, respectively . According to [3],

$$\mathbf{R}_{na} = \text{E}[\mathbf{n}_{\text{AR}}\mathbf{n}^H_{\text{AR}}] = 4\text{k}_{\text{B}}B_W(T_{\text{AE}}\Re\{\mathbf{Z}_{\text{AR}}\} + T_{\text{AL}}\mathbf{R}_{\text{AR}}), \tag{8}$$

where k_B is the Boltzmann constant, B_W is the bandwidth.

The third noise source is the LNA. An LNA is usually assumed to be a unilateral device. This means that the signal at its output port does not affect the signal at the input port. An LNA is characterized by its impedance parameters

$$\mathbf{Z}_{\text{LNA}} = \begin{bmatrix} c & 0 \\ d & g \end{bmatrix}. \qquad (9)$$

As discussed in [4], [5], the noise of each LNA is modeled by a series voltage source and a parallel current source at the input of the LNA. Such sources have the statistical properties:

$$\begin{aligned} \text{E}[\mathbf{i}_n\mathbf{i}_n^H] &= \beta\mathbf{I}_M, \quad \text{E}[\mathbf{v}_n\mathbf{v}_n^H] = \beta R_N^2\mathbf{I}_M \\ \text{E}[\mathbf{v}_n\mathbf{i}_n^H] &= \rho\beta R_N\mathbf{I}_M. \end{aligned} \qquad (10)$$

Each LNA has an optimal matching impedance z_{opt} for which its noise contribution is minimal. Note that there is no correlation between noise generated from different LNAs.

The fourth noise source is the downstream noise which is generated by all the circuitry after the LNA [7]. It is modeled by the voltage source $\tilde{\mathbf{v}}_n$, which has the following statistical property:

$$\text{E}[\tilde{\mathbf{v}}_n\tilde{\mathbf{v}}_n^H] = \psi\mathbf{I}_M. \qquad (11)$$

C. Input Output Relationship

Using circuit theory relations similar to the ones used in [5], [7], we write the final input-output equation of the MIMO system similar to (1)

$$\mathbf{v}_l = \mathbf{A}\mathbf{v}_g + \mathbf{u}_n, \qquad (12)$$

with equivalent channel matrix $\mathbf{A}$ and noise vector $\mathbf{u}_n$ given as

$$\begin{aligned} \mathbf{A} &= j\frac{z_l d}{z_l + g}\mathbf{D}\mathbf{F}_R\mathbf{Z}_{\text{SRT}}\mathbf{M}_T & (13) \\ \mathbf{u}_n &= \frac{z_l d}{z_l + g}(\mathbf{D}(\mathbf{Z}_R\mathbf{i}_n - \mathbf{v}_n + \mathbf{F}_R\mathbf{n}_{\text{AR}}) + \frac{1}{d}\tilde{\mathbf{v}}_n), & (14) \end{aligned}$$

where the matrices $\mathbf{Z}_R$, $\mathbf{F}_R$ and $\mathbf{M}_T$ are defined in (4) and (5) and the matrix $\mathbf{D} = (c\mathbf{I}_M + \mathbf{Z}_R)^{-1}$.

For notational convenience, we define the following

$$\begin{aligned} \mathbf{G} &= \mathbf{Z}_{\text{SRT}}\mathbf{M}_T\text{E}[\mathbf{v}_g\mathbf{v}_g^H]\mathbf{M}_T^H\mathbf{Z}_{\text{SRT}}^H \\ \mathbf{P} &= \mathbf{F}_R\mathbf{G}\mathbf{F}_R^H. \end{aligned} \qquad (15)$$

The covariance matrix of the desired signal at the receiver is

$$\mathbf{K}_s = \left|\frac{z_l d}{z_l + g}\right|^2 \mathbf{D}\mathbf{P}\mathbf{D}^H. \qquad (16)$$

