Max-Min Fairness in Compact MU-MIMO Systems: Can the Matching Network Play a Role?

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***Abstract*— With the two conflicting demands of increasing the number of antennas in terminals and decreasing the devices sizes, considering compact MIMO systems became inevitable. Such systems suffer from both antenna coupling and noise correlation. This has led to the observation that the impedance matching network fundamentally affects the performance of such MIMO enabled communications systems. In this paper we consider a multi-user MIMO system, for which we design the matching network at the receiver given that it uses the optimal successive interference cancellation receiver. For the two cases of time sharing and no time sharing we design the matching network to alter the capacity region in order to maximize the minimum user rate. We compare to both the conventional matching, which ignores antenna coupling, and to the case of choosing the matching network such that the achievable rate is maximized. In both cases we observe a performance enhancement. We view our results as a new step in connecting the two worlds of information theory and circuit theory. Such results are of very relevant practical interest to wireless communication systems that adopt a large number of antennas while maintaining a small size.**

I. Introduction

As a result of the increasing demand on higher data rates and enhancing spectral efficiency, MIMO systems gained very high consideration. Increasing the number of antennas is known to boost the performance of MIMO communication systems. This fact was the main motivation behind the major attention that massive MIMO systems gained recently [1]. On one hand, new communication systems are expected to be equipped with larger number of antennas, on the other hand, a small form factor of devices is highly required. This entails antenna arrays with inter-element spacing of fraction of a wavelength (λ). Such systems are denoted by compact MIMO systems. Another important aspects in designing future communication systems are ubiquity and fairness.

It is well known that the compact antenna systems suffer from spatial correlation and antenna coupling. As studied in [2], [3], due to antenna coupling the additive noise at the receiver, which originates from different sources becomes spatially correlated. Coupling furthermore introduces impedance mismatch, which leads to a reduction of the signal power absorbed by the receiver and to an increase of the noise generated by the LNAs. Hence, neglecting coupling in studying compact MIMO systems leads to misleading results, as demonstrated e.g. in [1]. The optimum matching technique in the presence of antenna coupling requires a coupled matching network, i.e. a multiport matching network [2], [4]. This complex network is very hard to implement and suffers from very low operation bandwidth [5]. Consequently in this paper we focus on uncoupled (UC) matching networks. In UC matching, each antenna has a *separate* matching network and there are no connections between the antenna elements.

Designing the UC matching network for compact MIMO was studied in various publications, most of them were concerned with *maximizing the achievable rate*. In [6] an algorithm for UC matching network optimization is given for a 2×2 system, which is later extended to any $N \times N$ configuration in [7]. Both papers assume the knowledge of the statistical properties of the channel. A random search algorithm to maximize the achievable rate given instantaneous channel state information is introduced in [8]. Spatially uncorrelated noise is assumed in [6], [7], [8]. In [9], an algorithm is introduced for maximizing the achievable rate in the presence of *any* noise source. It is designed for either instantaneous or statistical channel knowledge.

The capacity region for a multi-user (MU) MIMO uplink channel is known to form a polymatroid [10]. Successive interference cancellation (SIC) is the optimal receiving strategy in terms of sum rate [11]. For N single antenna users, the capacity facet has $N!$ vertices. Any point within this facet can be achieved through time sharing. In [10] an algorithm is presented to get the time shares leading to the so called leximin maximal rate vector, which also maximizes the minimum user rate (MaxMin) criterion. However, time sharing may not be feasible. In [10], [12] an algorithm is shown to find the optimal vertex point that achieves the (MaxMin) criterion. Effects of coupling, noise correlation and spatial correlation on the minimum user rate of SIC receivers were not studied before.

In this paper we fill this gap. For the two cases of the time-sharing and non time sharing we design the UC matching network to maximize the minimum user rate. We compare our results to the cases of the conventional matching network that ignores the coupling, and for the matching network that maximizes the achievable rate. We study the performance of the different matching networks in two different scenarios, (i) low number of users N and small number of M receive antennas and (ii) larger number of users N and larger number of receive antennas M. Our results show a considerable performance enhancement when the proposed matching network is used, especially for large number of antennas. Such results can be seen as a new step in exploring potentials of optimizing the RF front end components for different performance criteria.

