Cooperative Distributed Multiuser MMSE Relaying in Wireless Ad-Hoc Networks

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***Abstract*— We consider a wireless ad-hoc network with single antenna nodes under a two-hop traffic pattern. Two system architectures are investigated in this paper: Either linear amplify-and-forward relays (LinRel) or a distributed antenna system with linear processing (LDAS) serve as repeaters. The gain factors of the repeaters are assigned such that the mean squared error (MSE) of the signal at the destinations is minimised (*multiuser MMSE relaying*). A scalar multiplier $\gamma_m \in \mathbb{C}$ at each destination m allows for received signals that are scaled and rotated versions of the transmitted symbols. We distinguish two cases: 1) The factors are equal for all destinations: $\gamma_m = \gamma$. 2) An individual factor γ_m is chosen for each destination m. Multiuser MMSE relaying essentially realizes a distributed spatial multiplexing gain with single antenna nodes as all source/destination pairs can communicate concurrently over the same physical channel. The main contribution of this paper is the derivation of the MMSE gain factors. We evaluate the relaying scheme in comparison to *multiuser zero forcing* (ZF) [1] and show that it can outperform the latter in terms of average sum rate and diversity gain.**
Keywords – cooperative relaying, ad-hoc networks, distributed spatial multiplexing, minimum mean squared error (MMSE)

I. Introduction

In wireless cooperative networks distributed relay nodes assist the communication between a number of sources and destinations. Mostly it is assumed that the terminals are not able to transmit and receive at the same time (half-duplex constraint). We focus on two-hop strategies where the sources transmit their data in a first phase while relays and destinations receive. In a second phase, the relays or a subset of them retransmit a signal which is related to their previously received data. We speak of an *amplify-and-forward* or *nonregenerative* scheme if the relay nodes simply retransmit an amplified and possibly rotated version of their received signal and of a *decode-and-forward* or *regenerative* scheme if they fully decode and re-encode before retransmission. *Compress-and-forward* denotes the case where the relays forward a compressed estimation of their received signal.

Due to spatial separation of the relaying terminals, such schemes can provide a *distributed spatial diversity gain* which can be used to enhance the link reliability by averaging over independent channel realisations. Another approach is to increase the capacity of the system by using the relays in order to achieve a *distributed spatial multiplexing gain*. This essentially means that a number of nodes can communicate concurrently over the same physical channel.

The fundamental diversity-multiplexing tradeoff, which is well-known for multiple-input multiple-output (MIMO) wireless system, has been analysed for *repetition-based* as well as *space-time coded* cooperative schemes in distributed wireless networks utilising *decode-and-forward* relays [2]. While the latter scheme achieves a higher spectral efficiency, both protocols are shown to provide full spatial diversity in the order of cooperating nodes. *Opportunistic relaying*, where only one relay out of the whole possible set is chosen to assist the communication of a single source/destination pair, can also achieve this diversity order [3]. The authors point out that it even exhibits exactly the same diversity-multiplexing tradeoff as the space-time coded protocol presented in [2]. *Decode-and-forward* and *compress-and-forward* strategies for relay networks are developed in [4]. Considering full-duplex terminals, the authors evolve capacity theorems and provide achievable rates and rate regions for a number of wireless channel models. Amplify-and-forward schemes which apply the idea of space-time coding originally devised for multiple-antenna systems to the problem of communication over a distributed wireless relay network are analysed in [5]. The authors focus on the diversity achievable without requiring the relay nodes to decode.

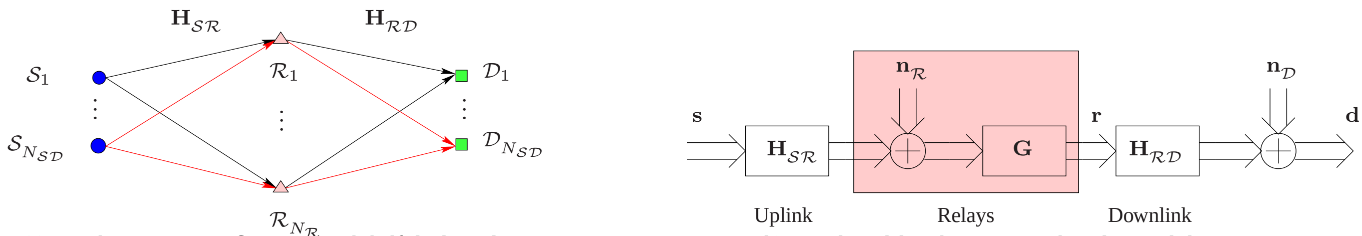
As stated in the beginning, distributed spatial multiplexing can be used to directly increase the data rate of a wireless network. In [6] the authors show that the capacity of a wireless relay network can scale linearly with the total number of transmit antennas. They present a zero-forcing based scheme for the case of infinite number of relays which exhibits this gain. For finite number of single-antenna relays, a scheme which is based on zero-forcing by performing a nullspace projection is presented in [1]. The gain factors of the amplify-and-forward relays are chosen such that multi-access interference at the destinations is cancelled. A global phase reference as well as channel knowledge are however needed at the relays. In [7] the degrading effect of noisy channel state information, phase noise, and quantisation noise is investigated.

