Optimum Time-Division in MIMO Two-Way Decode-and-Forward Relaying Systems

Jian Zhao, Marc Kuhn and Armin Wittneben
Communication Technology Laboratory, ETH Zurich
CH-8092 Zurich, Switzerland
Email: {zhao, kuhn, wittneben}@nari.ee.ethz.ch

Gerhard Bauch
DoCoMo Euro-Labs
D-80687 Munich, Germany
Email: bauch@docomolab-euro.com

***Abstract*—We consider a multiple-input multiple-output (MIMO) two-way relaying system, where two wireless terminals exchange data via a half-duplex decode-and-forward (DF) relay. The problem of optimum time-division (TD) between the multiple access (MAC) phase and the broadcast (BRC) phase is addressed in this paper. We characterize the achievable rate regions with optimum TD strategies between the two phases for the following two cases: One is under peak power constraint and the other is under average power constraint. The simulation results show that the achievable rate regions with optimum TD strategies are much larger than those with equal TD strategy. Furthermore, the ergodic sum rates of the system using optimum and equal TD strategies are compared for different antenna configurations.**

I. Introduction

We consider the scenario that two wireless terminal nodes exchange data via a decode-and-forward (DF) relay terminal. Here all the terminals are half-duplex and are equipped with multiple antennas. *Two-way relaying* is a spectral efficient relaying scheme proposed for this scenario. It was shown in [1] that the two-way relaying scheme can recover a significant portion of the spectral efficiency loss that is due to the half-duplex constraint. Compared to traditional relaying schemes where the two terminal nodes transmit and receive data alternatively in four phases, the two-way relaying scheme exchanges the data of the two nodes in just two phases: the multiple access (MAC) phase and the broadcast (BRC) phase. In the MAC phase, the two terminal nodes transmit their data simultaneously to the relay and the relay decodes the received data. After that, the relay combines the decoded data and retransmits them in the BRC phase.

The MAC phase is a conventional multiple access scenario. Its capacity region and optimal coding schemes have been summarized in [2]. The encoding schemes at the relay in the BRC phase has aroused much research interest in recent years. The authors of [1] proposed the *superposition coding scheme*, where the relay remodulates the decoded data separately, adds them up and retransmits them. The receiving terminals cancel the back-propagated data symbols before decoding. A network coding approach, the *XOR scheme*, was considered in [3], where the relay combines the decoded data on the bit level by means of the XOR operation, modulates the XOR-ed data bits and retransmits them. The terminal nodes demodulate the received symbols and reveal the unknown data by XOR-ing the decoded data with their own transmitted data on the bit level. The capacity region and the optimal coding strategies for the BRC phase have been recently characterized in [4] from the information theoretic perspective, where the authors assume that both receiving terminals have perfect side information about the messages intended for the other terminal in the BRC phase.

The achievable information rates of the two-way relaying system are subject to the constraints of both the MAC phase and the BRC phase. Up to now, most papers, e.g., [5], allocate equal time lengths to the MAC and BRC phases. The author of [6], [7] considered the problem of finding the optimum time-division (TD) strategies between the two phases, and solved it for systems with single antenna. However, how to determine the optimum TD strategies for such a system with multiple antennas, so that the achievable rate region is maximized, has not yet been solved. In this case, both the TD strategies and the transmit signal covariance matrices have to be optimized if all the terminals have channel knowledge.

In this paper, we propose algorithms for calculating the achievable rate regions with optimum TD strategies between the MAC and BRC phases for the following two cases: One is under peak power constraint and the other is under average power constraint. The increase of the achievable rate regions compared to the equal TD case is shown by simulations.

This paper is organized as follows: The system model and the capacity regions of the MAC and BRC phases are shown in Section II. An algorithm to find the optimum TD strategy under peak power constraint is presented in Section III. Section IV presents an algorithm to find the optimum TD strategy under average power constraint. Comprehensive simulation results are presented in Section V, where the achievable rate regions and the ergodic sum rates of two-way relaying systems with optimum TD strategies are compared to those with equal TD strategy. After that, conclusions are drawn in Section VI.