II. SYSTEM MODEL

In communication theory, a MIMO system with N transmit antennas and M receive antennas, is often described by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{H}$ is the channel matrix, and $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{n}$ are the vectors of the transmit signals, received signals and noise, respectively. In the sequel, we summarize the equivalent relation between the transmitter generator voltages $\mathbf{v}_g$, and the receiver load voltages $\mathbf{v}_l$ and noise $\mathbf{u}_n$ at the impedance z_l behind the low noise amplifier (LNA). The model used is based on [3] and extended in [9] to capture circuitry effects behind the LNA.

A. MIMO Circuit Model

Since we are only interested in the receiver side, we are going to focus on the circuitry on the receiver side. At the transmitter side, the relationship between the antennas currents $\mathbf{i}_{\text{AT}}$ and the generators voltages $\mathbf{v}_g$ is

$$\mathbf{i}_{\text{AT}} = \mathbf{M}_T\mathbf{v}_g. \tag{2}$$

Details about the matrix $\mathbf{M}_T$ can be found in [3].

We introduce the transimpedance matrix $\mathbf{Z}_{\text{SRT}}$ describing the effect of the physical propagation channel between the transmitter and the receiver, i.e. the spatial channel. It relates the currents flowing in the transmitter antenna array $\mathbf{i}_{\text{AT}}$ to the open circuit voltage at the receiver antenna array $\mathbf{v}_o$

$$\mathbf{v}_o = \mathbf{Z}_{\text{SRT}}\mathbf{i}_{\text{AT}}. \tag{3}$$

The transimpedance matrix is modeled using the Kronecker model

$$\mathbf{Z}_{\text{SRT}} = \mathbf{R}_{\text{RX}}^{\frac{1}{2}}\mathbf{Z}_p\mathbf{R}_{\text{TX}}^{\frac{1}{2}}. \tag{4}$$

The entries of the matrix $\mathbf{Z}_p$ are independent identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. The two matrices $\mathbf{R}_{\text{TX}}$ and $\mathbf{R}_{\text{RX}}$ are the spatial correlation matrices at the transmit antenna array and, respectively, the receive antenna array.

The circuit model for the receiver can be seen in Fig. 1. The matrix $\mathbf{Z}_{\text{CR}}$ denotes the impedance matrix of the antenna array of the receiver. We only consider lossless antennas. For arrays of half wavelength dipoles, the matrix $\mathbf{Z}_{\text{CR}}$ can be calculated using formulas in [13]. In case of the presence of ohmic losses in the antennas, corresponding positive real values are added to the diagonal of $\mathbf{Z}_{\text{CR}}$.

The matrix $\mathbf{Z}_{\text{MR}}$ denotes the *lossless* matching network at the receiver, and has the form:

$$\mathbf{Z}_{\text{MR}} = \begin{bmatrix} j\mathbf{Z}_{\text{MR},11} & j\mathbf{Z}_{\text{MR},12} \\ j\mathbf{Z}_{\text{MR},21} & j\mathbf{Z}_{\text{MR},22} \end{bmatrix}. \tag{5}$$

For a matching network to be reciprocal and lossless it has to be pure imaginary and symmetric [3]. This means that $\mathbf{Z}_{\text{MR},11} = \mathbf{Z}_{\text{MR},11}^T$, $\mathbf{Z}_{\text{MR},22} = \mathbf{Z}_{\text{MR},22}^T$and $\mathbf{Z}_{\text{MR},12} = \mathbf{Z}_{\text{MR},21}^T$. Note that UC matching means that the submatrices of the matching network in (5) are *diagonal*. For notational convenience, we are going to drop the subindex MR of the matching network submatrices, for example we denote $\mathbf{Z}_{\text{MR},11}$ by $\mathbf{Z}_{11}$.

