Decentralized Target Rate Optimization for MU-MIMO Leakage Based Precoding

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***Abstract*—In this paper we propose a decentralized target rate precoding for multi-user multiple-input multiple-output downlink setups. It is well suited for joint transmission and works for arbitrary cooperation sets. The precoding is optimized for each link separately with respect to the transmit and leakage power. This allows to control the interference into the network and to design the precoding flexibly with respect to energy efficiency and outage minimization. A closed form solution for the optimization problem is presented and the trade-off between the used transmit power and generated leakage power is discussed. Based on the proposed target rate precoding, different strategies for the iterative optimization of a network are presented and evaluated in numerical simulations.**

I. Introduction

In multi user (MU) multiple input multiple output (MIMO) systems, with one or multiple transmitters and multiple receivers, the distribution of the achievable rates among the users is of crucial interest from a fairness point of view. Different beamforming strategies can be thought of. While sum rate maximization in a system maximizes the total throughput, the achievable rates can be distributed very unevenly among the different users. Strong users get very high rates and weak users very low rates or even nothing at all [1]. Hence, it is unpractical to be applied in a real MU system in terms of user satisfaction. In contrast to sum rate maximization, maximizing the minimal rate (Max-Min) among all users promises to be a very fair approach. It leads to equal rates for all users, as the rate of the strong users is reduced for the benefit of the weak ones [1], [2]. However, this equality could lead to very low rates and total throughput if one user has a very poor channel. Hence, this scheme is unpractical in real systems as well.

An interesting alternative is target rate precoding. The precoding is designed such that a user just achieves a certain predefined target rate. This rate could be different for every user and adapted according to applications and/or user priorities. In contrast to Max-Min, the rates of the strong users are only reduced down to their target rate. If a weak user is not able to achieve his target rate, he is considered to be in an outage. However, all other users are not further affected. This way, the transmitters can efficiently use their resources to serve as many receivers as possible with their desired target rate, while using as little transmit power as possible.

In the literature, various approaches have been proposed for target rate precoding such as [3]–[5]. In [3], two methods are proposed to find the transmit covariance matrices which minimize the transmit power subject to target dirty-paper coding rates for single antennas as well as multi antenna receivers. In [4], a transmitter design for single antenna receivers is suggested, based on Tomlinson-Harashima precoding. It minimizes the transmit power under target rate and bit error probability constraints. Ref. [5] in the end studies the weighted sum-power minimization problem under rate constraints and the admission problem. All these approaches state a global optimization problem over all users and iteratively solve it. Furthermore, all of them consider transmit power minimization under the rate constraints without constraints on the transmit power (except [5] with the admission problem). However, in real networks, global optimization is unpractical and the transmit power is generally limited.

Therefore, we propose a decentralized target rate precoding. In decentralized precoding, each link is considered separately without knowledge about the precoding of all others. Therefore, no global cooperation and only limited channel state information (CSI) is required. However, the whole network is inherently coupled, as the precoding of each MS affects the performance of all others via the generated interference (so called leakage). Therefore, iterative adaption of the precoding is necessary for reliable target rate precoding. Furthermore, it is important to consider the leakage power in the precoding design. Leakage based precoding (LBP) [6] has shown to be a promising concept thereof, reducing the leakage to all unintended users for each link separately. However, under limited transmit power, leakage minimization comes at the price of lower quality of service at the desired user [7]. Hence, it is important to consider both, the transmit power as well as the leakage power in the precoding design and to find a good trade-off between leakage reduction and serving the desired user.

In this context, we derive a decentralized target rate precoding considering the transmit as well as the leakage power and derive a closed form solution to it. We provide thorough analysis on the interdependency of the transmit and leakage power in the proposed precoding which helps to find an optimal trade-off. Three different strategies are discussed to iteratively optimize the network performance in terms of energy efficiency and outage minimization, and their performance is analyzed with numerical simulations on the downlink of a cellular network where multiple base stations (BSs) serve multiple mobile stations (MSs). However, it could be applied to any other MU setup.

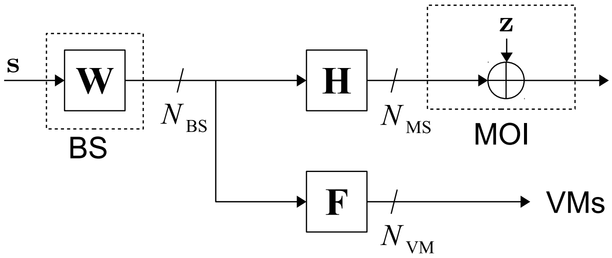

Fig. 1. System model.