The covariance matrix of the noise voltages $\mathbf{u}_n$ is given as

$$\mathbf{K}_n = \text{E}[\mathbf{u}_n\mathbf{u}_n^H] = \left|\frac{z_l d}{z_l + g}\right|^2 \mathbf{D}\Phi\mathbf{D}^H, \qquad (17)$$

with

$$\begin{aligned} \Phi = & \overbrace{\beta\left(\mathbf{Z}_R\mathbf{Z}_R^H - 2R_N\Re\{\rho^*\mathbf{Z}_R\} + R_N^2\mathbf{I}_M\right)}^{\text{LNA noise}} + \overbrace{\mathbf{F}_R\mathbf{R}_{\text{na}}\mathbf{F}_R^H}^{\text{Ant. noise}} + \\ & \underbrace{\frac{1}{|d|^2}\mathbf{D}^{-1}\psi\mathbf{I}_M\mathbf{D}^{-H}}_{\text{Downstream noise}}, \end{aligned} \qquad (18)$$

where $(.)^*$ denotes the complex conjugate operation and $\Re\{.\}$ denotes the real part of the input matrix.

III. Matching Network Optimization

One of the main metrics used for evaluating the performance of a MIMO system is the achievable rate. The achievable sum rate r in [Bits/ Channel Use] of the system described by (12) is

$$\begin{aligned} r &= \log_2(\det(\mathbf{K}_s\mathbf{K}_n^{-1} + \mathbf{I}_M)) \\ &= \log_2(\det(\mathbf{K}_s + \mathbf{K}_n)) - \log_2(\det(\mathbf{K}_n)) \end{aligned} \qquad (19)$$

From equations (4), (12), (16) and (17) we can deduce the following facts. i) The matching network affects both the signal of interest and the antenna noise in the same way. ii) The matching network affects the LNA noise in a different way than the signal. iii) As the LNAs are unilateral devices, the matching network does not affect the downstream noise. These facts show that a smart choice of the matching network leads to a performance enhancement. Since the matrix $\mathbf{D}$ is invertible and affects both the noise and the signal in the same way, then the achievable rate in (19) is identical to

$$r = \log_2(\det(\mathbf{P} + \mathbf{\Phi})) - \log_2(\det(\mathbf{\Phi})). \qquad (20)$$

A. Achievable Rate Gradient

In this subsection, we derive the analytical gradient of the rate in (20) with respect to the submatrices of the matching network $\mathbf{Z}_{\text{MR}}$ in (2). Using the matrices relations in [17], for any element in the row i and the column j of the submatrix $\mathbf{Z}_{\text{lk}}$, where l and k can be either 1 or 2

$$\frac{\partial r}{\partial \mathbf{Z}_{\text{lk,ij}}} = \frac{1}{\ln 2}\text{Tr}\left((\mathbf{P}+\mathbf{\Phi})^{-1}\left(\frac{\partial \mathbf{P}}{\partial \mathbf{Z}_{\text{lk,ij}}} + \frac{\partial \mathbf{\Phi}}{\partial \mathbf{Z}_{\text{lk,ij}}}\right) - \mathbf{\Phi}^{-1}\left(\frac{\partial \mathbf{\Phi}}{\partial \mathbf{Z}_{\text{lk,ij}}}\right)\right). \qquad (21)$$

Before starting the derivations, we define the matrix $\mathbf{E}_{ij}$ to be the unit differential matrix with respect to the entry of the i^{th} row and j^{th} column for a non symmetric matrix [17]. It is an all zero matrix except for the element in the i^{th} row and j^{th} column which is one. Similarly, $\mathbf{S}_{ij}$ is the unit differential matrix with respect to the entry of the i^{th} row and j^{th} column for a symmetric matrix [17]. All its elements are zero except the ones in the i^{th} row and j^{th} column and in the j^{th} row and i^{th} column which are one. We use the symmetry properties of the components discussed in section II-A.