Notation We use bold uppercase letters to denote matrices and bold lowercase letters to denote vectors. $\mathbf{\Omega} \succeq 0$ means $\mathbf{\Omega}$ is a positive semidefinite matrix. $\mathbf{I}_N$ is an $N \times N$ identity matrix. $\mathbb{C}^{M \times N}$ denotes the space of $M \times N$ matrices with complex entries. $\mathcal{CN}(0, \mathbf{K})$ denotes a circularly symmetric complex normal zero mean random vector with covariance matrix $\mathbf{K}$. Furthermore, $\mathbb{E}[\cdot]$, $\mathrm{tr}(\cdot)$, $\det(\cdot)$, $(\cdot)^T$ and $(\cdot)^H$ denote the expectation, the trace, the determinant, the transpose

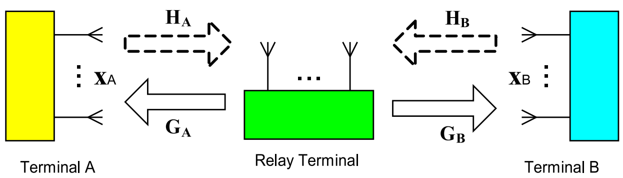

Fig. 1. MIMO two-way DF relaying system. The dashed arrows and solid arrows represent the transmissions in the MAC and BRC phases, respectively.

and the conjugate transpose, respectively.

II. System Model

We consider a two-way DF relaying system as shown in Fig. 1, where the number of antennas at terminal A, the relay and terminal B are denoted as N_A, N_R and N_B, respectively. Each terminal operates in time division duplex (TDD) mode, i.e., it transmits and receives data consecutively in time. $\mathbf{H}_k \in \mathbb{C}^{N_\mathsf{R}\times N_k}$ and $\mathbf{G}_k \in \mathbb{C}^{N_k \times N_\mathsf{R}}$, where $k \in \{\mathsf{A},\mathsf{B}\}$, denote the channel matrices between terminal k and the relay in the MAC and BRC phases, respectively. All the channels are frequency-flat and remain constant during its corresponding transmission phase. Both the transmitters and receivers have their corresponding channel knowledge. We define the *time-division (TD) factor* α as the portion of the total transmission time assigned to the MAC phase as shown in Fig. 2, where $0 \le \alpha \le 1$. The BRC phase occupies $1-\alpha$ portion of the total transmission time. The transmit power constraints at terminal A, the relay and terminal B are P_A, P_R and P_B, respectively. We distinguish between two cases: One is *peak power constraint* where the actual transmit power in the MAC and BRC phases cannot exceed P_k, $k \in \{\mathsf{A},\mathsf{B},\mathsf{R}\}$; the other is *average power constraint* where the transmitter k can vary its actual transmit power in the MAC and BRC phases according to α if the transmit power averaged over the total transmission time does not exceed P_k, $k \in \{\mathsf{A},\mathsf{B},\mathsf{R}\}$. In the following, we use P_k^* to denote the actual transmit power in the MAC or BRC phase for transmitter k. Furthermore, we use R_A to denote the information rate of the data to be transmitted from terminal A to B, and use R_B to denote the information rate of the data to be transmitted from terminal B to A.

In the MAC phase, terminal A and B transmit their data simultaneously to the relay, and the relay decodes the received data. The received signal $\mathbf{y} \in \mathbb{C}^{N_\mathsf{R}\times 1}$ at the relay in the MAC phase is

$$\mathbf{y} = \sqrt{\frac{P_\mathsf{A}^*}{N_\mathsf{A}}}\mathbf{H}_\mathsf{A}\mathbf{x}_\mathsf{A} + \sqrt{\frac{P_\mathsf{B}^*}{N_\mathsf{B}}}\mathbf{H}_\mathsf{B}\mathbf{x}_\mathsf{B} + \mathbf{n}, \tag{1}$$

where $P_k^* = P_k$ under peak power constraint and $P_k^* = P_k/\alpha$ under average power constraint for $k \in \{\mathsf{A},\mathsf{B}\}$. $\mathbf{x}_\mathsf{A} \in \mathbb{C}^{N_\mathsf{A}\times 1}$ and $\mathbf{x}_\mathsf{B} \in \mathbb{C}^{N_\mathsf{B}\times 1}$ are the transmit signal vectors at terminal A and B in the MAC phase, respectively. We denote $\mathbf{\Omega}_\mathsf{A} = \mathbb{E}(\mathbf{x}_\mathsf{A}\mathbf{x}_\mathsf{A}^H)$ and $\mathbf{\Omega}_\mathsf{B} = \mathbb{E}(\mathbf{x}_\mathsf{B}\mathbf{x}_\mathsf{B}^H)$. In order to satisfy the power constraint, we have $\mathrm{tr}(\mathbf{\Omega}_\mathsf{A}) \le N_\mathsf{A}$ and $\mathrm{tr}(\mathbf{\Omega}_\mathsf{B}) \le N_\mathsf{B}$. $\mathbf{n} \sim \mathcal{CN}(0, \sigma^2\mathbf{I}_{N_\mathsf{R}})$ is the additive white Gaussian noise (AWGN) at the relay in the MAC phase.