An MMSE-based distributed gain allocation scheme for a system comprising a single multi-antenna source/destination pair and several multi-antenna relaying terminals is presented in [8]. Compared to that, our work focuses on the case of a multi-user system utilising single-antenna terminals. We present a distributed gain allocation where the complex relay gain factors are calculated such that the mean squared error at all destinations is minimised jointly.

Notation: The operators $\odot$, $\mathrm{E}_{\{x\}}[\cdot]$, $\mathrm{tr}(\cdot)$ and $(\cdot)^{\mathrm{H}}$ denote the elementwise product, expectation with respect to x, trace operation, and conjugate complex transpose, respectively. $\mathbf{I}_N$ is the identity matrix of size $N \times N$. The terms $\mathbf{X}[i,j]$, $\mathbf{X}[:,i]$, and $\mathbf{X}[i,:]$ denote the element (i,j), the ith column, and the ith row of a matrix $\mathbf{X}$, respectively. A vector whose entries are taken from a complex normal distribution with mean 0 and variance σ^2 is denoted by $\mathbf{x} \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2\mathbf{I}\right)$. The operator $\mathrm{diag}(\cdot)$ has two meanings: When the argument is a matrix, it takes the diagonal elements and puts them into a column vector. When the argument is a vector, it puts the elements of the vector into a diagonal matrix. Finally, the conjugate complex transpose of a matrix inverse is denoted by $(\cdot)^{-\mathrm{H}}$.

II. System Model

We consider a wireless network where $N_{\mathcal{SD}}$ source/destination pairs want to communicate concurrently over the same physical channel. $N_{\mathcal{R}}$ amplify-and-forward relay nodes assist the communication in a half-duplex scheme. They multiply the signals they receive from the sources with a complex gain factor before retransmitting them. All nodes in the network employ only one antenna. The communication follows a two-hop relay traffic pattern, i.e., each



(a) Two-hop system configuration with half-duplex relays

(b) Signal model with AWGN at the relays and destinations

Fig. 1. System configuration and compound signal model for a two-hop relay network

transmission cycle includes two channel uses: one for the *uplink* transmission from the sources to all relays and one for the *downlink* transmission from the relays to the destinations. The direct link is not taken into account in the following considerations. **Figs. 1(a)** and **1(b)** show the system configuration and the compound signal model, respectively. The scalar transmit symbols are stacked in the vector $\mathbf{s} \in \mathbb{C}^{N_{\mathcal{SD}}}$. No channel state information (CSI) is present at the sources. Consequently, no power- or bit loading is performed. Each source is assumed to use the same transmit power $\sigma_s^2 = \frac{P_{\mathcal{S}}}{N_{\mathcal{SD}}}$, where $P_{\mathcal{S}}$ is the total transmit power of all sources. The source signal vector $\mathbf{s}$ is first transmitted over the uplink matrix channel $\mathbf{H}_{\mathcal{SR}} \in \mathbb{C}^{N_{\mathcal{R}} \times N_{\mathcal{SD}}}$ to the relays. The vector $\mathbf{n}_{\mathcal{R}} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_{n_{\mathcal{R}}}^2 \mathbf{I}_{N_{\mathcal{R}}}\right)$ comprises the additive, white, Gaussian noise (AWGN) contributions at the relay nodes. After multiplication with the gain matrix $\mathbf{G} \in \mathbb{C}^{N_{\mathcal{R}} \times N_{\mathcal{R}}}$, the signal $\mathbf{r}$ is passed through the downlink matrix channel $\mathbf{H}_{\mathcal{RD}} \in \mathbb{C}^{N_{\mathcal{SD}} \times N_{\mathcal{R}}}$ to the $N_{\mathcal{SD}}$ destination nodes. The vector $\mathbf{n}_{\mathcal{D}} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_{n_{\mathcal{D}}}^2 \mathbf{I}_{N_{\mathcal{SD}}}\right)$ comprises the AWGN contribution at the destinations. For all numerical results we let $\sigma_{n_{\mathcal{R}}}^2 = \sigma_{n_{\mathcal{D}}}^2 := \sigma_n^2$ and use the same sum transmit power at the sources and at the relays. We apply this constraint because one of our goals is to study the influence of the number of relays on the performance of the system. Therefore, to have a fair comparison, additional relays shall not increase the total transmit power of the system. All channel coefficients are assumed to be independent, complex Gaussian random variables with circular symmetric probability density function (frequency flat Rayleigh fading).

The signals at the destinations are stacked in the vector

$$\mathbf{d} = \mathbf{H}_{\mathcal{RD}}\mathbf{G}\mathbf{H}_{\mathcal{SR}} \cdot \mathbf{s} + \mathbf{H}_{\mathcal{RD}}\mathbf{G} \cdot \mathbf{n}_{\mathcal{R}} + \mathbf{n}_{\mathcal{D}} = \mathbf{H}_{\mathcal{SD}}\mathbf{s} + \mathbf{n}, \quad (1)$$

where $\mathbf{H}_{\mathcal{SD}}$ is called the *equivalent channel matrix* and $\mathbf{n}$ the *equivalent noise vector*. The components of $\mathbf{n}$ are spatially no longer white.