The LNA is the first noisy component in (18). We start by the first term $\beta\mathbf{T}_1 = \beta\mathbf{Z}_R\mathbf{Z}_R^H$.

$$\begin{aligned} \frac{\partial \mathbf{T}_1}{\partial \mathbf{Z}_{11,ij}} &= j\mathbf{S}_{ij}\mathbf{Z}_R^H + \mathbf{Z}_R(-j\mathbf{S}_{ij}) \\ \frac{\partial \mathbf{T}_1}{\partial \mathbf{Z}_{12,ij}} &= (\mathbf{E}_{ij}\mathbf{F}_R^T + \mathbf{F}_R\mathbf{E}_{ij}^T)\mathbf{Z}_R^H + \mathbf{Z}_R(\mathbf{E}_{ij}\mathbf{F}_R^H + \mathbf{Z}_{12}\mathbf{B}_1^H\mathbf{E}_{ij}^T) \\ \frac{\partial \mathbf{T}_1}{\partial \mathbf{Z}_{22,ij}} &= \mathbf{F}_R(-j\mathbf{S}_{ij})\mathbf{F}_R^T\mathbf{Z}_R^H + \mathbf{Z}_R\mathbf{Z}_{12}\mathbf{B}_1^H(j\mathbf{S}_{ij})\mathbf{F}_R^H, \end{aligned} \qquad (22)$$

where we define the matrix $\mathbf{B}_1 = (j\mathbf{Z}_{22} + \mathbf{Z}_{\text{CR}})^{-1}$. Since both matrices $\mathbf{Z}_{22}$ and $\mathbf{Z}_{\text{CR}}$ are symmetric, $\mathbf{B}_1$ is also symmetric.

The derivatives of the second term of the LNA noise contribution $-\beta R_N\mathbf{T}_2 = -\beta R_N(\rho^*\mathbf{Z}_R + \rho\mathbf{Z}_R^*)$ are

$$\begin{aligned} \frac{\partial \mathbf{T}_2}{\partial \mathbf{Z}_{11,ij}} &= \rho^* j\mathbf{S}_{ij} + \rho(-j\mathbf{S}_{ij}) \\ \frac{\partial \mathbf{T}_2}{\partial \mathbf{Z}_{12,ij}} &= \rho^*(\mathbf{E}_{ij}\mathbf{F}_R^T + \mathbf{F}_R\mathbf{E}_{ij}^T) + \rho(\mathbf{E}_{ij}\mathbf{F}_R^H + \mathbf{F}_R^*\mathbf{E}_{ij}^T) \\ \frac{\partial \mathbf{T}_2}{\partial \mathbf{Z}_{22}, ij} &= \rho^*\mathbf{F}_R(-j\mathbf{S}_{ij})\mathbf{F}_R^T + \rho\mathbf{Z}_{12}\mathbf{B}_1^H(j\mathbf{S}_{ij})\mathbf{F}_R^H. \end{aligned} \qquad (23)$$

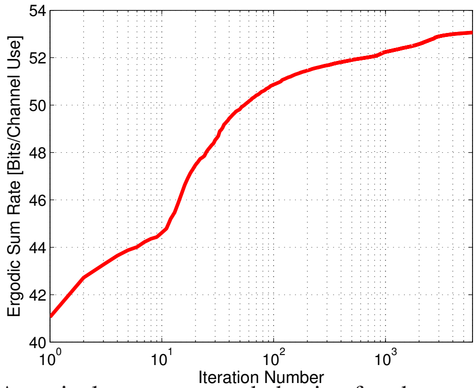


Fig. 2: A typical convergence behavior for the gradient algorithm for $N=12$ and $M=30$ antennas at SNR=12dB.

The derivatives of the downstream noise components $\frac{\psi}{|d|^2}\mathbf{T}_3=\frac{\psi}{|d|^2}\mathbf{D}^{-1}\mathbf{I}_M\mathbf{D}^{-H}$ with respect to the matching network submatrices are