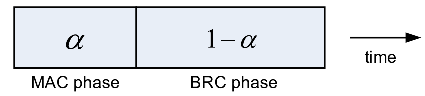

Fig. 2. Time-division between the MAC and BRC phases

Given the TD factor α, the MAC phase capacity region $\mathcal{C}_{\mathrm{MAC}}(\alpha) = (R_\mathsf{A}, R_\mathsf{B})$ can be characterized as [2]

$$R_\mathsf{A} \le \alpha \log_2 \det\left(\mathbf{I}_{N_\mathsf{R}} + \frac{P_\mathsf{A}^*}{N_\mathsf{A}\sigma^2}\mathbf{H}_\mathsf{A}\mathbf{\Omega}_\mathsf{A}\mathbf{H}_\mathsf{A}^H\right), \tag{2}$$

$$R_\mathsf{B} \le \alpha \log_2 \det\left(\mathbf{I}_{N_\mathsf{R}} + \frac{P_\mathsf{B}^*}{N_\mathsf{B}\sigma^2}\mathbf{H}_\mathsf{B}\mathbf{\Omega}_\mathsf{B}\mathbf{H}_\mathsf{B}^H\right), \tag{3}$$

$$\sum_{k\in\{\mathsf{A},\mathsf{B}\}} R_k \le \alpha \log_2 \det\left(\mathbf{I}_{N_\mathsf{R}} + \sum_{k\in\{\mathsf{A},\mathsf{B}\}} \frac{P_k^*}{N_k\sigma^2}\mathbf{H}_k\mathbf{\Omega}_k\mathbf{H}_k^H\right), \tag{4}$$

where $\mathrm{tr}(\mathbf{\Omega}_k) \le N_k$ and $\mathbf{\Omega}_k \succeq 0$ for $k \in \{\mathsf{A},\mathsf{B}\}$. For simplicity reasons, we denote the special case $\mathcal{C}_{\mathrm{MAC}}(\alpha = 1)$ as $\mathcal{C}_{\mathrm{MAC}}(1)$.

In the BRC phase, the relay combines the decoded data into the data symbol vector $\mathbf{x} \in \mathbb{C}^{N_\mathsf{R}\times 1}$ and sends it to the two terminals. The received signal vectors at terminal A and B are $\mathbf{y}_\mathsf{A} \in \mathbb{C}^{N_\mathsf{A}\times 1}$ and $\mathbf{y}_\mathsf{B} \in \mathbb{C}^{N_\mathsf{B}\times 1}$, respectively. We have

$$\mathbf{y}_\mathsf{A} = \sqrt{\frac{P_\mathsf{R}^*}{N_\mathsf{R}}}\mathbf{G}_\mathsf{A}\mathbf{x} + \mathbf{n}_\mathsf{A}, \tag{5}$$

$$\mathbf{y}_\mathsf{B} = \sqrt{\frac{P_\mathsf{R}^*}{N_\mathsf{R}}}\mathbf{G}_\mathsf{B}\mathbf{x} + \mathbf{n}_\mathsf{B}, \tag{6}$$

where $P_\mathsf{R}^* = P_\mathsf{R}$ under peak power constraint and $P_\mathsf{R}^* = P_\mathsf{R}/(1-\alpha)$ under average power constraint. $\mathbf{n}_\mathsf{A} \sim \mathcal{CN}(0, \sigma_\mathsf{A}^2\mathbf{I}_{N_\mathsf{A}})$ and $\mathbf{n}_\mathsf{B} \sim \mathcal{CN}(0, \sigma_\mathsf{B}^2\mathbf{I}_{N_\mathsf{B}})$ are the AWGN at the receivers of terminal A and B, respectively. The transmit signal covariance matrix at the relay is denoted as $\mathbf{\Omega} = \mathbb{E}(\mathbf{x}\mathbf{x}^H)$. Furthermore, we require $\mathrm{tr}(\mathbf{\Omega}) \le N_\mathsf{R}$ in order to satisfy the power constraint at the relay. Assuming both terminals A and B have perfect side information about the messages intended for the other terminal, the capacity region of the BRC phase $\mathcal{C}_{\mathrm{BRC}}(\alpha) = (R_\mathsf{A}, R_\mathsf{B})$ can be characterized as [4]

$$R_\mathsf{A} \le (1-\alpha) \log_2 \det\left(\mathbf{I}_{N_\mathsf{B}} + \frac{P_\mathsf{R}^*}{N_\mathsf{R}\sigma_\mathsf{B}^2}\mathbf{G}_\mathsf{B}\mathbf{\Omega}\mathbf{G}_\mathsf{B}^H\right), \tag{7}$$

$$R_\mathsf{B} \le (1-\alpha) \log_2 \det\left(\mathbf{I}_{N_\mathsf{A}} + \frac{P_\mathsf{R}^*}{N_\mathsf{R}\sigma_\mathsf{A}^2}\mathbf{G}_\mathsf{A}\mathbf{\Omega}\mathbf{G}_\mathsf{A}^H\right), \tag{8}$$

where $\mathrm{tr}(\mathbf{\Omega}) \le N_\mathsf{R}$ and $\mathbf{\Omega} \succeq 0$. For simplicity reasons, we denote the special case $\mathcal{C}_{\mathrm{BRC}}(\alpha = 0)$ as $\mathcal{C}_{\mathrm{BRC}}(0)$.