II. System Model

In decentralized precoding, the beamforming matrix is computed for each MS separately. That is, we split up the system into several single user MIMO (SU-MIMO) setups as denoted in Fig. 1. One BS serves the mobile of interest (MOI) with the desired signal, while several victim mobiles (VMs) suffer from the generated interference. The number of antennas at the BS is $N_{\rm BS}$, the number of antennas at the MOI $N_{\rm MS}$, and the VMs are assumed to be equipped with in total $N_{\rm VM}$ antennas. Instead of one BS, also a cluster of BSs can be considered serving the MOI jointly. The channel from the BS to the MOI is expressed by $\mathbf{H} \in \mathbb{C}^{N_{\rm MS}\times N_{\rm BS}}$ and the channel from the BS to the VMs by $\mathbf{F} \in \mathbb{C}^{N_{\rm VM}\times N_{\rm BS}}$. The transmit signal vector is denoted by $\mathbf{s} \in \mathbb{C}^{N_{\rm MS}}$ with signal covariance matrix $\mathbf{\Lambda_s} = \mathsf{E}\left[\mathbf{s}\mathbf{s}^{\mathsf{H}}\right]$ and the precoding matrix by $\mathbf{W} \in \mathbb{C}^{N_{\rm BS}\times N_{\rm MS}}$. Thus, the transmit covariance matrix is $\mathbf{Q} = \mathbf{W}\mathbf{\Lambda_s}\mathbf{W}^{\mathsf{H}}$ and the transmit power can be found as $P_{\rm BS} = \mathrm{tr}\,(\mathbf{Q})$, where $\mathrm{tr}\,(\cdot)$ denotes the trace of a matrix. At the MOI we assume additive white Gaussian noise plus interference summarized in the term $\mathbf{z} \in \mathbb{C}^{N_{\rm MS}}$ with covariance matrix $\mathbf{K_z}$. Hence, the received signal can be written as

$$\mathbf{r}_{\rm MOI} = \mathbf{H}\mathbf{W}\mathbf{s} + \mathbf{z}, \tag{1}$$

and the achievable rate thereof as

$$\mathbf{R}_{\rm MOI} = \log\det\left(\mathbf{I} + \tilde{\mathbf{H}}\mathbf{Q}\tilde{\mathbf{H}}^{\mathsf{H}}\right), \tag{2}$$

with $\mathbf{I}$ the identity matrix and the substitution $\tilde{\mathbf{H}} = \mathbf{K_z}^{-1/2}\mathbf{H}$. The leaked signal received at the VMs is

$$\mathbf{r}_{\rm VM} = \mathbf{F}\mathbf{W}\mathbf{s}. \tag{3}$$

The leakage power can thus be written as $P_{\rm L} = \mathrm{tr}\left(\mathbf{F}\mathbf{Q}\mathbf{F}^{\mathsf{H}}\right)$. We assume block fading channels. For the precoding, the BS needs to perfectly know the channel matrix $\mathbf{H}$ and the interference plus noise covariance matrix at the MOI $\mathbf{K_z}$ through feedback. From the VMs only the matrix $\mathbf{F}^{\mathsf{H}}\mathbf{F}$ needs to be known at the BS.

III. Target Rate Precoding

LBP has shown to be a promising approach for decentralized precoding in a multi user system [6]. It does not require any cooperation or information about the precoding of other MSs but still allows to control the interference into the network.

In [7] the trade-off between transmit power and leakage power for rate optimal MIMO LBP has been investigated. Thereby, it has been shown that there exists a fundamental trade-off between serving the MOI with a rate as high as possible and generating as low leakage as possible. Both is important for the network performance, however generally contradicting. To reduce the leakage power, transmit power has to be invested into the leakage reduction, reducing the available power for the desired signal. Hence, it is reasonable to consider both, the transmit and leakage power in the precoding design to optimize the network performance.