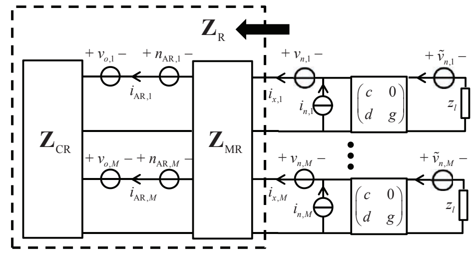


Fig. 1: Circuit model for the receiver of compact MIMO.

We denote the input impedance matrices looking into the matching networks at the receiver side by $\mathbf{Z}_R$. From circuit theory [3], the impedance matrix $\mathbf{Z}_R$ is related to the matrices $\mathbf{Z}_{\text{MR}}$ and $\mathbf{Z}_{\text{CR}}$ by

$$\begin{aligned} \mathbf{Z}_R &= j\mathbf{Z}_{11} + \mathbf{Z}_{12}(j\mathbf{Z}_{22} + \mathbf{Z}_{\text{CR}})^{-1}\mathbf{Z}_{12}^T \\ &= j\mathbf{Z}_{11} + \mathbf{F}_R\mathbf{Z}_{12}^T. \end{aligned} \tag{6}$$

B. Noise

For space limitations, we restrict the noise to be generated from the LNAs. This means that we ignore the antenna noise $\mathbf{n}_{\text{AR}}$ and the noise from the circuitry behind the LNAs $\tilde{\mathbf{v}}_n$, i.e. $\mathbf{n}_{\text{AR}} = 0$ and $\tilde{\mathbf{v}}_n = 0$. The LNA noise is usually modeled by a series voltage source and a parallel current source at the input of the LNA [2], [3]. Such sources have the following statistical properties

$$\begin{aligned} \text{E}[\mathbf{i}_n\mathbf{i}_n^H] &= \beta\mathbf{I}_M, \quad \text{E}[\mathbf{v}_n\mathbf{v}_n^H] = \beta R_N^2\mathbf{I}_M \\ \text{E}[\mathbf{v}_n\mathbf{i}_n^H] &= \rho\beta R_N\mathbf{I}_M. \end{aligned} \tag{7}$$

Note that each LNA has an optimal matching impedance z_{opt} for which its contribution is minimal. An LNA is usually modeled as a unilateral device i.e. signal at the output port does not affect the signal at the input port. The impedance parameters for each LNA is given by

$$\mathbf{Z}_{\text{LNA}} = \begin{bmatrix} c & 0 \\ d & g \end{bmatrix}. \tag{8}$$

C. Input Output Relationship

Using circuit theory relations similar to the ones used in [2], [3], we write the final input-output equation of the MIMO system similar to (1)

$$\mathbf{v}_l = \mathbf{A}\mathbf{v}_g + \mathbf{u}_n, \tag{9}$$

with equivalent channel matrix $\mathbf{A}$ and noise vector $\mathbf{u}_n$:

$$\mathbf{A} = j\frac{z_l d}{z_l + g}\mathbf{D}\mathbf{F}_R\mathbf{Z}_{\text{SRT}}\mathbf{M}_T \tag{10}$$

$$\mathbf{u}_n = \frac{z_l d}{z_l + g}\mathbf{D}(\mathbf{Z}_R\mathbf{i}_n - \mathbf{v}_n), \tag{11}$$

where the matrices $\mathbf{Z}_R$, $\mathbf{F}_R$ and $\mathbf{M}_T$ are defined in (6) and (2) and the matrix $\mathbf{D} = (c\mathbf{I}_M + \mathbf{Z}_R)^{-1}$. The covariance matrix of the noise voltages $\mathbf{u}_n$ is given as

$$\mathbf{K}_n = \text{E}[\mathbf{u}_n\mathbf{u}_n^H] = \left|\frac{z_l d}{z_l + g}\right|^2 \mathbf{D}\mathbf{\Phi}\mathbf{D}^H, \tag{12}$$