$\mathcal{C}_{\mathrm{MAC}}(\alpha) \bigcap \mathcal{C}_{\mathrm{BRC}}(\alpha)$ depicts the achievable rate region of the two-way relaying system for the given TD factor α. Considering all possible TD factors, we have the following achievable rate region of the two-way relaying system

$$\mathcal{R}_{\mathrm{OPT}} = \bigcup_{0\le\alpha\le 1}\left(\mathcal{C}_{\mathrm{MAC}}(\alpha)\bigcap\mathcal{C}_{\mathrm{BRC}}(\alpha)\right). \tag{9}$$

Here we restrict ourselves to the case that the transmit signal covariance matrices $\mathbf{\Omega}_\mathsf{A}$, $\mathbf{\Omega}_\mathsf{B}$ and $\mathbf{\Omega}$ remain unchanged in their corresponding transmission phase. Furthermore, we have the following conjecture:

Conjecture 1: $\mathcal{R}_{\rm OPT}$ is always convex for Gaussian multiple-input multiple-output (MIMO) two-way relaying channels.

It has been proved in [7] that $\mathcal{R}_{\rm OPT}$ is always convex for two-way relaying systems with only single-antenna terminals under both peak and average power constraints. Whether $\mathcal{R}_{\rm OPT}$ is always convex in MIMO two-way relaying systems is still an open problem to the best of our knowledge. If $\mathcal{R}_{\rm OPT}$ is not always convex, *time-sharing* between two TD strategies is further required to achieve certain boundary points of $\mathcal{C}_{\rm OPT} = \mathbf{conv}\mathcal{R}_{\rm OPT}$, where **conv** denotes the convex hull. However, this time-sharing strategy requires the transmit signal covariance matrices $\mathbf{\Omega}_{\sf A}$, $\mathbf{\Omega}_{\sf B}$ and $\mathbf{\Omega}$ to be changed in different time-sharing phases, which is not considered in this paper.

Before we present algorithms in Section III–IV to determine $\mathcal{R}_{\rm OPT}$ regardless of its convexity, we first consider the case that the rate region $\mathcal{R}_{\rm OPT}$ is convex. In this case, each of its boundary points can be determined by solving the following optimization problem (**Q0**):

$$\begin{array}{ll} \text{maximize} & \lambda R_{\sf A} + (1-\lambda) R_{\sf B} \\ \text{subject to} & (R_{\sf A}, R_{\sf B}) \in \mathcal{C}_{\rm MAC}(\alpha) \bigcap \mathcal{C}_{\rm BRC}(\alpha) \\ & 0 \le \alpha \le 1 \\ \text{variables} & R_{\sf A}, R_{\sf B}, \alpha, \mathbf{\Omega}_{\sf A}, \mathbf{\Omega}_{\sf B}, \mathbf{\Omega}, \end{array}$$

where λ is a weighting constant and $0 \le \lambda \le 1$. After solving the problem **Q0** for all possible values of λ, the boundary of the rate region $\mathcal{R}_{\rm OPT}$ can be fully characterized.

For a fixed weighting constant λ, the problem **Q0** can be solved by decomposition methods [8]: Firstly, we observe that for a given TD factor α, where $0 \le \alpha \le 1$, the following is a convex optimization problem (**Q1**):

$$\begin{array}{ll} \text{maximize} & \lambda R_{\sf A} + (1-\lambda) R_{\sf B} \\ \text{subject to} & (R_{\sf A}, R_{\sf B}) \in \mathcal{C}_{\rm MAC}(\alpha) \bigcap \mathcal{C}_{\rm BRC}(\alpha) \\ & \alpha \text{ is given} \\ \text{variables} & R_{\sf A}, R_{\sf B}, \mathbf{\Omega}_{\sf A}, \mathbf{\Omega}_{\sf B}, \mathbf{\Omega}. \end{array}$$