$$\begin{aligned}\frac{\partial \mathbf{T}_3}{\partial \mathbf{Z}_{11,ij}} &= j\mathbf{S}_{ij}(c^*\mathbf{I}_M+\mathbf{Z}_R^H)+(c\mathbf{I}_M+\mathbf{Z}_R)(-j\mathbf{S}_{ij})\\ \frac{\partial \mathbf{T}_3}{\partial \mathbf{Z}_{12,ij}} &= (\mathbf{E}_{ij}\mathbf{F}_R^T+\mathbf{F}_R\mathbf{E}_{ij}^T)(c^*\mathbf{I}_M+\mathbf{Z}_R^H)+\\ &\quad (c\mathbf{I}_M+\mathbf{Z}_R)(\mathbf{E}_{ij}\mathbf{F}_R^H+\mathbf{Z}_{12}\mathbf{B}_1^H\mathbf{E}_{ij}^T)\\ \frac{\partial \mathbf{T}_3}{\partial \mathbf{Z}_{22,ij}} &= (\mathbf{F}_R(-j\mathbf{S}_{ij})\mathbf{F}_R^T)(c^*\mathbf{I}_M+\mathbf{Z}_R^H)+\\ &\quad (c\mathbf{I}_M+\mathbf{Z}_R)(\mathbf{Z}_{12}\mathbf{B}_1^H(j\mathbf{S}_{ij})\mathbf{F}_R^H). \end{aligned}\quad (24)$$

Since the matching network affects the signal components and the external noise components identically, it is enough to give the derivative of any of them with respect to the matching network submatrices. The derivatives of the desired signal components are

$$\begin{aligned}\frac{\partial \mathbf{P}}{\partial \mathbf{Z}_{12,ij}} &= \mathbf{E}_{ij}\mathbf{B}_1\mathbf{G}\mathbf{F}_R^H+\mathbf{F}_R\mathbf{G}\mathbf{B}_1^H\mathbf{E}_{ij}^T\\ \frac{\partial \mathbf{P}}{\partial \mathbf{Z}_{22,ij}} &= \mathbf{F}_R\left((-j\mathbf{S}_{ij})\mathbf{B}_1\mathbf{G}+\mathbf{G}\mathbf{B}_1^H(j\mathbf{S}_{ij})\right)\mathbf{F}_R^H. \end{aligned}\quad (25)$$

The derivative of the antenna noise is identical to (25), but with replacing $\mathbf{G}$ by $\mathbf{R}_{\text{na}}$. Now we derived the derivatives needed to calculate the gradient in (21) with respect to different components of the matching network. This gradient is valid for any noise contribution and for any matching network either multiport or SP. For the case of SP matching, we only have the diagonal elements in the submatrices, i.e. $i=j$.

The gradient is then fed into a conjugate gradient algorithm to find the optimal matching network. The search direction of the conjugate gradient algorithm is updated according to the Polak-Ribiere formula discussed in [18]. We initialize the algorithm using the optimal matching network for the uncoupled antennas case. Fig. 2 shows the typical development of the rate as a function of the iterations. In the first 100 iterations the performance gain is very high reaching around 10 [Bits /Channel Use] for this simulation scenario. Afterwards, the gain develops slowly.

B. *Matching Network Design for Unknown Spatial Channel*

In mobile communications, sometimes the spatial channel changes very fast. Hence optimizing the matching network given a full knowledge of the channel state may not be feasible. However, second order statistics of the transimpedance matrix may be known. In this case, we choose to design a *fixed* matching network that maximizes the upper bound for the achievable rate which is known as Jensen's bound. In words, Jensen's bound states that the average achievable rate can not exceed the rate of the channel equivalent to the average channels. This bound comes from the concavity of the $\log(\det((X)))$ formula in (20) [14]. It is presented mathematically by the following inequality

$$\mathrm{E}[\log_2(\det(\mathbf{P}+\mathbf{\Phi}))]-\log_2(\det(\mathbf{\Phi}))\leq \log_2(\det(\mathrm{E}[\mathbf{P}]+\mathbf{\Phi})) - \log_2(\det(\mathbf{\Phi})). \quad (26)$$

If we know the covariance matrix of the signal components without matching, i.e. $\mathrm{E}[\mathbf{G}]$ in (15), and the noise properties of the receiver, then we choose the matching network that maximizes the Jensen's bound in (26). An intuitive explanation of what we are doing, is the following. We choose the matching network that alters both the signal and the noise covariance matrices such that we maximize an upper bound on the mean achievable rate. The solution of (26) is found using the gradient search algorithm. The gradient of Jensen's bound with respect to different components of the matching network is identical to the gradients in III-A, but with replacing $\mathbf{G}$ with $\mathrm{E}[\mathbf{G}]$.

