Localization via Taylor Series Approximation for UWB based Human Motion Tracking

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Abstract—In this paper, we propose and investigate a low-complexity human motion tracking system which is based on ultra-wideband (UWB) radio nodes. The maximum likelihood (ML) solution of the localization problem that arises in such a system has been presented in [1]. However, the ML solution has a computational complexity that prohibits cost-effective real-time human motion tracking. In this paper, we present an iterative solution of the localization problem, which is based on the first order Taylor series (TS) approximation of the ranges between the anchors and the agents, and the idea presented there works only for the case when the ranges have zero ranging offsets. However, it has been shown that using these predicted positions as an initial guess for the TS based approximation scheme greatly improves performance.

I. INTRODUCTION

Human motion tracking has a wide range of applications in the fields ranging from medicine to virtual reality [2]. In clinical medicine, for example, it can be used to assist patients who undergo a stroke rehabilitation process [3]. An accurate performance can be improved by analyzing the data recorded during training. In virtual reality, motion tracking plays a vital role by feeding the action of the person (the user) to the computer in real-time [4]. Motion tracking systems are also employed in the film industry to record the motion of human and animal actors. The recorded information can then be used to animate fictional characters.

An “ideal” motion tracking system which meets the requirements of all the aforementioned applications should be [5]:

1) Accurate - estimate the position and orientation (pose) of different body parts with a very high precision;
2) Fast - update pose estimates at high frequency;
3) No line-of-sight (LOS) restriction - pose estimates should be immune to occlusion of different body parts [6];
4) Robust - performance should not degrade due to external effects such as light, magnetic field and heat; and
5) Cheep.

Owing to the difficulty of finding such a “have-it-all” system that meets all the above requirements, there exist a variety of systems which are designed to suit to a specific application. The most prominent systems are those based on optical, inertial, magnetic sensing and time-of-flight based technologies (or a combination of them). Optical systems, which are based on cameras, provide accurate tracking with high sampling rate (e.g. [7], [8]). The main disadvantage of these systems is that they require dedicated infrastructure, complex setting, highly skilled operators and LOS restriction problem [9]. Inertial systems, which are based on accelerometers and gyroscopes, and magnetic systems (e.g. [2], [10]), on the other hand do not have LOS restriction but they are prone to drift errors and interference from near-by ferromagnetic materials.

For time-of-flight based systems, electronic-textile (e-textile), which are fabrics that have interconnected electronic sensors and processors as an intrinsic part of the cloth, can be used [11]. Motion tracking such systems is either based on ultrasonic (e.g. [11]) or radio-wave propagation. For the latter case, one approach is motion tracking by means of ultra-wideband (UWB) radio communication, which is one of the most promising technologies for future body area networks [12], [13]. By exploiting the very large signal bandwidth (from 3.1 to 10.6 GHz) body mounted UWB nodes can determine their relative distance with very high spatio-temporal resolution and perform motion tracking based on this metric. Combined with the fact that such a system needs no external infrastructure, UWB positioning systems are particularly interesting for human motion tracking and can even be implemented as secondary application of an existing communication centric UWB body area network [1].

While UWB based human motion tracking systems may find application in several areas, substantial research is still required to prove its feasibility [14]. In [1], we have proposed a maximum likelihood (ML) positioning method for UWB based human motion tracking which takes into account the node topology constraints that are imposed by the kinematics of the body. Both simulation and experimental results assert that taking into account the node topology constraints alleviate the performance loss due to multipath propagation around the human body. Even if it has shown the feasibility of such a system, the ML solution has a computational complexity that prohibits cost-effective real-time human motion tracking.