This is because $\mathcal{C}_{\rm MAC}(\alpha)$ and $\mathcal{C}_{\rm BRC}(\alpha)$ are both convex regions for the fixed value α. Their intersection is thus also convex. For the fixed α, we denote the optimum objective value of **Q1** as $r(\alpha) = \lambda R^\star_{\sf A}(\alpha) + (1-\lambda) R^\star_{\sf B}(\alpha)$. Since **Q1** is a convex optimization problem, $r(\alpha)$ can be determined efficiently using convex optimization methods, e.g., interior-point methods [9]. That is, $r(\alpha)$ can be considered as a function of the TD factor α and the value of $r(\alpha)$ can be calculated efficiently for each given value of α. Secondly, we solve the following problem (**Q2**):

$$\begin{array}{ll} \text{maximize} & r(\alpha) = \lambda R^\star_{\sf A}(\alpha) + (1-\lambda) R^\star_{\sf B}(\alpha) \\ \text{subject to} & 0 \le \alpha \le 1 \\ \text{variables} & \alpha. \end{array}$$

Since there is only one variable α in **Q2**, it can be solved by using the *bisection* method or the *subgradient* method [8]. The solution of α and the optimum objective value of **Q2** are also respectively the optimum TD factor and the optimum objective value of **Q0** for the given weighting constant λ. In this way, the original problem **Q0** is decomposed into two subproblems **Q1** and **Q2**. However, this decomposition method is slow in practice and is subject to certain implementation constraints [10].

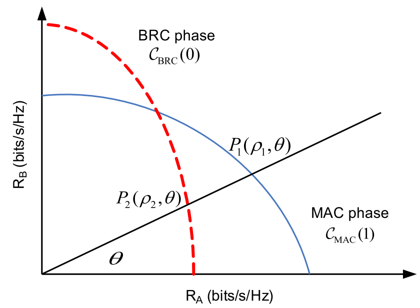

Fig. 3. The MAC and BRC capacity regions $\mathcal{C}_{\rm MAC}(1)$ and $\mathcal{C}_{\rm BRC}(0)$ of a two-way relaying system. The ray θ intersects with $\mathcal{C}_{\rm MAC}(1)$ and $\mathcal{C}_{\rm BRC}(0)$ on the point P_1 and P_2, respectively. The two points are indicated in polar coordinates.

In the following, we propose two implementable algorithms to determine the boundary points of the achievable rate region $\mathcal{R}_{\rm OPT}$ regardless of its convexity. They find both the optimum values of α and the signal covariance matrices for each boundary point of $\mathcal{R}_{\rm OPT}$. Section III and Section IV consider the problem under peak and average power constraints, respectively.

III. Optimum Time-Division Under Peak Power Constraint

Under peak power constraint, we have $P^*_k = P_k$ for $k \in \{{\sf A}, {\sf R}, {\sf B}\}$ in (2)–(4) and (7)–(8). Fig. 3 shows the capacity regions $\mathcal{C}_{\rm MAC}(1)$ and $\mathcal{C}_{\rm BRC}(0)$ of a two-way relaying system. For a given TD factor α, the boundary points of $\mathcal{C}_{\rm MAC}(1)$ and $\mathcal{C}_{\rm BRC}(0)$ are scaled respectively by the factors of α and $1-\alpha$ along the line through the origin to get the boundary points of $\mathcal{C}_{\rm MAC}(\alpha)$ and $\mathcal{C}_{\rm BRC}(\alpha)$. We can write $\mathcal{C}_{\rm MAC}(\alpha) = \alpha\mathcal{C}_{\rm MAC}(1)$ and $\mathcal{C}_{\rm BRC}(\alpha) = (1-\alpha)\mathcal{C}_{\rm BRC}(0)$. It is more convenient for us to use polar coordinates to represent the rate pair $(R_{\sf A}, R_{\sf B})$ in our discussions of this section.

The algorithm to characterize the boundary points of the achievable rate region $\mathcal{R}_{\rm OPT}$ under peak power constraint consists of the following two steps: Firstly, for a given angle θ, we determine the two intersection points $P_1(\rho_1, \theta)$ and $P_2(\rho_2, \theta)$ on the boundaries of $\mathcal{C}_{\rm MAC}(1)$ and $\mathcal{C}_{\rm BRC}(0)$ as shown in Fig. 3, where they are indicated by polar coordinates. That will be discussed in Section III-A. Secondly, we calculate the optimum TD factor $\alpha^\star$ between P_1 and P_2, and get the boundary point on $\mathcal{R}_{\rm OPT}$ for the given angle θ. That will be discussed in Section III-B. Without causing confusions, we use θ to denote both the angle and the ray that forms the angle with the $R_{\sf A}$-axis.