The same is true for target rate precoding as considered in this paper. A signal generating a lot of leakage harms all the VMs. However, we want to use as little transmit power as possible to achieve the target rate. Therefore, we consider both, the transmit as well as the leakage power in the precoding design. Hence, the optimization problem is to minimize transmit and leakage power under a target rate constraint. To capture both objective functions in one, we introduce the weighted sum power, analogously to [7].

$$\begin{aligned}\tilde{P} &= cP_{\rm L} + (1-c)\,P_{\rm BS} \\ &= c\,\mathrm{tr}\left(\mathbf{W}^{\mathsf{H}}\mathbf{F}^{\mathsf{H}}\mathbf{F}\mathbf{W}\mathbf{\Lambda}_{\rm s}\right) + (1-c)\,\mathrm{tr}\left(\mathbf{W}^{\mathsf{H}}\mathbf{W}\mathbf{\Lambda}_{\rm s}\right) \\ &= \mathrm{tr}\left(\mathbf{W}^{\mathsf{H}}\left(c\mathbf{F}^{\mathsf{H}}\mathbf{F} + (1-c)\,\mathbf{I}\right)\mathbf{W}\mathbf{\Lambda}_{\rm s}\right) \\ &= \mathrm{tr}\left(\mathbf{W}^{\mathsf{H}}\tilde{\mathbf{F}}^{\mathsf{H}}\tilde{\mathbf{F}}\mathbf{W}\mathbf{\Lambda}_{\rm s}\right) \\ &= \mathrm{tr}\left(\tilde{\mathbf{F}}\mathbf{Q}\tilde{\mathbf{F}}^{\mathsf{H}}\right),\end{aligned} \tag{4}$$

with $c \in [0,1]$. With this transformation and for a fixed c, the optimization problem can be stated as

$$\min_{\mathbf{Q}} \mathrm{tr}\left(\tilde{\mathbf{F}}\mathbf{Q}\tilde{\mathbf{F}}^{\mathsf{H}}\right) \text{ s.t. } \log\det\left(\mathbf{I} + \tilde{\mathbf{H}}\mathbf{Q}\tilde{\mathbf{H}}^{\mathsf{H}}\right) \geq R_{\rm t}. \tag{5}$$

That is, we design the precoding matrix $\mathbf{W}$ and the signal covariance matrix $\mathbf{\Lambda_s}$ such that the weighted sum power is minimized under the target rate constraint. To solve this problem we resort to the generalized eigenvalue decomposition (GEVD), similar as in [6]. The GEVD of the matrix tuple $\left(\tilde{\mathbf{H}}^{\mathsf{H}}\tilde{\mathbf{H}}, \tilde{\mathbf{F}}^{\mathsf{H}}\tilde{\mathbf{F}}\right)$ provides a diagonal matrix $\mathbf{D}$ containing the generalized eigenvalues of the tuple and a matrix $\mathbf{Z}$ containing the corresponding generalized eigenvectors as columns, such that

$$\begin{aligned}\mathbf{Z}^{\mathsf{H}}\tilde{\mathbf{H}}^{\mathsf{H}}\tilde{\mathbf{H}}\mathbf{Z} &= \mathbf{D} \\ \mathbf{Z}^{\mathsf{H}}\tilde{\mathbf{F}}^{\mathsf{H}}\tilde{\mathbf{F}}\mathbf{Z} &= \mathbf{I}.\end{aligned} \tag{6}$$

Hence, by substituting $\mathbf{W} = \bar{\mathbf{Z}}$, where $\bar{\mathbf{Z}}$ corresponds to the nonzero GEVs $\bar{\mathbf{D}}$ in $\mathbf{D}$, we can find the equivalent optimization problem

$$\min_{\mathbf{\Lambda_s}} \mathrm{tr}\left(\mathbf{\Lambda}_{\rm s}\right) \text{ s.t. } \log\det\left(\mathbf{I} + \bar{\mathbf{D}}^{\frac{1}{2}}\mathbf{\Lambda}_{\rm s}\bar{\mathbf{D}}^{\frac{1}{2}}\right) \geq R_{\rm t}. \tag{7}$$

This problem is optimized by a diagonal $\mathbf{\Lambda_s}$. The proof thereof works analogously to the proof of the point to point MIMO capacity in [8]. Due to the diagonal structure of $\mathbf{\Lambda_s}$, its elements can be found by splitting up the system into parallel channels and applying the method of Lagrange multipliers:

$$\lambda_i = \left(\mu - \frac{1}{\delta_i}\right)^+ \tag{8}$$

$$\mu = \sqrt[N_{\rm MS}]{\frac{2^{R_{\rm t}}}{\delta_1\cdots\delta_{\rm MS}}}, \tag{9}$$