IV. System Setup and Performance Evaluation

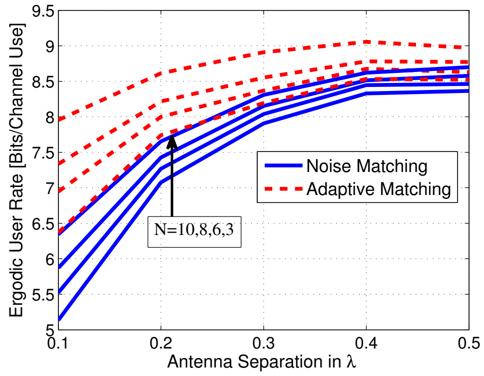
In our system we have N single antenna users that communicate with a single receiver, e.g. a base station with M antennas that are organized in a planar array form with minimum distance between two adjacent antennas being d_r. One of the practical motivations of our work is having a large number of receive antennas. We consider the receiver to have $M=30$ antennas that are organized in two rows each having 15 antennas. All the antennas used are half wavelength dipoles and are assumed to be *lossless*. The users are widely spaced in terms of wavelength, i.e. uncoupled and uncorrelated. This means that $\mathbf{M}_T=\alpha\mathbf{I}_N$ and $\mathbf{R}_{\text{TX}}=\mathbf{I}_N$. We also assume that all the users transmit with the same average power, i.e. $\mathrm{E}[\mathbf{v}_g\mathbf{v}_g^H]=P_s\mathbf{I}_N$.

The LNA input impedance c defined in (9), the impedance R_N defined in (10) and the optimal matching impedance z_{opt} are all assumed to be 75Ω. The correlation coefficient ρ in (10) is assumed to be 0. We scale the parameters β, d and ψ such that in (18) the effect of LNA noise is equal to the effect of antenna noise and to the effect of downstream noise in the case of a single antenna system:

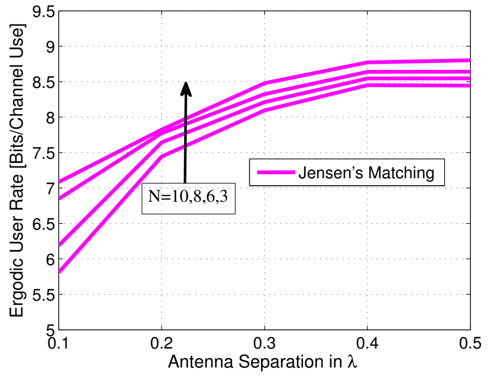
$$\begin{aligned}\Gamma &= \beta(|z_{\text{opt}}|^2-2R_N\Re\{\rho z_{\text{opt}}\}+R_N^2)\\ &= 4k_B T_A B_W\Re\{z_{\text{opt}}\}=\psi\left|\frac{(c+z_{\text{opt}})}{d}\right|^2. \end{aligned}\quad (27)$$

This simply means that in the case of widely spaced antennas, noise generated from different sources have the same effect. We define the signal to noise ratio to be $\text{SNR}=P_s/\Gamma$.

The scattering profile at the receiver is assumed to be wide-sense stationary uncorrelated scattering in 2D. As discussed



(a) Using noise matching and adaptive matching.



(b) Using Jensen's matching.

Fig. 3: Ergodic per user rate against antenna separation for different matching networks for $N \in \{10, 8, 6, 3\}$ users (bottom to top) and $M = 30$ antennas at SNR $= 27$ dB.

in [16], the entries of the correlation matrices are given as $R_{x,i,j} = J_0(2\pi d_{i,j})$, where $d_{i,j}$ is the separation between the i^{th} and j^{th} antenna, and J_0 is the first kind Bessel function of order 0.