III. MMSE Relaying Gain Factors

The gain matrix $\mathbf{G}$ is to be designed such that the mean squared error of the signal at the destinations is minimised. In addition we demand the transmit power of all relays, $P_{\mathcal{R}}$ (averaged over the transmit symbols and noise), to be equal to the total transmit power of all sources ($P_{\mathcal{S}}$). The constrained optimisation we have to perform in order to find the gain matrix $\mathbf{G}_{\text{MMSE}}$ can then be formulated as

$$\mathbf{G}_{\text{MMSE}} = \arg\min_{\mathbf{G}} \mathrm{E}_{\{\mathbf{s}, \mathbf{n}_{\mathcal{R}}, \mathbf{n}_{\mathcal{D}}\}} \left[\|\mathbf{s} - \mathbf{\Gamma}\mathbf{d}\|_2^2\right] \quad (2)$$

subject to

$$\mathrm{E}_{\{\mathbf{s}, \mathbf{n}_{\mathcal{R}}\}} \left[\|\mathbf{r}\|_2^2\right] = P_{\mathcal{S}}. \quad (3)$$

In the remainder of the section all expectations $\mathrm{E}[\cdot]$ are take with respect to transmit symbols $\mathbf{s}$ as well as noise $\mathbf{n}_{\mathcal{R}}$ and $\mathbf{n}_{\mathcal{D}}$. The matrix $\mathbf{\Gamma}$ is diagonal and contains factors $\gamma_m \in \mathbb{C}$, $m \in \{1, \ldots, N_{\mathcal{SD}}\}$ that allow for received signals which are a scaled and rotated version of the transmitted symbols. In order to solve the constrained optimisation problem ((2), (3)) we use the method of Lagrangian Multipliers (e.g. [9]). The Lagrangian function is

$$L(\mathbf{G}, \mathbf{\Gamma}, \lambda) = \mathrm{E}\left[\|\mathbf{s} - \mathbf{\Gamma}\mathbf{d}\|_2^2\right] + \lambda\left(\mathrm{E}\left[\|\mathbf{r}\|_2^2\right] - P_{\mathcal{S}}\right) \quad (4)$$

with

$$\frac{\partial}{\partial \mathbf{G}} L(\mathbf{G}, \mathbf{\Gamma}, \lambda) \stackrel{!}{=} \mathbf{0}, \quad (5)$$

$$\frac{\partial}{\partial \mathbf{\Gamma}} L(\mathbf{G}, \mathbf{\Gamma}, \lambda) \stackrel{!}{=} \mathbf{0}, \quad (6)$$

$$\text{and} \quad \frac{\partial}{\partial \lambda} L(\mathbf{G}, \mathbf{\Gamma}, \lambda) \stackrel{!}{=} 0. \quad (7)$$

Solving (5) for $\mathbf{G}$ we can write

$$\mathbf{G} = \lambda^{-\frac{1}{2}} \cdot \tilde{\mathbf{G}}, \quad (8)$$

where $\lambda^{-\frac{1}{2}}$ is a real-valued scaling factor and $\tilde{\mathbf{G}}$ is a function of

1) $\mathbf{\Psi} = \lambda^{-\frac{1}{2}} \cdot \mathbf{\Gamma}$ and
2) known parameters $\mathcal{P} = \{\mathbf{H}_{\mathcal{SR}}, \mathbf{H}_{\mathcal{RD}}, \mathbf{R}_s, \mathbf{R}_{n_{\mathcal{R}}}, \mathbf{R}_{n_{\mathcal{D}}}\}$.

The matrices $\mathbf{R}_s = \mathrm{E}_{\{\mathbf{s}\}}\left[\mathbf{s}\mathbf{s}^{\mathrm{H}}\right]$, $\mathbf{R}_{n_{\mathcal{R}}} = \mathrm{E}_{\{\mathbf{n}_{\mathcal{R}}\}}\left[\mathbf{n}_{\mathcal{R}}\mathbf{n}_{\mathcal{R}}^{\mathrm{H}}\right]$, and $\mathbf{R}_{n_{\mathcal{D}}} = \mathrm{E}_{\{\mathbf{n}_{\mathcal{D}}\}}\left[\mathbf{n}_{\mathcal{D}}\mathbf{n}_{\mathcal{D}}^{\mathrm{H}}\right]$ denote the covariance matrices of the transmit signal, the noise at the relays and the noise at the destinations, respectively. With (7) we can calculate λ such that the power constraint (3) is always met if $\mathbf{G}$ is chosen according to (8). This means that we essentially adapt the domain of the gain matrix according to our constraint and thus reduce the number of equations from $N_{\mathcal{SD}} + N_{\mathcal{R}} + 1$ to $N_{\mathcal{SD}}$. The Lagrangian Function (4) now reduces to