A. Boundary Point on Capacity Regions for Given θ

The capacity regions $\mathcal{C}_{\rm MAC}(1)$ and $\mathcal{C}_{\rm BRC}(0)$ are both convex. Efficient algorithms for calculating the maximum

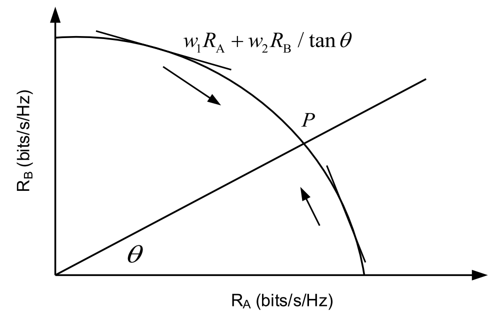

Fig. 4. Finding the point P where the ray θ intersects with the capacity region $\mathcal{C}$'s boundary

weighted sum rate $\sum_k \mu_k R_k$, $k \in \{\mathsf{A}, \mathsf{B}\}$, for the MAC phase $\mathcal{C}_{\mathrm{MAC}}(1)$ are available in e.g., [11], [12]. Here μ_k, $k \in \{\mathsf{A}, \mathsf{B}\}$ are non-negative weighting constants. Similar algorithms can be applied to calculate the maximum weighted sum rate for the BRC phase $\mathcal{C}_{\mathrm{BRC}}(0)$. Let $\mathcal{C}$ represent the capacity region $\mathcal{C}_{\mathrm{MAC}}(1)$ or $\mathcal{C}_{\mathrm{BRC}}(0)$. For a given angle $0 \leq \theta \leq \pi/2$ as shown in Fig. 4, we denote the intersection point of the ray θ with the boundary of the capacity region $\mathcal{C}$ as P. Since $\mathcal{C}$ is convex, there is only one intersection point. Furthermore, the following lemma can be utilized to determine the intersection point P.

Lemma 2 ([13]): Let w denote the weighting factor, where $0 \leq w \leq 1$. The intersection point $P = (R^\star_{\mathsf{A}}, R^\star_{\mathsf{B}})$ of the ray θ with the boundary of the capacity region $\mathcal{C}$ satisfies

$$R^\star_{\mathsf{A}} = \frac{R^\star_{\mathsf{B}}}{\tan\theta} \tag{10}$$

$$= \max_{(R_{\mathsf{A}}, R_{\mathsf{B}}) \in \mathcal{C}} \min\left(R_{\mathsf{A}}, \frac{R_{\mathsf{B}}}{\tan\theta}\right) \tag{11}$$

$$= \max_{(R_{\mathsf{A}}, R_{\mathsf{B}}) \in \mathcal{C}} \min_{0 \leq w \leq 1} \left[w R_{\mathsf{A}} + (1-w) \frac{R_{\mathsf{B}}}{\tan\theta} \right] \tag{12}$$

$$= \min_{0 \leq w \leq 1} \max_{(R_{\mathsf{A}}, R_{\mathsf{B}}) \in \mathcal{C}} \left[w R_{\mathsf{A}} + (1-w) \frac{R_{\mathsf{B}}}{\tan\theta} \right]. \tag{13}$$

Furthermore, we have $\forall w$,

$$w R^\star_{\mathsf{A}} + (1-w) \frac{R^\star_{\mathsf{B}}}{\tan\theta} = R^\star_{\mathsf{A}} = \frac{R^\star_{\mathsf{B}}}{\tan\theta}. \tag{14}$$

Proof: Equation (10) is because the point $P = (R^\star_{\mathsf{A}}, R^\star_{\mathsf{B}})$ is on the ray θ. Equation (11) is due to the fact that the ray θ divides the first quadrant into two sectors. In the sector below the ray θ, $R_{\mathsf{B}}/R_{\mathsf{A}} \leq \tan\theta$ and thus $R_{\mathsf{B}}/\tan\theta \leq R_{\mathsf{A}}$. In that sector, $\min(R_{\mathsf{A}}, R_{\mathsf{B}}/\tan\theta) = R_{\mathsf{B}}/\tan\theta$ and (11) is equivalent to maximizing R_{B}, which is achieved by the intersection point $P = (R^\star_{\mathsf{A}}, R^\star_{\mathsf{B}})$. The same argument also applies to the sector above the ray θ. Equation (12) follows from the fact that $wx + (1-w)y \geq (w + 1 - w)\min(x, y) = \min(x, y)$, $\forall x, y \geq 0$ and $0 \leq w \leq 1$. The equality is achieved when

$$w = \begin{cases} 1 & \text{if } x \leq y \\ 0 & \text{if } x > y. \end{cases}$$