Inspired by the results in [1], in this paper, we investigate an iterative solution of the localization problem which is based on the first order Taylor series (TS) approximation of the ranges between the anchors and the agents. The idea of this iterative scheme, which has the potential to greatly reduce the complexity of the tracking system, was first proposed in [15]. The scheme as presented there works only for the case when the range measurements have zero ranging offsets.
The limb on which the agent $A_i$ is located emanates from the joint located at position $S_i$ (for e.g. the upper arm, on which agent $A_1$ is located, emanates from the shoulder joint located at $S_1$). The distance from joint $S_i$ to agent $A_i$ is denoted by $l_i$. In the spherical coordinate system, we note that $S_i, A_i$ and $l_i$ are related by

$$A_i = l_i \cdot [\cos \theta_i \sin \phi_i, \sin \theta_i \sin \phi_i, \cos \phi_i]^T + S_i$$

(1)

where $[\theta_i, \phi_i]^T$ represents the orientation (azimuth and polar angles) of the $i$th limb. Hence, estimating the positions of the $N_a$ agents corresponds to estimating the unknown parameter vector $\omega \triangleq [\theta_1, \phi_1, \ldots, \theta_{N_a}, \phi_{N_a}]^T$.

In general, the range measurement between the $i$th anchor and the $j$th agent can be expressed as

$$d_{ij} = d_{ij} + e_{ij} + b_j,$$

(2)

where $d_{ij} = \|R_i A_j\|$ is the true distance between anchor $i$ and agent $j$, $e_{ij}$ is the range measurement error, and $b_j$ is the range measurement offset due to the asynchronism between the clocks of the $j$th agent to that of the anchors. Hence, for the case when the agents are not synchronized with the anchors, the unknown (nuisance) ranging offsets need to be estimated along with the unknown parameter vector $\omega$.

We consider four clock synchronization cases

1) **Synchronous**: all the agents are synchronized with the anchors. For this case, all the range measurement offsets are zero and known. Hence, we only need to estimate the unknown parameter vector $\omega$.

2) **Group synchronous**: all the agents are synchronized with each other but not with the anchors. For this case, all the range measurement offsets are equal but unknown. Hence, along with $\omega$, we need to estimate one unknown nuisance parameter.

3) **Partial group synchronous**: some of the agents form a group and agents which belong to a group share the same clock. This is a practically relevant scenario because it is easier to synchronize groups of agents (for example agents on the arm or agents on the leg) than synchronizing all the agents with each other. If there are $N_g$ groups of agents, we need to estimate $N_g$ unknown nuisance parameters along with $\omega$.

4) **Asynchronous**: the agents are synchronized neither with each other nor with the anchors. This is also a practically relevant scenario as it has the potential to greatly reduce the system complexity. For this case, we need to estimate $N_a$ unknown nuisance parameters along with $\omega$.

Hence, for the clock synchronization of case 2, 3 and 4 (which we collectively call the non-synchronous cases in the following), equation (2) can be re-written as follows

$$\hat{d}_{ij} = d_{ij} + e_{ij} + b_k,$$

(3)

where $\mathcal{G}_k$ is the $k$th group of agents. We note that for the group synchronous case, $N_g = 1$ and $\mathcal{G}_1 = \{A_1, A_2, \ldots, A_{N_a}\}$. And for the asynchronous case, $N_g = N_a$ and $\mathcal{G}_k = \{A_k\}$.

\footnote{Note that $N_g \leq N_a$.}
Define a vector \( \mathbf{b} \triangleq [b_1, b_2, \ldots, b_{N_e}]^T \) which represents the unknown ranging offsets. With this formulation, the problem that we want to solve is the following: given the range measurements \( \mathbf{d} \), estimate \( \omega \) (for the synchronous case) or \( \{\omega, \mathbf{b}\} \) (for the non-synchronous cases) that fulfill the constraint in (1).

### III. Maximum Likelihood Solution

In this section, we derive the maximum likelihood (ML) solution of the localization problem presented in Section II. We assume that the ranging errors are modeled as zero mean and statistically independent Gaussian random variables with variance \( \sigma_{ij}^2 \), i.e.

\[
e_{ij} \sim \mathcal{N}(0, \sigma_{ij}^2).
\]