A. *Performance Evaluation*

Since the LNA is the main noise contributor in the receiver chain and as its effect was not studied before, and also due to space limitations in this paper, we only consider LNA noise in the upcoming results. We study how our algorithms perform with respect to different parameters such as the SNR, number of users N and antenna separation distance. We are going to compare the ergodic achievable rate performance for three different cases. First, the case of no optimization, which is using the matching network optimum for the uncoupled antennas. We call this *noise matching*. The second case is when we have the full knowledge of $\mathbf{Z}_{\text{SRT}}$, and we optimize the matching network accordingly. We call this procedure *adaptive matching*. We choose the matching network that maximizes

$$\log_2(\det(|\alpha|^2 P_s \mathbf{F}_R \mathbf{Z}_{\text{SRT}} \mathbf{Z}_{\text{SRT}}^H \mathbf{F}_R^H + \mathbf{\Phi})) - \log_2(\det(\mathbf{\Phi})). \quad (28)$$

Third, is the case of knowing only the covariance matrix of the spatial channel $N\mathbf{R}_{\text{RX}} = \text{E}\left[\mathbf{Z}_{\text{SRT}} \mathbf{Z}_{\text{SRT}}^H\right]$. We use $N\mathbf{R}_{\text{RX}}$ in (28) instead of $\mathbf{Z}_{\text{SRT}} \mathbf{Z}_{\text{SRT}}^H$ and choose the matching network that maximizes Jensen's bound. We are going to call this fixed matching network *Jensen's matching* network.

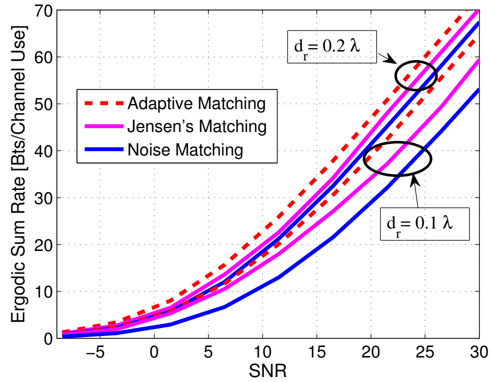


Fig. 4: Rate scaling against SNR for $N = 8$ users, and $M = 30$ antennas at the receiver for antenna spacings of $d_r = 0.1\lambda$ and $d_r = 0.2\lambda$.

We examine the effect of using the optimized matching networks for different antenna spacings. In Fig. 3(a) and Fig. 3(b) the average rate per user $\frac{\text{E}[r]}{N}$ is shown for different number of users $N \in \{3, 6, 8, 10\}$. We choose a high SNR regime because the differences between the curves are easier to see. As we can see from Fig. 3(a) and Fig. 3(b), ignoring antenna coupling in the design of the matching networks leads to severe performance degradation, specially in the case of lower antenna separation. This is attributed to the fact that due to coupling there is impedance mismatch between the LNAs and the antennas. This leads to a lower signal absorption by the receiver, and higher noise generated by the LNAs. In Fig. 3(a) we can see that the lower the number of the users is, the larger the performance gain due to the use of the adaptive matching network.

Comparing the rates in Fig. 3, we can have an interesting observation. Rates that are achieved by using conventional approaches of having 0.5λ spacing between the antennas and ignoring coupling, can be achieved at smaller distances if we use other matching networks. This leads to substantial decrease of the planar array area. For $N = 3$, such a decrease reaches a maximum of about 80% for adaptive matching network and about 51% for Jensen's matching network. In Fig. 3(a) and Fig. 3(b) it can be seen that, the lower the number of users is, the smaller is the gap between having antenna separations of 0.1λ and 0.2λ. This is due to the fact that the lower the separation is, the higher is the correlation of $\mathbf{Z}_{\text{SRT}}$. Mathematically, correlation decreases the strength of the smaller eigenvalues of the channel matrix. For a lower number of users this is not a big problem as there are enough strong eigenvalues that are used for communication. However, for a larger number of users this is a critical problem.

The sum rate is plotted against SNR in Fig. 4 for $N = 8$ users and antenna separations of 0.1λ and 0.2λ. It can be seen that at antenna separation of 0.1λ using Jensen's matching network yields a performance near the one of the optimized matching network for instantaneous knowledge of $\mathbf{Z}_{\text{SRT}}$ at low and moderate SNR, however at higher SNR the gap between the curves increase. For a separation distance of 0.2λ, the Jensen's matching network performance is nearer to the one of the noise matching network. Hence, at separation distance of 0.2λ, it pays off more to use the instantaneous knowledge of $\mathbf{Z}_{\text{SRT}}$.