Thus $\min_{0 \leq w \leq 1}[wx + (1-w)y] = \min(x, y)$. Equation (13) follows from Fan's Minimax Theorem [14]. This is because the capacity region $\mathcal{C}$ and the set $W = \{w | 0 \leq w \leq 1\}$ are both convex and compact. Moreover, the function $f(w, R_{\mathsf{A}}, R_{\mathsf{B}}) = wR_{\mathsf{A}} + (1-w)R_{\mathsf{B}}/\tan\theta$ is continuous and linear on the set W and $\mathcal{C}$. So the strong max-min property holds. ■

The algorithm to determine the intersection point of the given ray θ with the boundary of the capacity region $\mathcal{C}$ is presented in Algorithm 1. Here we utilize the bisection method [8] to determine the optimum weighting factor w.

Algorithm 1 Calculating the intersection point $P = (R^\star_{\mathsf{A}}, R^\star_{\mathsf{B}})$ for the given ray θ

Initialize $w^{\min} = 0$, $w^{\max} = 1$ and $w = (w^{\max} + w^{\min})/2$.
repeat
Determine the rate pair $(R_{\mathsf{A}}, R_{\mathsf{B}}) \in \mathcal{C}$ that maximizes the weighted sum rate using the algorithms described in [11], [12], i.e., determine

$$(R_{\mathsf{A}}(w), R_{\mathsf{B}}(w)) = \arg\max_{(R_{\mathsf{A}}, R_{\mathsf{B}}) \in \mathcal{C}} wR_{\mathsf{A}} + (1-w)\frac{R_{\mathsf{B}}}{\tan\theta}.$$

if $R_{\mathsf{A}}(w) < R_{\mathsf{B}}(w)/\tan\theta$ **then**
$w^{\min} = w$; $w^{\max}$ unchanged; $w = (w^{\max} + w^{\min})/2$;
else if $R_{\mathsf{A}}(w) > R_{\mathsf{B}}(w)/\tan\theta$ **then**
$w^{\max} = w$; $w^{\min}$ unchanged; $w = (w^{\max} + w^{\min})/2$;
end if
until $|R_{\mathsf{A}} - R_{\mathsf{B}}/\tan\theta| < \epsilon$ or $|w^{\max} - w^{\min}| < \zeta$
return $R^\star_{\mathsf{A}} = wR_{\mathsf{A}}(w) + (1-w)R_{\mathsf{B}}(w)/\tan\theta$, $R^\star_{\mathsf{B}} = R^\star_{\mathsf{A}}\tan\theta$.

B. Calculating the Optimum Time-Division Factor

For a given angle θ, Algorithm 1 can determine the intersection points $P_1 = (R^\star_{\mathsf{A}1}, R^\star_{\mathsf{B}1})$ and $P_2 = (R^\star_{\mathsf{A}2}, R^\star_{\mathsf{B}2})$ of the ray θ with the capacity region boundaries $\mathcal{C}_{\mathrm{MAC}}(1)$ and $\mathcal{C}_{\mathrm{BRC}}(0)$. In order to calculate the optimum TD factor between P_1 and P_2, we first convert them into polar coordinate representations $P_1(\rho_1, \theta)$ and $P_2(\rho_2, \theta)$ as shown in Fig. 3. The following lemma can be utilized to determine the optimum TD factor $\alpha^\star$ between P_1 and P_2:

Lemma 3: We assume the ray θ intersects with the boundaries of $\mathcal{C}_{\mathrm{MAC}}(1)$ and $\mathcal{C}_{\mathrm{BRC}}(0)$ respectively on $P_1(\rho_1, \theta)$ and $P_2(\rho_2, \theta)$. The point $(\rho^\star, \theta)$ in polar coordinates is on the boundary of $\mathcal{R}_{\mathrm{OPT}}$ with optimum TD factor $\alpha^\star$, where

$$\rho^\star = \frac{\rho_1\rho_2}{\rho_1 + \rho_2} \tag{15}$$

$$\alpha^\star = \frac{\rho_2}{\rho_1 + \rho_2}. \tag{16}$$

The transmit covariance matrices of the MAC and BRC phases for that point are the corresponding ones obtained for P_1 and P_2, respectively.