**ML solution for the synchronous case:**

When the agents are synchronized with the anchors, the likelihood function of \( \omega \) takes the form [1]

\[
p \left( \mathbf{d} \mid \omega = \omega' \right) = \prod_{j=1}^{N_a} \prod_{i=1}^{N_r} \frac{1}{\sqrt{2\pi\sigma_{ij}^2}} \exp \left( -\frac{(d_{ij} - ||\mathbf{R}_i \mathbf{A}_j||)^2}{2\sigma_{ij}^2} \right),
\]

where \( x' \) represent the trial value of parameter \( x \). Hence, the ML estimate of \( \omega \) is given by

\[
\hat{\omega} = \arg \min_{\omega' \in \Omega} \sum_{j=1}^{N_a} \sum_{i=1}^{N_r} \frac{(d_{ij} - ||\mathbf{R}_i \mathbf{A}_j||)^2}{\sigma_{ij}^2}.
\]

**ML solution for the non-synchronous cases:**

For the case when there is an asynchronism between the clock of the agents and the anchors, the ranging offsets \( \mathbf{b} \) need to be estimated along with \( \omega \). The likelihood function of \( \{\omega, \mathbf{b}\} \) is given by [1]

\[
p \left( \mathbf{d} \mid \omega = \omega', \mathbf{b} = \mathbf{b}' \right) = \prod_{k=1}^{N_g} \prod_{i=1}^{N_r} \prod_{j=1}^{N_a} \frac{1}{\sqrt{2\pi\sigma_{ij}^2}} \exp \left( -\frac{(d_{ij} - ||\mathbf{R}_i \mathbf{A}_j||)^2}{2\sigma_{ij}^2} \right).
\]

The joint ML estimate of \( \{\omega, \mathbf{b}\} \) then takes the form

\[
\{
\hat{\omega}, \hat{\mathbf{b}}\} = \arg \min_{\omega', \mathbf{b}' \in \Omega} \sum_{k=1}^{N_g} \sum_{i=1}^{N_r} \sum_{j=1}^{N_a} \frac{(d_{ij} - ||\mathbf{R}_i \mathbf{A}_j||)^2}{\sigma_{ij}^2}.
\]

Optimum estimate of the ranging offsets:

For a given trial value of \( \omega' \) the ML estimate of the offsets can be computed analytically. We first note that for a given set of trial positions \( \{\mathbf{A}_j\}_{j=1}^{N_a} \) which correspond to the trial orientations \( \omega' \), the cost function in (5) takes the following form [1]

\[
\Psi(\mathbf{b}') = \sum_{k=1}^{N_g} \sum_{i=1}^{N_r} \sum_{j=1}^{N_a} \frac{(\psi_{ij} - b_k)^2}{\sigma_{ij}^2},
\]

where \( \psi_{ij} = d_{ij} - ||\mathbf{R}_i \mathbf{A}_j|| \). Hence, the ranging offset of the \( k \)th group of agents that minimizes the cost function in (5) satisfies

\[
\frac{\partial \Psi(\mathbf{b}')}{\partial b_k} = 0 \quad \Rightarrow \quad \sum_{i=1}^{N_r} \sum_{j=1}^{N_a} \frac{(\psi_{ij} - b_k)}{\sigma_{ij}^2} = 0.
\]

Thus, the optimum estimate of the \( k \)th group ranging offset that corresponds to the trial positions \( \{\mathbf{A}_j\}_{j=1}^{N_a} \) is given by

\[
b_k = \frac{1}{\gamma_k} \tilde{d}_k - \frac{1}{\gamma_k} \sum_{i=1}^{N_r} \sum_{j \in \mathcal{G}_k} \frac{||\mathbf{R}_i \mathbf{A}_j||}{\sigma_{ij}^2},
\]

where we have defined \( \gamma_k = \sum_{i=1}^{N_r} \sum_{j \in \mathcal{G}_k} \frac{1}{\sigma_{ij}^2} \) and \( \tilde{d}_k = \sum_{i=1}^{N_r} \sum_{j \in \mathcal{G}_k} d_{ij} / \sigma_{ij}^2 \).