$$L(\mathbf{G}, \mathbf{\Gamma}, \lambda) = \underbrace{\mathrm{E}\left[\|\mathbf{s} - \mathbf{\Gamma}\mathbf{d}\|_2^2\right]}_{:=\epsilon} + \lambda\underbrace{\left(\mathrm{E}\left[\|\mathbf{r}\|_2^2\right] - P_{\mathcal{S}}\right)}_{=0}, \quad (9)$$

where the cost function ϵ can now be written as function of $\mathbf{\Psi}$ and the parameters $\mathcal{P}$. Due to the form of $\mathbf{G}$ in (8) the power constraint in the original formulation of our optimisation problem is never active and we end up with an unconstrained optimisation with respect to $\mathbf{\Psi}$. Considering (8) and (9) we get

$$\mathbf{G}_{\text{MMSE}} = \lambda^{-\frac{1}{2}} \cdot \tilde{\mathbf{G}}(\mathbf{\Psi})\Big|_{\mathbf{\Psi}=\mathbf{\Psi}_{\text{MMSE}}} \quad (10)$$

$$\text{where} \quad \mathbf{\Psi}_{\text{MMSE}} = \arg\min_{\mathbf{\Psi}} \epsilon(\mathbf{\Psi}) \quad (11)$$

Note that λ is a function of $\tilde{\mathbf{G}}(\mathbf{\Psi})$ and that solving (11) also implicitly ensures that (6) is met. In the following we investigate the gain allocation (10) with unconstrained optimisation problem (11) for a *linear relaying* (LinRel) architecture as well as for a *linear distributed antenna system* (LDAS).

A. Linear Relaying (LinRel)

In the linear relaying case, the relays are assumed to have perfect *global channel knowledge*, i.e., all relays know the instantaneous uplink channel matrix $\mathbf{H}_{\mathcal{SR}}$ as well as the instantaneous downlink channel matrix $\mathbf{H}_{\mathcal{RD}}$ perfectly. However, they only have *local signal* *knowledge*, which means that they only know the signals they receive. They have no knowledge about the signals at the other relays and can consequently apply a complex gain factor to their locally received signal only. This essentially means that any gain matrix $\mathbf{G}$ for this type of relay cooperation is diagonal. It comprises the $N_{\mathcal{R}}$ gain factors on its main diagonal: $\mathbf{G} = \mathrm{diag}(\mathbf{g})$. In accordance with (10) we can write the vector that contains the complex gain factors which solve the original optimisation problem ((2), (3)) as

$$\mathbf{g}_{\mathrm{MMSE}} = \lambda^{-\frac{1}{2}} \cdot \tilde{\mathbf{g}}_{\mathrm{MMSE}}, \tag{12}$$

where

$$\tilde{\mathbf{g}}_{\mathrm{MMSE}} = (\mathbf{B} \odot \mathbf{A}^*)^{-1} \cdot \mathrm{diag}\left(\mathbf{C}^{\mathrm{H}}\right) \tag{13}$$

and

$$\lambda = \frac{\tilde{\mathbf{g}}_{\mathrm{MMSE}}^{\mathrm{H}} \left(\mathbf{A} \odot \mathbf{I}_{N_{\mathcal{R}}}\right) \tilde{\mathbf{g}}_{\mathrm{MMSE}}}{P_{\mathcal{S}}}. \tag{14}$$

We used the substitutions

$$\mathbf{A} := \mathbf{H}_{\mathcal{SR}} \mathbf{R}_s \mathbf{H}_{\mathcal{SR}}^{\mathrm{H}} + \mathbf{R}_{n_{\mathcal{R}}} = \mathbf{A}^{\mathrm{H}}, \tag{15}$$

$$\mathbf{B} := \mathbf{H}_{\mathcal{RD}}^{\mathrm{H}} \boldsymbol{\Psi}_{\mathrm{MMSE}}^{\mathrm{H}} \boldsymbol{\Psi}_{\mathrm{MMSE}} \mathbf{H}_{\mathcal{RD}} + \mathbf{I}_{N_{\mathcal{R}}} = \mathbf{B}^{\mathrm{H}}, \tag{16}$$

$$\text{and} \quad \mathbf{C} := \mathbf{H}_{\mathcal{SR}} \mathbf{R}_s \boldsymbol{\Psi}_{\mathrm{MMSE}} \mathbf{H}_{\mathcal{RD}}. \tag{17}$$

In the following we distinguish between the case where all destinations use the same multiplication factor γ ($\gamma_m = \gamma \ \forall \ m \in \{1, \ldots, N_{\mathcal{SD}}\}$) and the case where each destination m applies an individual factor γ_m to their received signals (see (2)).