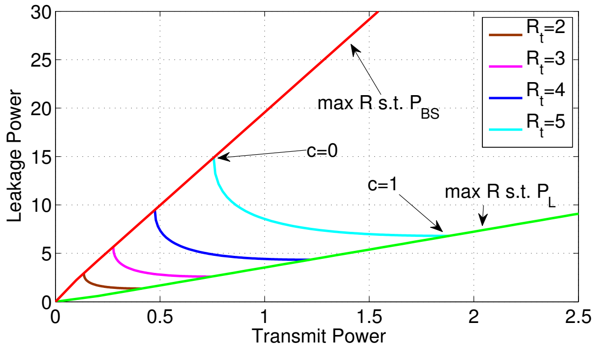

Fig. 2. $P_{\mathrm{BS}} - P_{\mathrm{L}}$ plane.

where λ_i are the entries of the diagonal matrix $\mathbf{\Lambda_s}$, δ_i the generalized eigenvalues, and $(x)^+ = \max(0, x)$.

To get a better understanding of this optimization problem we have a look at the $P_{\mathrm{BS}} - P_{\mathrm{L}}$ plane in Fig. 2. It shows an example outcome of the relation between the leakage power and the transmit power. The solid red and green curves show the resulting leakage power respectively transmit power if the rate at the MOI is maximized under a transmit respectively leakage power constraint [7]. For a fixed c, the optimization minimizes the weighted sum power subject to the target rate. The choice of c allows to control the relation of the transmit power and leakage power minimization and different points in the $P_{\mathrm{BS}} - P_{\mathrm{L}}$ plane can be achieved. For $c = 0$, the transmit power is minimized and hence, the resulting transmit and leakage power lie on the red curve. For $c = 1$, the leakage power is minimized and the resulting transmit and leakage power lie on the green curve. For any c in between, the resulting $(P_{\mathrm{BS}}, P_{\mathrm{L}})$ tuples lie somewhere in between and describe a curve, the so called target rate curve. That is, R_{t} can be achieved with any of these transmit and leakage power levels. It can be observed, that the lower the transmit power, the higher the generated leakage power and vice versa. However, in the lower regime the curve flattens out. That is, in this area further reduction of the leakage power comes at a high price of additional transmit power. This relation is shown for different target rates ($R_{\mathrm{t}} \in \{2, 3, 4, 5\}$) in Fig. 2. Obviously, a higher target rate requires higher transmit and leakage power. Hence, the target rate curve is shifted up and to the right.

IV. The $P_{\mathrm{BS}} - P_{\mathrm{L}}$ Trade-Off

For a MS in a SU-MIMO setup as shown in Fig. 1 with fixed channels and $\mathbf{K_z}$, a given target rate can be achieved by choosing any point on the target rate curve, i.e. for any $c \in [0, 1]$. However, in practical systems, the transmit power is constraint, thus not all points on the curve might be feasible or non at all. Furthermore, the whole network is inherently coupled. The precoding of each MS affects the performance of all others due to the generated leakage power. That is, their target rate curve is shifted with each precoding adaption of another MS. This leads to the fact that MSs might be unable to achieve the target rate under the current interference plus noise level and the transmit power constraint. To optimize the performance of the network in terms of outage minimization and energy efficiency, it is important to find a good trade-off (i.e. point on the R_{t}-curve) between the used transmit power and the generated leakage power and to adapt this trade-off to the network properties such as user density, regulations etc. In different words: we have to reduce the leakage power as much as necessary, but as little as possible.

As the interference at each MS changes with every update of the precoding for the other MSs, feedback of the interference plus noise covariance matrix $\mathbf{K_z}$ is introduced. After each feedback, the precoding is updated. That is, depending on the new interference and noise level, the precoding is adapted using more or less transmit power and generating more or less leakage power, depending on the strategy for the trade-off (choice of c). Therefore, convergence of the precoding is not guaranteed. However, it is shown in Section V that the system stabilizes already after a few iterations if a reasonable strategy is chosen. That is, due to the decentralized approach, iterative adaption of the precoding is necessary with a careful choice of the transmit power - leakage power trade-off for reasonable results.