Note that for the group synchronous case, since we have only one group (and all agents belong to this group), the optimum estimate of the ranging offset takes the form

\[
b_i = \frac{1}{\gamma_i} \bar{d}_i - \frac{1}{\gamma_i} \sum_{j=1}^{N_a} \frac{||\mathbf{R}_i \mathbf{A}_j||}{\sigma_{ij}^2},
\]

with \( \gamma_i = \sum_{j=1}^{N_a} \frac{1}{\sigma_{ij}^2} \) and \( \bar{d}_i = \sum_{j=1}^{N_a} \frac{d_{ij} / \sigma_{ij}^2}{\sum_{i=1}^{N_a} \frac{d_{ij} / \sigma_{ij}^2}} \).

### IV. Taylor-Series Approximation

In this section we present an iterative solution of the localization problem which is based on the first order TS approximation of \( d_{ij} = ||\mathbf{R}_i \mathbf{A}_j|| \). We first re-write (3) by replacing \( d_{ij} \) with its first order TS approximation. This will result in \( N_r N_a \) simultaneous linear equations. The weighted least square solution of the system of equations is then presented.

The first order Taylor series expansion of \( d_{ij} = ||\mathbf{R}_i \mathbf{A}_j|| \) around a trial orientation \( [\theta_i, \phi_i]^T \) is given by

\[
d_{ij} \approx d_{ij}' + a_{ij}' \delta_{\theta_i} + a_{ij}'' \delta_{\phi_i},
\]
where \( a_{ij}^{(\theta_i)} = \frac{\partial d_{ij}}{\partial \theta_i} \mid \theta_i = \theta'_i, \phi_i = \phi'_i \), \( a_{ij}^{(\phi_i)} = \frac{\partial d_{ij}}{\partial \phi_i} \mid \theta_i = \theta'_i, \phi_i = \phi'_i \), and \([\delta_{\theta_i}, \delta_{\phi_i}]^T\) is the difference between the true orientation and the trial orientation, i.e. \([\hat{\theta}_i, \hat{\phi}_i]^T = [\theta'_i, \phi'_i]^T + [\delta_{\theta_i}, \delta_{\phi_i}]^T\).

Note that for the synchronous case, plugging (7) into (3) results in the linear equation
\[
a_{ij}^{(\theta_i)} \delta_{\theta_i} + a_{ij}^{(\phi_i)} \delta_{\phi_i} \approx d_{ij} - d_{ij}' - e_{ij}.
\]

However, for the non-synchronous cases, the ranging offset needs to be subtracted from the range measurement. Hence, we have the following linear equation
\[
a_{ij}^{(\theta_i)} \delta_{\theta_i} + a_{ij}^{(\phi_i)} \delta_{\phi_i} \approx \tilde{d}_{ij} - b'_k - d_{ij}' - e_{ij}, \quad \text{if} \ A_j \in \mathcal{G}_k.
\]

The approach we propose to handle the unknown offsets in the range measurements is the following: For each trial orientation \([\theta'_i, \phi'_i]^T\), estimate the optimum offset \(b'_k\) using (6) and subtract it from the range measurement.

Rewriting the system of \(N_c N_a\) linear equations in matrix form, we get
\[
\mathbf{\Gamma} \delta \approx z - e, \quad (8)
\]
where we have defined the following matrices and vectors:
\[
\mathbf{\Gamma} = \text{diag} \{ \Gamma_1, \Gamma_2, \ldots, \Gamma_{N_a} \},
\]
\[
e = [e_{11}, e_{21}, \ldots, e_{N_c N_a}]^T,
\]
\[
\delta = [\delta_{\theta_1}, \delta_{\phi_1}, \delta_{\theta_2}, \delta_{\phi_2}, \ldots, \delta_{\theta_{N_a}}, \delta_{\phi_{N_a}}]^T,
\]
\[
\mathbf{\Gamma}_i = \begin{bmatrix}
a_{ij}^{(\theta_i)} & a_{ij}^{(\phi_i)} \\
\vdots & \vdots \\
\end{bmatrix},
\]
\[
z = \begin{bmatrix}
\tilde{d}_{11} - d_{11}' \\
\vdots \\
\tilde{d}_{N_c, N_a} - d_{N_c, N_a}' \\
\end{bmatrix},
\]
and \(\tilde{d}_{ij} = \tilde{d}_{ij}\) for the synchronous case, however, for the non-synchronous cases \(\tilde{d}_{ij} = d_{ij} - b'_k\) if \(A_j \in \mathcal{G}_k\).