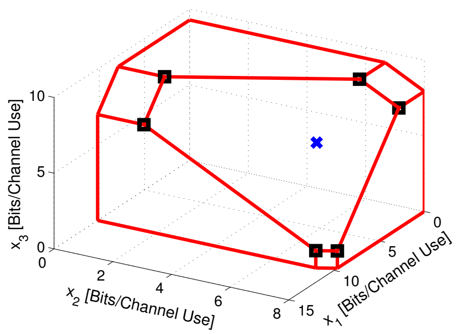


Fig. 2: Rate region example for N=3. x_i is the rate of user i.

with $\Phi = \beta\left(\mathbf{Z}_R\mathbf{Z}_R^H - 2R_N\Re\{\rho^*\mathbf{Z}_R\} + R_N^2\mathbf{I}_M\right)$. The operator $(.)^*$ denotes the complex conjugate operation. Since we are assuming a MU-MIMO scenario with single antenna users that are widely spaced, then the matrix $\mathbf{M}_T = \sqrt{\alpha}\mathbf{I}_N$ is a scaled identity matrix. We also assume that all the users use the same power, i.e. $\mathrm{E}[\mathbf{v}_g\mathbf{v}_g^H] = P\mathbf{I}_N$.

III. MaxMin Rate Optimization

The maximum achievable rate of a MIMO system is given by

$$S_R = \log_2(\det(P\mathbf{A}^H\mathbf{K}_n^{-1}\mathbf{A} + \mathbf{I}_N)). \tag{13}$$

It is known that the SIC receiver is the one that results in a sum of user rates equal to S_R [11]. In SIC the receiver applies a linear minimum mean square error (MMSE) filter to the received signal and chooses one stream to decode. After successful decoding of the stream, the receiver removes its effect and proceeds to decode the information of the next user. The receiver keeps doing the same procedure until the information of all the users are decoded. Each user decoded at stage i is decoded with $N-i$ interfering streams. There are $N!$ possible user rates corresponding to all the possible decoding orders. The sum-capacity facet is the convex hull of these $N!$ vertices. Fig. 2 shows an example of the rate region for $N=3$. The 6 black squares are the vertices, and the 6 edges connecting them form the rate facet.

Any point on this facet can be achieved by time sharing. The time share given to each vertex does not affect the sum rate. However, each user rate strongly depends on the time share given to each vertex point. We define the matrix $\mathbf{R}$ as the $N \times N!$ matrix that contains the coordinates of each vertex point as its column vectors. The rates of user i for the possible decoding orders are in the i^{th} row of $\mathbf{R}$. The $N!$ vector that has the time share of each vertex is $\mathbf{t} = [t_1 t_2 \ldots t_{N!}]^T$. The vector $\mathbf{w} = [w_1 w_2 \ldots w_N]^T$ of the user rates is given as

$$\mathbf{w} = \mathbf{R}\mathbf{t}. \tag{14}$$

A. MaxMin Matching Network Optimization

As we discussed above, each column of $\mathbf{R}$ in (14) corresponds to the user rates given one of the $N!$ possible decoding sequences. To get the elements of each column, we first define the noise whitened channel matrix as $\mathbf{A}_e = \mathbf{K}_n^{-1/2}\mathbf{A}$. At each iteration the receiver applies a linear MMSE filter to the received signal after removing the effect of the previously decoded layers. In the MMSE filter it uses the matrix $\mathbf{A}_e$ after removing the columns corresponding to the previously decoded layers. i.e. at iteration i the matrix $\mathbf{A}_e^{(i)}$ has dimensions $M \times (N-i+1)$. The corresponding linear filter is then:

$$\mathbf{G}^{(i)} = (\underbrace{P\mathbf{A}_e^{(i)^H}\mathbf{A}_e^{(i)} + \mathbf{I}_{N-i+1}}_{\Psi})^{-1} P\mathbf{A}_e^{(i)^H}\mathbf{K}_n^{-1/2}. \tag{15}$$

Assume that the layer to be decoded in iteration i corresponds to the user having column j in $\mathbf{A}_e^{(i)}$, then the signal to interference and noise ratio (SINR) of this user is given by

$$\gamma_j = \frac{1}{(\Psi^{-1})_{jj}} - 1. \tag{16}$$

The corresponding rate of this user is $r_j = \log_2(1+\gamma_j)$. As it can be seen, the rates in $\mathbf{R}$ depend on both the matrices $\mathbf{A}$ and $\mathbf{K}_n$, which both *depend on the matching network*. This means that for different matching networks, we would get a different rate matrix $\mathbf{R}$.