In the following we consider three different strategies for such a trade-off:

1) In strategy 1 (S_1), we always choose the $(P_{\mathrm{BS}}, P_{\mathrm{L}})$ tuple with minimal transmit power. That is, we always choose $c = 0$, and obtain the solution on the red curve in Fig. 2. If that violates the transmit power constraint $\tilde{P}_{\mathrm{BS}}$, the obtained transmit signal is scaled to the maximal allowed transmit power.
2) In strategy 2 (S_2), we always choose the $(P_{\mathrm{BS}}, P_{\mathrm{L}})$ tuple with minimal leakage power. That is, we always choose $c = 1$, and obtain the solution on the green curve in Fig. 2. Again, if this solution violates the transmit power constraint $\tilde{P}_{\mathrm{BS}}$, the obtained transmit signal is scaled to the maximal allowed transmit power.
3) In strategy 3 (S_3), we try to find a good compromise between S_1 and S_2. The idea is to reduce the transmit power as long as we are in the flat regime of the target rate curve, i.e. as long as the reduction of the transmit power comes at a low price of increased leakage power. Therefore, we always compute the precoding for $c = 1$, i.e. the solution on the green curve in Fig. 2 and then reduce the used transmit power by a certain percentage by walking back on the curve of constant rate. If it is not possible to reduce the transmit power by that amount or if this point violates the transmit power constraint, we take the obtained solution for $c = 1$ and scale it to the allowed transmit power. This strategy allows to reduce the transmit power compared to S_2 and to reach the target rate for some MSs where S_2 would not succeed. However, it also generates more leakage.

S_1 and S_2 are simple to implement. We just calculate the solutions for $c = 0$, respectively $c = 1$, check whether the transmit power constraint is fulfilled and scale it if necessary. The solution of S_3 can be found with the bisection method. That is, we first compute the precoding for $c = 1$ and then iteratively search for the c leading to the desired transmit power.

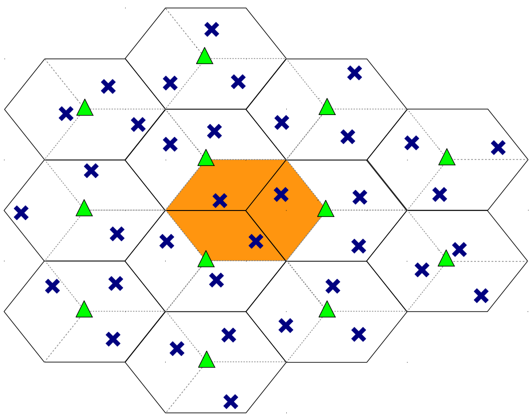

Fig. 3. Cellular network layout.

V. Numerical Simulations

To evaluate the performance of the proposed target rate precoding, numerical simulations have been performed in a cellular network structure as shown in Fig. 3, which was motivated in [2]. The 12 hexagonal cells with 3 sectors each are regularly arranged with a BS in the center of each cell consisting of 3 (one per sector) independent antenna arrays with $N_{\mathrm{BS}} = 8$ elements. These antenna arrays have a directional pattern according to [2] (120 degrees beam width). In every sector, one MS with $N_{\mathrm{MS}} = 2$ omnidirectional antennas is randomly placed. These MSs are then served by 3 cooperating BSs applying joint beam forming, i.e. these 3 BSs form a large virtual antenna array to serve the MS. The reuse factor in the network is 1, all MSs are served in the same frequency band and time slot. A backhaul with infinite capacity is assumed between the cooperating BSs to share the CSI and the transmit symbols. The cooperating BSs are grouped into fixed clusters, always consisting of 3 neighboring sectors of 3 different cells as highlighted in Fig. 3. Each MS is served with at most $\tilde{P}_{\mathrm{BS}} = 10$ W over the entire LTE Advanced bandwidth of 100 MHz. The distribution of this transmit power among the cooperating BSs can be arbitrary. The noise variance is assumed to be $\sigma^2 = 5 \cdot 10^{-12}$ W. The channels are considered frequency flat fading and modeled by Rayleigh-fading with pathloss and shadowing according to the WINNER II channel model, scenario C2 [9].

In the following, the performance of target rate precoding is investigated and the three different strategies are compared. For reasons of simplicity, the target rate is assumed to be the same for all MSs. In S_3, the reduction of the transmit power has been set to 40 percent for all MSs ($\mathrm{S}_3(0.4)$). The precoding is adapted iteratively. In each iteration, the precoding is computed for all MSs, the resulting $\mathbf{K}_{\mathbf{z}}$s determined and fed back. The initial $\mathbf{K}_{\mathbf{z}}$ is assumed to be $\mathbf{I} \cdot 10^{-10}$ W for all MSs.