The weighted least squares solution of (8) is then given by
\[
\hat{\delta} = \arg \min_{\delta} \left\| W^{1/2} (z - \mathbf{\Gamma} \delta) \right\|
\]
where \(W\) is an \(N_c N_a \times N_c N_a\) weighting matrix. For the case when the measurements are independent, \(W\) becomes a diagonal matrix, where each diagonal element represents the weight for the corresponding measurement. If the variance of the measurements are known, then the ideal weight for each measurement is the reciprocal of the variance. This is intuitive because the weighting makes sure that good measurements are emphasized while bad measurements are deemphasized.

Since the the range measurements are independent, we note that the weighted least squares solution of (8) takes the form [15]
\[
\hat{\delta} = (\mathbf{\Gamma}^T \Sigma^{-1} \mathbf{\Gamma})^{-1} \mathbf{\Gamma}^T \Sigma^{-1} z, \quad (9)
\]
where \(\Sigma = \text{diag} \{ \sigma_{11}^2, \ldots, \sigma_{N_c N_a}^2 \}\) is the covariance matrix of the range measurements.

The localization algorithm then works as follows

1) Choose an initial guess of the agents orientation \(\{[\theta'_i, \phi'_i]^T\}\).
2) Approximate \(\left\| R_i \mathbf{A}^T \right\|\) around \(\{[\theta'_i, \phi'_i]^T\}\) with its first order TS expansion and determine the matrix \(\mathbf{\Gamma}\) in (8).
3) For the non-synchronous cases, subtract the optimum ranging offset estimate using (6) from the range measurement to result in \(d_{ij} = d_{ij} - b'_k\). However, for the synchronous case, \(d_{ij} = d_{ij}\).
4) Calculate \(z\).
5) Estimate \(\hat{\delta}\) using (9).
6) Update the orientations using the newly estimated \(\hat{\delta}\), i.e. \(\theta_i' \leftarrow \theta_i' + \delta_{\theta_i}, \phi_i' \leftarrow \phi_i' + \delta_{\phi_i}\).
7) If \(\delta\) is above a threshold value, go back to 2.
8) \(\{[\theta'_i, \phi'_i]^T\}\) are the orientation estimates.

Note that the weighted least squares solution in (9) only requires the covariance matrix of the range measurement errors but it does not assume any distribution. Hence, the localization algorithm works for range measurements taking any type of distribution. In [15], it has been stated that the algorithm converges almost always independently of the chosen initial guess. In the next section, we will show that for not carefully chosen initial guesses the algorithm converges very slowly.

V. SIMULATION-BASED PERFORMANCE ANALYSIS

In this section, we discuss simulation results that assess the performance of the localization algorithms presented in Sections III and IV. For all the simulations, four anchors were mounted on the torso at locations \(R_1 = [-15 \text{ cm}, 0, 5 \text{ cm}]^T\), \(R_2 = [-25 \text{ cm}, 20 \text{ cm}, 0]^T\), \(R_3 = [-5 \text{ cm}, 20 \text{ cm}, 0]^T\) and \(R_4 = [-15 \text{ cm}, 40 \text{ cm}, 15 \text{ cm}]^T\). Two agents were mounted at the left arm (one at the elbow joint and the other one at the wrist joint). The length of the upper arm and the forearm is set to 30 cm. The standard deviation of all the ranging errors is the same and is set to 0.3612 cm (which corresponds to a variance of 0.1 cm²).

Figure 2 shows the cumulative density function (CDF) of the root mean squared error (RMSE) of both the ML solution and the TS based approximate solution. The initial guess for the TS based approximate solution is chosen randomly and the maximum number of iterations is set to 100. Figure 2(a) shows the CDF plots of the RMSE of the elbow position estimates. From the figure, we first note that for all the clock synchronization cases – synchronous (black lines), group synchronous (green lines) and asynchronous (yellow lines) – the TS based approximate solution performs reasonably close to its ML counter part.