1) Equal γ_m: For the case that all destinations are assigned the same multiplication factor γ we have $\boldsymbol{\Gamma} = \gamma \mathbf{I}_{N_{\mathcal{SD}}}$ and consequently $\boldsymbol{\Psi} = \lambda^{-\frac{1}{2}} \gamma \mathbf{I}_{N_{\mathcal{SD}}}$. Solving (11) delivers

$$\boldsymbol{\Psi}_{\mathrm{MMSE}} = \sqrt{\frac{P_{\mathcal{S}}}{\mathrm{tr}\left(\mathbf{R}_{\mathrm{n}_{\mathcal{D}}}\right)}} \cdot \mathbf{I}_{N_{\mathcal{SD}}}. \tag{18}$$

With (14) we finally get

$$\boldsymbol{\Gamma}_{\mathrm{MMSE}} = \sqrt{\frac{\tilde{\mathbf{g}}_{\mathrm{MMSE}}^{\mathrm{H}} \left(\mathbf{A} \odot \mathbf{I}_{N_{\mathcal{R}}}\right) \tilde{\mathbf{g}}_{\mathrm{MMSE}}}{\mathrm{tr}\left(\mathbf{R}_{\mathrm{n}_{\mathcal{D}}}\right)}} \cdot \mathbf{I}_{N_{\mathcal{SD}}}, \tag{19}$$

where $\tilde{\mathbf{g}}_{\mathrm{MMSE}}$ is calculated according to (13). Note that γ is real-valued. This means that a multiplication with γ at the destinations corresponds to a simple scaling of the received signals.

2) Individual γ_m: Opposed to the case of using the same factor γ for all destinations, we now consider the situation where each destination m features an individual factor γ_m. Recall that by definition $\boldsymbol{\Gamma}_{\mathrm{MMSE}} = \lambda^{\frac{1}{2}} \cdot \boldsymbol{\Psi}_{\mathrm{MMSE}}$, where λ is calculated according to (14). We cannot provide a closed-form solution of (11) for this case yet. Instead, $\boldsymbol{\Psi}_{\mathrm{MMSE}}$ is found by a numerical search algorithm for the moment.

It turns out that the entries of $\boldsymbol{\Psi}_{\mathrm{MMSE}}$ and consequently also of $\boldsymbol{\Gamma}$ are complex numbers. This means that the received signal at each destination is scaled and rotated individually.

B. Linear Distributed Antenna System (LDAS)

Now, all relays are connected via a wired backbone and can thus share their received signals. We say that they have *global signal knowledge*. In contrast to the LinRel architecture, the gain matrix $\mathbf{G}$ does not have to be diagonal for this case. We also assume perfect *global channel knowledge* at all relays. Considering (10) we get

$$\mathbf{G}_{\mathrm{MMSE}} = \lambda^{-\frac{1}{2}} \cdot \tilde{\mathbf{G}}_{\mathrm{MMSE}}, \tag{20}$$

where

$$\tilde{\mathbf{G}}_{\mathrm{MMSE}} = \mathbf{B}^{-1} \mathbf{C}^{\mathrm{H}} \mathbf{A}^{-\mathrm{H}} \tag{21}$$

and

$$\lambda = \frac{\mathrm{tr}\left(\tilde{\mathbf{G}}_{\mathrm{MMSE}} \mathbf{A} \tilde{\mathbf{G}}_{\mathrm{MMSE}}^{\mathrm{H}}\right)}{P_{\mathcal{S}}}. \tag{22}$$

We again used the substitutions (15), (16), and (17). As in section III-A we distinguish between the case where there is a common factor γ at all destinations and the case where the factor γ_m is assigned individually to each destination.

1) Equal γ_m: Solving the unconstrained optimisation (11) for $\boldsymbol{\Psi}$ delivers the same result as in (III-A.1):

$$\boldsymbol{\Psi}_{\mathrm{MMSE}} = \sqrt{\frac{P_{\mathcal{S}}}{\mathrm{tr}\left(\mathbf{R}_{\mathrm{n}_{\mathcal{D}}}\right)}} \cdot \mathbf{I}_{N_{\mathcal{SD}}}. \tag{23}$$

Recalling that $\boldsymbol{\Gamma}_{\mathrm{MMSE}} = \lambda^{\frac{1}{2}} \cdot \boldsymbol{\Psi}_{\mathrm{MMSE}}$ and using (22) and (23) we get

$$\boldsymbol{\Gamma}_{\mathrm{MMSE}} = \sqrt{\frac{\mathrm{tr}\left(\tilde{\mathbf{G}}_{\mathrm{MMSE}} \mathbf{A} \tilde{\mathbf{G}}_{\mathrm{MMSE}}^{\mathrm{H}}\right)}{\mathrm{tr}\left(\mathbf{R}_{\mathrm{n}_{\mathcal{D}}}\right)}} \cdot \mathbf{I}_{N_{\mathcal{SD}}}. \tag{24}$$

The matrix $\tilde{\mathbf{G}}_{\mathrm{MMSE}}$ is calculated according to (21).

2) Individual γ_m: As for the linear relaying architecture, we cannot provide a closed-form solution to (11) for this case either. Consequently, we again find $\boldsymbol{\Psi}_{\mathrm{MMSE}}$ using a numerical search algorithm. The matrix $\boldsymbol{\Gamma}$ can then be computed by

$$\boldsymbol{\Gamma}_{\mathrm{MMSE}} = \lambda^{\frac{1}{2}} \cdot \boldsymbol{\Psi}_{\mathrm{MMSE}}, \tag{25}$$

with λ out of (22).