For a fixed time sharing vector $\mathbf{t}$, we want to choose the matching network that maximizes the minimum user rate. This optimization problem is the *main link* between information theory and circuit theory in the problem at hand. The problem can be stated formally as:

$$\begin{aligned} \underset{\mathbf{Z}_{11},\mathbf{Z}_{12},\mathbf{Z}_{22}}{\text{maximize}} \quad & \min_{i\in\{1,\ldots,N\}} w_i(\mathbf{Z}_{11},\mathbf{Z}_{12},\mathbf{Z}_{22}) \\ \text{subject to} \quad & \mathbf{Z}_{11} = \mathbf{Z}_{11}^T, \mathbf{Z}_{22} = \mathbf{Z}_{22}^T. \end{aligned} \tag{17}$$

The constraint in (17) is for the general case of any matching network. For our UC matching network, we only optimize for the diagonal elements of the submatrices as the other elements are all zeros, thus the constraint in (17) is implicitly satisfied.

We first define the function $f_p(\mathbf{w})$ as,

$$f_p(\mathbf{w}) = \left(\sum_{i=1}^{N} w_i^p\right)^{1/p}. \tag{18}$$

If we use a *large negative* value for p, the minimum element of $\mathbf{w}$ will strongly dominate the sum in (18). This leads to a good *approximation* of the minimum component of $\mathbf{w}$, i.e. we approximate the objective function in (17) by (18). To solve this problem we use a conjugate-gradient search algorithm, where we calculate the gradient of (18) with respect to different matching network components numerically.

B. MaxMin Time Sharing Vector

Since all the points share the same sum rate S_R, the optimal time sharing vector is the one leading to equal user rates i.e. $w_i = S_R/N \quad \forall i = 1,\ldots N$. An example for this point is given by the blue $\times$ in Fig. 2. However, this condition is not always feasible. The optimization problem to find the time vector that maximizes the minimum user rate is

$$\begin{aligned} \underset{\mathbf{t}}{\text{maximize}} \quad & \min_{i\in\{1,\ldots,N\}} w_i(\mathbf{t}) \\ \text{subject to} \quad & 0 \le t_l \le 1 \quad \forall l = 1,\ldots,N! \\ & \sum_{l=1}^{N!} t_l = 1 \end{aligned} \tag{19}$$

There are different algorithms to deal with this problem [10]. We adopt a straight forward algorithm based on the gradient search. First, we approximate the minimum element of $\mathbf{w}$ by a function similar to (18). The two constraints in (19) can be incubated by the following change of variables:

$$t_l = \frac{\tilde{t}_l^2}{\sum_{i=1}^{N!} \tilde{t}_i^2}. \quad (20)$$

Using the change of variables in (20), the constrained optimization in (19) can be turned into an unconstrained one. It is a function of $\tilde{\mathbf{t}} = [\tilde{t}_1 \tilde{t}_2 \dots \tilde{t}_{N!}]^T$ and written as

$$\underset{\tilde{\mathbf{t}}}{\text{maximize}} \quad f_p(\mathbf{w}(\tilde{\mathbf{t}})), \quad (21)$$

Due to its very high complexity, time sharing is not always preferred especially for the case of having large number of users. As discussed in [10], [12], the best strategy from fairness point of view is to decode the user with the highest SINR at every iteration. This ensures that the vertex point chosen is the one that maximizes the minimum user rate. This strategy is linked directly to (14) by restricting the cardinality of $\mathbf{t}$ to be 1. We will call this case "*no time sharing*".