Fig. 4 and Fig. 5 show the inherent coupling of the network. S_1 has been applied to all MSs for $R_{\mathrm{t}} = 5$ bps/Hz. After the system stabilized, one MS in the central cluster (orange in Fig. 3) achieving the target rate was chosen and the c adapted such that different points on the target rate curve $(P_{\mathrm{L},1}, \cdots, P_{\mathrm{L},8})$ were achieved (c.f. Fig. 4). That is, the generated leakage power has been decreased and the transmit power increased, while the achievable rate of this MS stayed constant. The effect

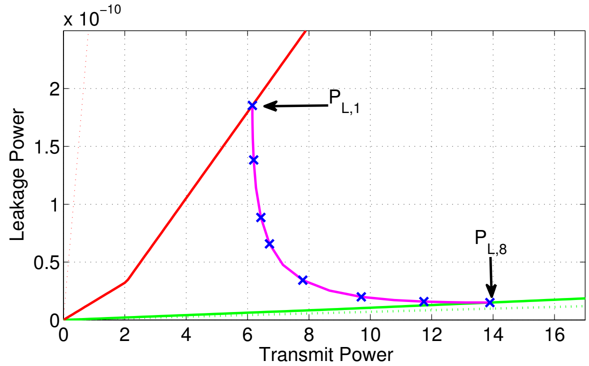

Fig. 4. $P_{\mathrm{BS}} - P_{\mathrm{L}}$ plane of MS 1.

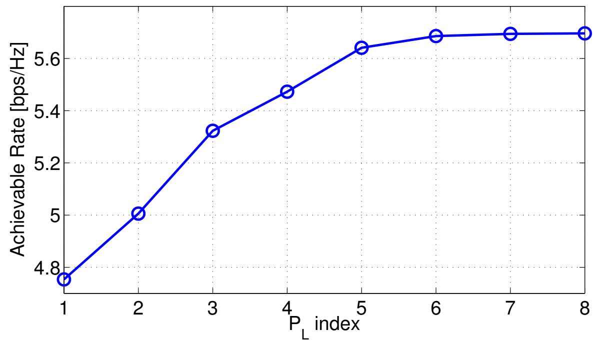

Fig. 5. Achievable Rate of MS 2.

of these changes on a second MS in this cluster can be observed in Fig. 5. The more the leakage at the first MS is decreased, the higher is the achievable rate at the second MS, however at the price of additional transmit power. Furthermore, it can be observed that at $P_{\mathrm{L},5}$ the achievable rate starts to saturate. That is, by further decreasing the leakage power and increasing the transmit power at the first MS, only little can be gained at the second MS for a high price of additional transmit power (more than 50 percent higher). This shows how crucial it is to find a good trade-off between used transmit power and generated leakage power for the network performance.

Fig. 6 shows the cumulative distribution function (CDF) of the convergence behaviour of the target rate precoding for strategy 3 ($\mathrm{S}_3(0.4)$) with $R_{\mathrm{t}} = 4$ bps/Hz. Expectedly, the performance of the initial precoding without any knowledge about the interference plus noise level does not lead to satisfying results. However, already after two iterations of feedback, very reasonable results are achieved. After 9 iterations the system stabilizes with low outage probability (the probability that a MS does not achieve the target rate). The knee of the CDF at the outage level is an effect of walking back on the target rate curve. By walking back, many MSs which would not have achieved the target rate before (as they would have violated the transmit power constraint) are now able to achieve it. Hence, all these MSs *jump up* to R_{t} in the CDF, leading to the flat interval. The deviation from the target rate at the top of the CDF comes from the fact, that although the system becomes very stable, still some $\mathbf{K}_{\mathbf{z}}$ change with the precoding update, and thus some MSs *overshoot* the target rate.

Fig. 7 shows the CDFs for various target rates after 9 iterations of feedback. It can be observed that for all target

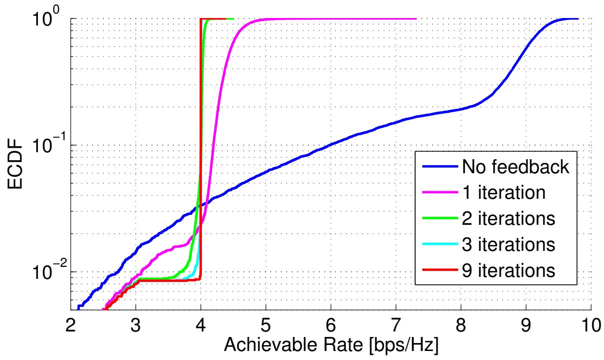

Fig. 6. Convergence behaviour of the achievable rates for $R_{\mathrm{t}} = 4$ bps/Hz.