IV. Performance Results

In this section, we present the results of Monte-Carlo simulations in order to evaluate the performance of the MMSE relaying scheme. We determine the gain matrix $\mathbf{G}$ of the four cases

1) LinRel, equal multiplication factors γ_m
2) LDAS, equal multiplication factors γ_m
3) LinRel, individual multiplication factors γ_m
4) LDAS, individual multiplication factors γ_m

for each channel realisation, and calculate the resulting signal-to-interference-and-noise ratios (SINRs) at each destination. The signal power, noise power, and interference power of source/destination link i with $i = 1, \ldots, N_{\mathrm{SD}}$ are given by

$$P_{\mathrm{Signal}}^{(i)} = \left|\mathbf{H}_{\mathcal{SD}}[i,i]\right|^2 \cdot \sigma_s^2 \tag{26}$$

$$P_{\mathrm{Noise}}^{(i)} = \sigma_{n_{\mathcal{R}}}^2 \cdot \|\mathbf{H}_{\mathcal{RD}}[i,:]\mathbf{G}_{\mathrm{MMSE}}\|_2^2 + \sigma_{n_{\mathcal{D}}}^2 \tag{27}$$

$$P_{\mathrm{Interference}}^{(i)} = \sigma_s^2 \cdot \sum_{j=1, j\neq i}^{N_{\mathcal{SD}}} \left|\mathbf{H}_{\mathcal{SD}}[i,j]\right|^2 \tag{28}$$

where σ_s^2, $\sigma_{n_{\mathcal{R}}}^2$, and $\sigma_{n_{\mathcal{D}}}^2$ denote the signal power of each source, the noise variance at the relays, and the noise variance at the destinations, respectively. The instantaneous sum rate can be calculated according to

$$I_{\mathrm{inst}} = \sum_{i=1}^{N_{\mathcal{SD}}} I_i = \frac{1}{2} \sum_{i=1}^{N_{\mathcal{SD}}} \log_2\left(1 + \mathrm{SINR}_i\right), \tag{29}$$

where SINR_i is the instantaneous SINR for source/destination link i. The factor $\frac{1}{2}$ comes from the fact that the transmission takes place over two timeslots (two-hop traffic pattern). Averaging the sum rate over all channel realisations delivers the average sum rate I_{avg}.

In order to calculate the average sum rate at a defined average signal-to-noise ratio $\mathrm{SNR}_{\mathrm{def}}$, we consider a reference scenario with a single source/destination pair and only one relay in between (denoted by $1 \times 1 \times 1$). The relay transmit power equals σ_s^2 in this case.

We determine the source transmit power which is needed to achieve SNR_{def} on the average by simulation. Then we apply this transmit power to the configuration that is to be evaluated at SNR_{def} and calculate the achieved sum rate.

We compare the performance of the presented multiuser MMSE relaying scheme with that of multiuser ZF relaying [10] to have an additional reference. In the following, MMSE relaying with equal and individual multiplication factors γ_m at the destinations will be denoted by „$\text{MMSE}_{\text{equal}}$"and „$\text{MMSE}_{\text{ind}}$", respectively, and ZF relaying by „ZF". For the simulations we assumed frequency flat Rayleigh fading on every single source-relay and relay-destination link. All channel coefficients are statistically independent and drawn from a complex normal distribution with zero mean and variance σ_{h}^2. The channel matrices are constant during each transmission cycle (block fading) and temporally independent. All relays are assumed to exhibit the same noise variance as do all the destinations.

A. Linear Relaying (LinRel)

In this section we evaluate the linear relaying (LinRel) architecture using multiuser MMSE relaying. On the average, all channel coefficients have unit power: $\sigma_{\text{h}}^2 = 1$ („symmetric links").

In **Fig. 2(a)** the average sum rate I_{avg} for a system featuring $N_{\mathcal{SD}} = 2$ and $N_{\mathcal{SD}} = 4$ source/destination pairs is plotted versus the number of relays $N_{\mathcal{R}}$. For the sake of orientation the average sum rate of a $1 \times 1 \times 1$ reference link, multiplied by 2 and 4, respectively, is also plotted. For $N_{\mathcal{R}} \leq N_{\mathcal{SD}}^2 - N_{\mathcal{SD}}$, $\text{MMSE}_{\text{equal}}$ with its slowly degrading performance towards small number of relays, achieves a higher average sum rate than ZF relaying. This is because the latter is not able to orthogonalise the subchannels below that limit [1]. For larger number of relays, the ZF relaying scheme will at first achieve a higher average sum rate because it favours good links while penalising bad links [1]. In contrast to that, $\text{MMSE}_{\text{equal}}$ always tries to make the equivalent channel matrix to be $\mathbf{H}_{\mathcal{SD}} = \gamma \mathbf{I}_{N_{\mathcal{SD}}}$. This is a very fair scheme as it provides all destinations with the same signal strength. MMSE_{ind} outperforms the other two schemes for any number of relays. Note also that the larger the number of relays, the better the MMSE relaying schemes are able to orthogonalise the equivalent channels. This shows in the fact that the average sum rate for $N_{\mathcal{SD}} = 2$ is larger than for $N_{\mathcal{SD}} = 4$ when the number of relays is small. For large number of relays, the linear relaying schemes achieve a higher average sum rate than 2 and 4 times the reference link, respectively. The reason is that they exploit a distributed array gain. This essentially means that they gain over the $1 \times 1 \times 1$ reference link from an SNR gain due to coherent combining of the signals at the destinations.