To solve our optimization problem, we adopt an alternating maximization approach. We handle the two cases of time sharing and no time sharing. For both cases, we fix the matching network and get a matrix $\mathbf{R}$, for which we optimize the time sharing vector $\mathbf{t}$, either by solving (19) or by selecting the decoding sequence that maximizes the minimum user rate without time sharing. Then we fix the time sharing vector, and optimize our matching network. We continuously perform this alternating maximization until the maximum number of iterations or reaching a maxima. We initialize the matching network with the one optimal for uncoupled antennas.

IV. System Setup and Performance Evaluation

In our system we have N single antenna users that communicate with a single receiver, e.g. a base station, with M antennas that are organized in a planar array form. The minimum distance between any two adjacent antennas being d_r. All the antennas used are half wavelength dipoles and are assumed to be *lossless*. The scattering profile at the receiver is assumed to be wide-sense stationary uncorrelated scattering in 2D. The entries of the correlation matrices are given as $\mathbf{R}_{x,i,j} = J_0(2\pi d_{i,j})$, where $d_{i,j}$ is the separation between the i^{th} and j^{th} antenna, and J_0 is the first kind Bessel function of order 0. The users are widely spaced in terms of wavelength. We assume the same pathloss for all users.

The LNA input impedance c defined in (8), the impedance R_N defined in (7) and the optimal matching impedance z_{opt} are all assumed to be 75Ω. The correlation coefficient ρ in (7) is assumed to be 0. Similar to classical MIMO papers that use directly (1), we normalize our signal to noise ratio (SNR) for the case $M = N = 1$, for which (9) would be $v_l = Av_g + u_n$. In this case the SNR is $\mathrm{E}[AA^H]P/\mathrm{E}[u_n u_n^H]$. We will only consider SNR= 15dB and $d_r = 0.1\lambda$. We assign $p = -30$ in all our optimization.

In our simulation results, we consider the two regimes of having little number of users and antennas at the receiver and having higher number of users and antennas at the receiver. We compare our results to two reference cases which are

- Ref1: Conventional noise matching, which is optimal for uncoupled antennas.
- Ref2: Matching network maximizing the sum rate [9].

We call the matching network maximizing the minimum user rate "Opt".

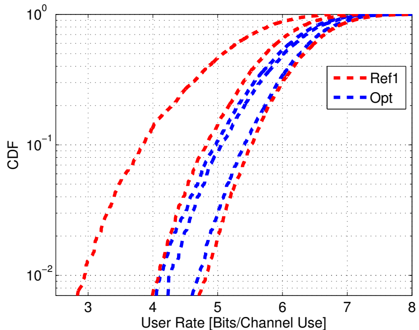
A. Low N and Low M

We consider the case of having $N = 3$ users and a receiver with $M = 4$ antennas organized as a 2×2 planar array. For this low number of users time sharing is feasible. We plot the CDFs of the sorted user rates in Fig. 3(a) and Fig. 3(b) for the case of time sharing and no time sharing, respectively. In Fig. 4 we plot the CDF of the user rates for both the time sharing case and the no time sharing case for the use of the Ref2 network. For the case of no time sharing, we can recognize that the minimum user rate is highly boosted when we use our proposed Opt matching. Regarding the minimum user 1% outage rate, it is 2.94 [bits/channel use] for Ref1 matching, and 2.75 [bits/channel use] for Ref2 matching, while it is 4.13 [bits/channel use]. Regarding fairness among users, it can be seen that in the case of using our Opt matching the difference between the users is much smaller than for the use of the other two reference networks.

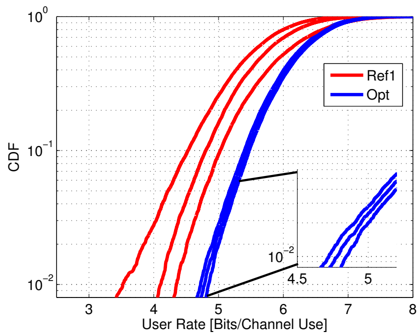
For the case of time sharing, we can see very interesting observations. First of all, the 1% outage rate is enhanced for the two reference matching networks strongly. The one which benefits the most from time sharing is Ref2, as its outage rate jumps from 2.75 to 4.19 [bits/channel use]. For the case of using the Opt matching with time sharing optimization, the outage rate becomes 4.72 [bits/channel use]. The most interesting result is that using the Opt matching leads to a very high fairness among the users, i.e. all the users have extremely minor rate variation.