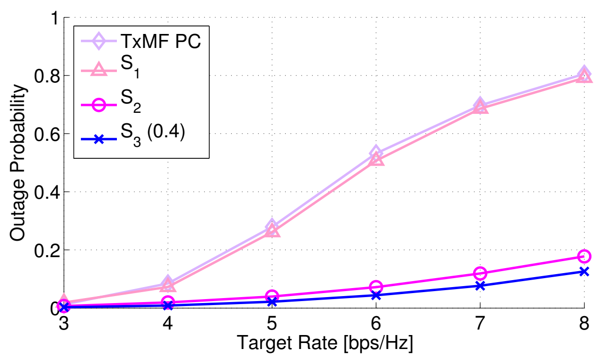

Fig. 8. Outage probabilities.

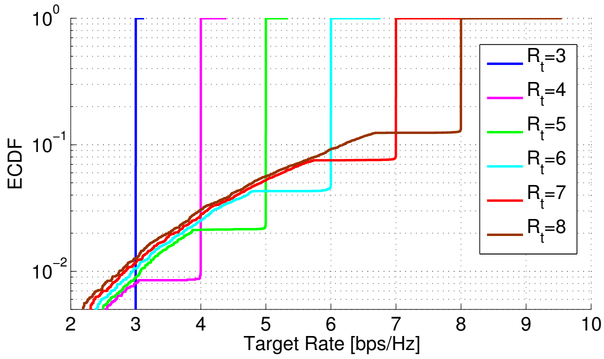

Fig. 7. CDFs of achievable rates for various R_{t}.

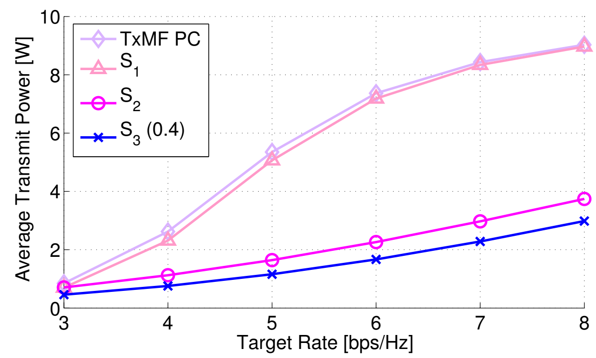

Fig. 9. Average transmit power.

rates the system converged. As expected, the outage probability continuously increases for increasing target rate.

Fig. 8 and Fig. 9 show the outage probability and the average transmit power of all three strategies. Additionally, the performance of transmit matched filter (TxMF) precoding with power control is included for comparison. The TxMF is scaled such that R_{t} is just achieved. If that is not feasible, it is scaled to $\tilde{P}_{\mathrm{BS}}$. It can be observed that S_1 and the TxMF lead to high outage probabilities and transmit power as they introduce strong leakage into the network. The increased interference level requires higher transmit power to achieve the target rate, which again generates more leakage, and so on, until the system stabilizes on a high transmit and leakage power level. Hence, although S_1 uses the least transmit power for a SU-MIMO system, it uses the most in a coupled MU system. S_2 and S_3 are close to each other. However, S_3 achieves up to 5 percent lower outage probability while using up to 30 percent less transmit power on average. By walking back on the target rate curve, more MSs achieve R_{t} although more leakage is generated. As long as we stay in the *flat* area of the curve, the benefit of achieving the target rate outweighs the loss of additional leakage power. Therefore, it is crucial to choose a reasonable value of transmit power reduction. The best option would be to adapt this value to each MS. Out of reasons of simplicity, this was not done so far and is the topic of future work.

VI. Conclusions

In this paper, we have proposed a decentralized target rate precoding considering the transmit as well as the leakage power, formulated the optimization problem for it and derived a closed form solution. The consideration of the leakage power helps to optimize the network performance in terms of outage probability and energy efficiency. However, a good trade-off between leakage reduction and transmit power reduction is crucial. We conclude that target rate precoding is a promising compromise between sum rate maximization and maximizing the minimum rate, achieving a low outage probability by serving the users with a desired rate.

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