Fig. 2(b) shows the cumulative distribution functions (CDFs) of the mutual information for one out of four source/destination pairs ($N_{\mathcal{SD}} = 4$). The number of relays is $N_{\mathcal{R}} = 20$. The small circles show the mean values of the curves. Disregarding the squeezed behaviour around zero mutual information the ZF relaying curve seems to be a right-shifted version of the CDF for the reference link that has essentially the same slope. This hints at an SNR gain of the zero-forcing scheme over the reference scenario without achieving additional diversity. The reason for that is the obtained distributed array gain already mentioned above. Furthermore, we see that with respect to ZF, both $\text{MMSE}_{\text{equal}}$ and MMSE_{ind} exhibit a higher slope. This hints at an additional diversity gain over the other two schemes (ZF relaying and reference case). $\text{MMSE}_{\text{equal}}$ outperforms ZF relaying when considering small outage probabilities. However, on the average (mean value) it performs worse. MMSE_{ind} outperforms the other schemes in terms of outage as well as average sum rate.

B. Linear Distributed Antenna System (LDAS)

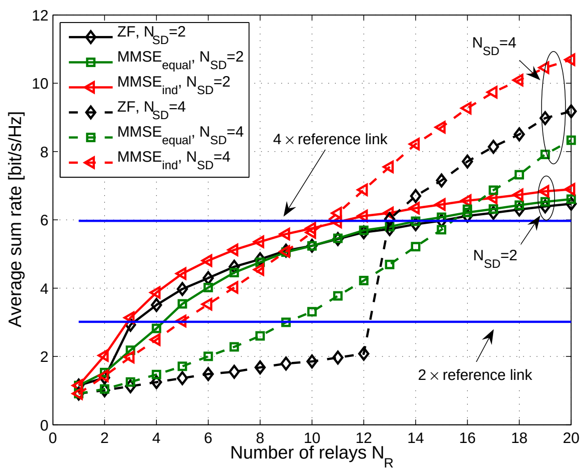
We now investigate the performance of MMSE relaying in an LDAS configuration. In order to highlight the performance gain of MMSE_{ind} over $\text{MMSE}_{\text{equal}}$ and ZF relaying, we consider an asymmetric system topology: The distance of source 1 and its corresponding destination 1 to the relays is assumed to be smaller than that of all other source/destination pairs. To include this in our simulations we choose the channel coefficients from source 1 to all relays and from all relays to destination 1 to be on the average 10 times as large as the other channel coefficients ($\sigma_{\text{h,link 1}}^2 = 100$ while $\sigma_{\text{h,link }i}^2 = 1,\ i \in \{2, .., N_{\mathcal{SD}}\}$; „asymmetric links").

Fig. 3(a) shows the average sum rate of $N_{\mathcal{SD}} = 2$ and $N_{\mathcal{SD}} = 4$ source/destination pairs versus the number of relays $N_{\mathcal{R}}$ for the present asymmetric system configuration. While $N_{\mathcal{R}} \geq N_{\mathcal{SD}}$ $\text{MMSE}_{\text{equal}}$ and ZF relaying perform virtually the same. In fact both schemes produce the same equivalent channel matrix $\mathbf{H}_{\mathcal{SD}}$ when the number of relays is large. When the number of relays is smaller than the number of source/destination pairs, ZF relaying does not work, because it cannot invert the uplink- and downlink channel matrices. As for the linear relaying case, MMSE_{ind} outperforms the other two schemes for any number of relays. As already observed in Section IV-A we see that for a small number of relays, the average sum rate for $N_{\mathcal{SD}} = 2$ is larger than for $N_{\mathcal{SD}} = 4$. The reason again is that the larger the number of relays, the better the schemes are able to orthogonalise the links. In **Fig. 3(b)** the CDFs of the mutual information of link 1 out of 4 links with 20 relays are depicted. The small circles denote the mean values of each distribution. The $1 \times 1 \times 1$ reference link this time denotes a reference system where $\sigma_{\text{h}}^2 = 100$ for both channel coefficients. As expected, $\text{MMSE}_{\text{equal}}$ and ZF relaying behave virtually the same for such a large number of relays. Compared to the linear relaying case, ZF relaying does not achieve a distributed array gain anymore. This is because the gain matrix $\mathbf{G}_{\text{ZF}}$ is simply chosen as product of the Moore-Penrose inverses of the downlink and uplink channel matrices without any power loading [11]. However, compared to the reference case, a large diversity gain is achieved for all three relaying schemes. MMSE_{ind} outperforms the other two schemes in terms of outage as well as mean value. The reason is that, compared to $\text{MMSE}_{\text{equal}}$ and ZF relaying, a distributed array gain additionally increases the SNR at the destinations.