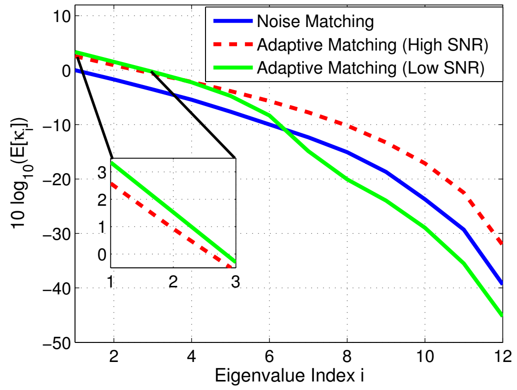


Fig. 5: Average eigenvalues for $N = 12$ users and $M = 30$ antennas for very high SNR and very low SNR.

B. Insights Revealing Simulations

In this subsection we want to get a stronger grasp of what the adaptive matching network does from a communication theoretic point of view. We define the matrix $\mathbf{K}_{\text{SN}} = \mathbf{\Phi}^{-1}\mathbf{F}_R\mathbf{Z}_{\text{SRT}}\mathbf{Z}_{\text{SRT}}^H\mathbf{F}_R^H$, whose ordered eigenvalues are $\mathbf{\Xi} = [\xi_1, \xi_2, \dots \xi_N]$. The achievable rate is then given as

$$r = \sum_{i=1}^{N} \log_2(1 + P_s\xi_i). \tag{29}$$

Looking at the expected value of the sorted eigenvalues for the *same* $\mathbf{Z}_{\text{SRT}}$ but for *different SNR* will help to get a deeper knowledge about the communication theoretic aspects of the algorithm. We define the normalized eigenvalues vector $\kappa = \frac{1}{\xi_{1,\text{NM}}}[\xi_1, \xi_2, \dots \xi_N]$, where $\xi_{1,\text{NM}}$ is the largest eigenvalue when we use the noise matching network. In Fig. 5 we plot the average value of κ_i for using the noise matching network and for using the adaptive matching network. We consider the two representative operation regimes, low SNR region (SNR $= -17$dB) and high SNR region (SNR $= 32$dB). The number of users $N = 12$ and the antenna separation is 0.1λ.

As it can be seen, for very low SNR the algorithm tends to maximize the strongest eigenvalues on the cost of minimizing the weaker ones. In other words, at low SNR it tries to maximize the array gain on the cost of the multiplexing gain. On the other hand, at high SNR the algorithm tries to maximize the weakest eigenvalue, which leads to a higher multiplexing gain. Note that at high SNR almost all the 12 eigenvalues can be effectively used for communication, however for low SNR the number of eigenvalues that are useful for communications are less. We can even get a deeper view of the algorithms' behavior by summing up the average of $\text{E}[\kappa_i]$, i.e. $\sum \text{E}[\kappa_i]$. Such a sum is an indication of the average received SNR for equal P_s. The curve for the SNR $= -17$dB gave slightly higher value than the one of the high SNR. These results give an intuition on the behavior of the algorithm at different transmit SNR. In summary, for low SNR it maximizes the received signal to noise ratio and the array gain (highest eigenvalue) while for high SNR it tries to maximize the number of eigenvalues useful for communications.

V. Conclusion

In this work, we proposed a unified framework for designing the receiver SP matching network to maximize the achievable rate for compact MIMO systems. We analytically derived the gradient of the achievable rate with respect to different matching network components. Our derivation captures effects of *any kind of noise*. The first part of our simulations revealed some issues of practical interest. For example, when the antennas separation distance is 0.1λ and the SNR is moderate or low, using the fixed Jensen's matching network yields a performance near to the one using the adaptive matching network which needs knowledge of the instantaneous spatial channel. However, when the separation distance increases to 0.2λ the gap between using the adaptive matching network and the Jensen's matching network increases. In comparison to conventional approaches that ignores coupling, our algorithms can yield the same performance for a substantially lower array size. We ended our simulations section by an eigenvalue distribution simulation for different SNRs that shed light on the communication theoretic aspect behind our algorithm.

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