The other point we note in Figure 2(a) is that for both localization techniques, the synchronous case performs better than the group synchronous case which in turn outperforms the asynchronous case. For the ML estimator, the result is very intuitive because we have to estimate one more variable in the group synchronous case (and \(N_a\) more variables in the

2Linear with respect to the unknown variables \(\delta_{\theta_i}\) and \(\delta_{\phi_i}\).

3The left shoulder joint is chosen as the center of the coordinate system.
From Figure 2(b), we further note that the estimates of the TS based approximation perform poorly as compared to the ML estimates. This poor performance of the TS based approximation is attributed to the random initial guesses and the restriction on the maximum number of allowed iterations (which is set to 100). If we let the algorithm iterate until a target accuracy is achieved, the TS based solution would have performed close to the ML solution. However, for real-time human motion tracking, the position of the agents has to be updated with a certain frequency, which means that we have a restriction on the maximum allowed number of iterations. If, on the other hand “good” initial guesses are used, the algorithm achieves the target accuracy with few iterations.

Figure 3 shows the CDF plot of the RMSE of the wrist position estimates using the TS approximation for the asynchronous case. Shown in the figure are estimates using true agent position as an initial guess (blue solid line) and estimates with random initial guess (yellow dotted lines). The number of iterations for both cases is set to 100. We note that a substantial performance gain is achieved when the initial guesses are chosen appropriately.

Next we shall discuss the performance improvement that can be achieved by the TS based approximation when we predict the position of the agents from a priori knowledge of the agents trajectory and estimates from the previous time-step, and then use this predicted positions as an initial guess in the next time-step.

Consider the scenario which is shown in Figure 4. The wrist follows the trajectory which is marked with “□”. For clarity reason, what is shown in the figure are the positions after 3 time-steps, i.e. between two plotted position there are three more position updates. In most of the human actions (for e.g. walking), the average speed of different limbs movement can be estimated. Furthermore, the update frequency of the motion tracking system is known. Equipped with the knowledge of the system update rate and the average speed of the limb's asynchronous case) as compared to that of the synchronous case. Similarly, for the TS based approximate solution, we use the estimates in (6) to suppress the offset in the range measurements. Since the offset estimates are not perfect, a performance loss is expected in the non-synchronous cases as compared to the synchronous counter part. We further note that since the offset estimate in the group synchronous case uses \( N_r N_a \) range measurements, it is more accurate than that of the asynchronous case which uses only \( N_r \) range measurements to estimate the offset for each agent.

The CDF plot of the RMSE of the wrist position estimates is shown in Figure 2(b). We first note that for all the synchronization cases, the wrist position estimates are worse than the elbow position estimates. This is expected because the elbow is connected to the shoulder joint which has a fixed and known position, however, the wrist is connected with the elbow joint whose position has to be estimated.
movement, distance between the position of the wrist between two time-steps can be estimated. In the simplest case, with \textit{a priori} knowledge of the agents trajectory as given in the figure, the position of the agent in the next time step can be determined from the estimate in the current time-step.

Figure 4 also shows the estimates of the TS based approximation at the 100th iteration for different initial guesses. We note that using random initial guess it is not possible to accurately follow the trajectory of the wrist with only 100 iterations. However, if we carefully chose the initial guess good enough positioning accuracy can be achieved. It is worthwhile to note that the estimates which use the position estimates from previous time-steps are very close to that of the estimates which use the predicted positions as their initial guess. Hence, if the system can afford fast update rates (which is the case for UWB based motion tracking systems) taking position estimates from previous time step will greatly improve performance.

VI. Conclusion

In this paper, we have presented a low-complexity UWB radio based human motion tracking system. An iterative solution of the localization problem that arise in such a system, which is based on first order TS approximation, has been discussed. It was shown that for the case when the clock of the agents and the anchors are synchronized with each other, the iterative solution preforms close to the ML solution. However, for the case when the agents are not synchronized with the anchors, which is the practically preferred approach as it has the potential to reduce the system complexity, the convergence speed of the localization algorithm heavily depends on the chosen initial guess. Predicting the agents position from previous time-step estimates and using them as an initial guess for the estimation in the next time-step has been shown to greatly improve localization performance.

REFERENCES