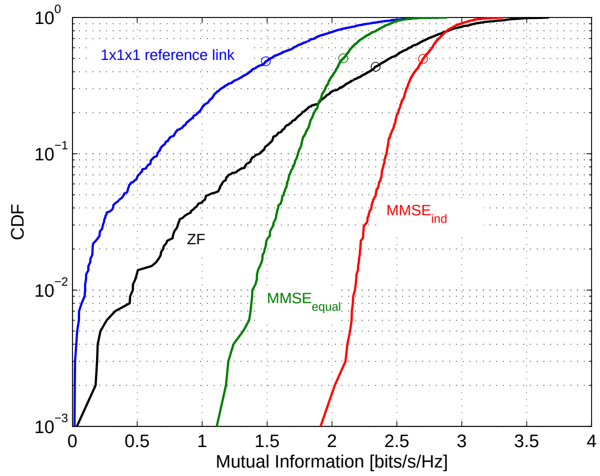
V. Conclusions

We considered a distributed relay network with two-hop traffic pattern. The amplify-and-forward (AF) relaying nodes are organised either in a linear distributed antenna system (LDAS) or a linear relaying (LinRel) architecture. All nodes employ only one antenna. As a main result of this work, the gain factors of the relays were derived such that the mean squared error (MSE) at the destinations is minimised jointly. A complex multiplication factor γ_m at each destination allows for received signals that are a scaled and rotated version of the transmitted symbols. We distinguished between the case where a common multiplication factor γ is used at all destinations and the case where each destination m is assigned an individual factor γ_m. A closed-form solution to the optimisation problem is provided for the first case. For the moment, the solution to the second case is found numerically.

In Monte-Carlo simulations we compared the derived MMSE relaying schemes with multiuser zero-forcing (ZF) relaying and a

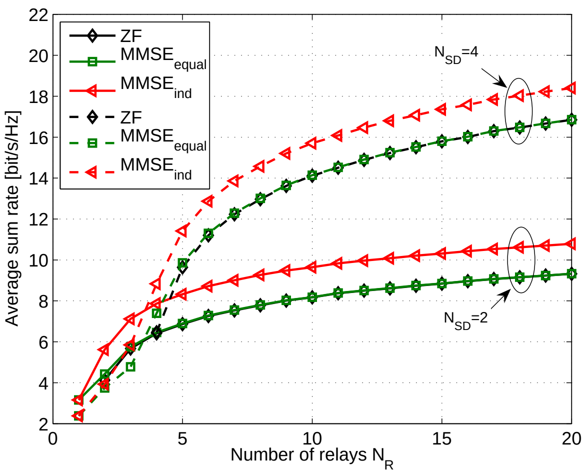


(a) Average sum rate for $N_{\mathcal{SD}} = 2$ and $N_{\mathcal{SD}} = 4$

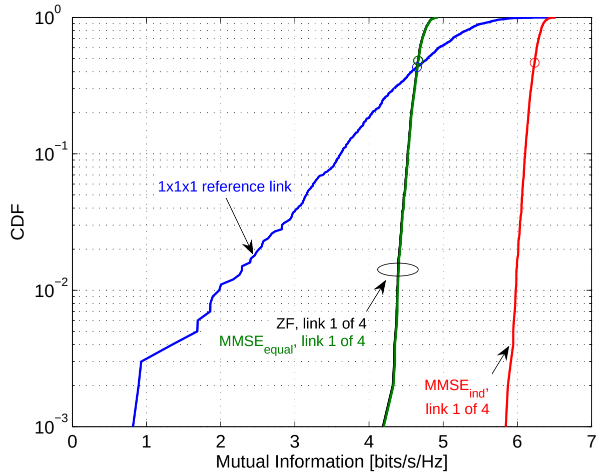


(b) CDF of the mutual information for a single link when $N_{\mathcal{SD}} = 4$

Fig. 2. LinRel scenario with symmetric links at $\mathrm{SNR}_{\mathrm{def}} = 10\,\mathrm{dB}$



(a) Average sum rate for $N_{\mathcal{SD}} = 2$ and $N_{\mathcal{SD}} = 4$



(b) CDF of the mutual information for a single link when $N_{\mathcal{SD}} = 4$

Fig. 3. LDAS scenario with asymmetric links

$1 \times 1 \times 1$ reference scenario. A linear relaying architecture (LinRel) where the relays only exchange channel information, and a linear distributed antenna system (LDAS) where the relays can also share their received signals, was considered. As figure of merit we investigated the average sum rate versus the number of relays and the CDFs of the mutual information of a single out of $N_{\mathcal{SD}}$ source/destination links.

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