B. High N and High M

We now switch to the case of larger system dimension, where time sharing is of very high complexity. We consider having $N = 10$ users and a receiver with $M = 30$ antennas organized as a 2×15 planar array. We will only consider optimizing the matching network given no time sharing. In Fig. 5 we plot the CDFs of the sorted users number $(1, 3, 5, 7, 10)$. We can see that using the Ref1 and Ref2 matching yield 1% outage rates of 4.37 and 4.85 [bits/channel use] compared to an outage rate of 6.33 [bits/channel use] if we use the Opt matching. This is definitely a substantial gain. With regards to fairness, we consider the difference between the 1% outage rates of the weakest user and both the fifth and the tenth (top) strongest user. We can see that in the case of Ref1 the two differences are 1.46 and 2.7 [bits/channel use]. For Ref2, the difference are higher reaching 2.4 and 3.51 [bits/channel use]. This is expected as the sum rate maximization does not care about fairness. However, if we use the Opt matching, the



(a) No time sharing



(b) Time Sharing

Fig. 3: CDF of per-user rate for $N = 3$ users and $M = 4$ receive antennas. (a) *no time sharing* and (b) *time sharing*.

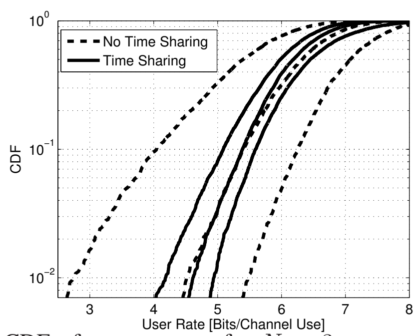
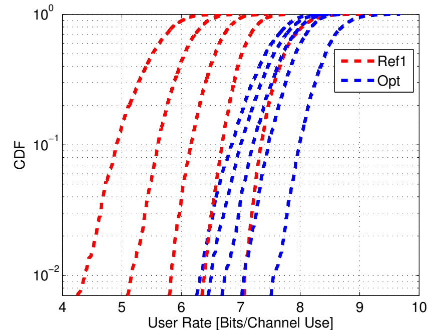


Fig. 4: CDF of per-user rate for $N = 3$ users and $M = 4$ receive antennas using the Ref2 matching.

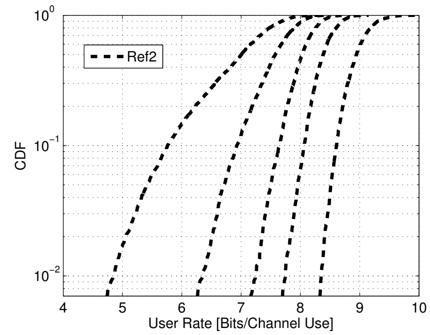
differences reduce strongly to 0.41 and 1.23 [bits/channel use].

V. Conclusion

In this paper, we presented a completely new approach of designing the UC matching networks for coupled MU-MIMO systems. Starting from the rate regions for MU-MIMO systems, we could design our UC matching network to maximize the minimum user rate. For the two cases of time sharing and no time sharing, our results show great performance superiority of our algorithm over conventional matching networks. Especially for the case of a large number of antennas we witnessed a substantial performance enhancement. From fairness point of view between users, our algorithm substantially reduced the differences between the user rates.



(a) Ref1 and Opt



(b) Ref2

Fig. 5: CDF of sorted users (1,3,5,7,10) rate for $N = 10$ users and $M = 30$ receive antennas. (a) *Ref1 and Opt matching* and (b)*Ref2 matching*.

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