Proof: For a given TD factor α, the point $Q(\rho(\alpha), \theta)$ represented in polar coordinates is on the boundary of the rate region $\mathcal{C}_{\mathrm{MAC}}(\alpha) \bigcap \mathcal{C}_{\mathrm{BRC}}(\alpha)$, where $\rho(\alpha) = \min(\alpha\rho_1, (1-\alpha)\rho_2)$. $\rho(\alpha)$ is maximized when $\alpha^\star\rho_1 = (1-\alpha^\star)\rho_2$, i.e., when $\alpha^\star = \rho_2/(\rho_1 + \rho_2)$. This is the same result as for two-way relaying systems with single antenna [7]. When we choose that TD factor $\alpha^\star$, the corresponding $\rho(\alpha^\star) = \rho_1\rho_2/(\rho_1 + \rho_2)$, which is the maximum value that $\rho(\alpha)$ can achieve for $0 \le \alpha \le 1$. ■

In summary, we have Algorithm 2 to calculate the boundary point of $\mathcal{R}_{\rm OPT}$ for the given angle θ.

Algorithm 2 Calculating the boundary point of $\mathcal{R}_{\rm OPT}$ for the given angle θ

1: For given θ, Use Algorithm 1 to determine the intersection points of the ray θ with $\mathcal{C}_{\rm MAC}(1)$ and $\mathcal{C}_{\rm BRC}(0)$;
2: Use (15) and (16) to determine the boundary point in polar coordinate $(\rho^\star, \theta)$.

Evaluating every boundary point of $\mathcal{R}_{\rm OPT}$ for $0 \le \theta \le \pi/2$ yields the whole boundary of the achievable rate region $\mathcal{R}_{\rm OPT}$.

IV. Optimum Time-Division Under Average Power Constraint

Under average power constraint, $\mathcal{C}_{\rm MAC}(\alpha)$ and $\mathcal{C}_{\rm BRC}(\alpha)$ are not simply the scaled versions of $\mathcal{C}_{\rm MAC}(1)$ and $\mathcal{C}_{\rm BRC}(0)$. In order to find the optimum TD strategies under average power constraint, we first prove the following lemma:

Lemma 4: For any given covariance matrices $\mathbf{\Omega}_{\sf A}$, $\mathbf{\Omega}_{\sf B}$ and $\mathbf{\Omega}$, the right-hand sides (RHS) of (2)–(4) are monotonically increasing functions of α, and the RHS of (7)–(8) are monotonically decreasing functions of α.

Proof: For any given transmit signal covariance matrix $\mathbf{\Omega}_{\sf A}$, the RHS of (2) under average power constraint can be written as

$$\alpha \log_2 \det\left(\mathbf{I}_{N_{\sf R}} + \frac{P_{\sf A}}{\alpha N_{\sf A}\sigma^2}\mathbf{H}_{\sf A}\mathbf{\Omega}_{\sf A}\mathbf{H}_{\sf A}^H\right) = \sum_{i=1}^{N_{\sf R}} \alpha \log_2(1+\frac{\lambda_i}{\alpha}),$$

where $\lambda_i \ge 0$, $i = 1, \cdots, N_{\sf R}$, denote the eigenvalues of $\frac{P_{\sf A}}{N_{\sf A}\sigma^2}\mathbf{H}_{\sf A}\mathbf{\Omega}_{\sf A}\mathbf{H}_{\sf A}^H$. $\forall \lambda_i \ge 0$, $f(\alpha) = \alpha\log_2(1 + \lambda_i/\alpha)$ is a monotonically increasing function of α when $0 \le \alpha \le 1$. Thus the RHS of (2) is a monotonically increasing function of α. The same discussion also applies to (3)–(4). Since the RHS of (7)–(8) are monotonically increasing functions of $1 - \alpha$, they are monotonically decreasing functions of α. ■

This lemma implies that the region $\mathcal{C}_{\rm MAC}(\alpha)$ swells, while the region $\mathcal{C}_{\rm BRC}(\alpha)$ diminishes as α increases. In this section, we propose an algorithm to determine the boundary points of $\mathcal{R}_{\rm OPT}$ under average power constraint. We first define

$$t_{\max} = \max_{{\rm tr}(\mathbf{\Omega})\le N_{\sf R}, \mathbf{\Omega}\succeq 0} \log_2 \det\left(\mathbf{I}_{N_{\sf A}} + \frac{P_{\sf R}}{N_{\sf R}\sigma_{\sf A}^2}\mathbf{G}_{\sf A}\mathbf{\Omega}\mathbf{G}_{\sf A}^H\right),$$

where $t_{\max}$ is the maximum possible value of $R_{\sf B}$ in the BRC phase. For a given value t, where $0 \le t \le t_{\max}$, the proposed algorithm finds the optimum value $p^\star$ and $\alpha^\star$ such that $(p^\star, t)$ is on the boundary of $\mathcal{R}_{\rm OPT}$ under average power constraint, and the TD factor for that point is $\alpha^\star$, i.e., $(p^\star, t) \in \mathcal{C}_{\rm MAC}(\alpha^\star) \bigcap \mathcal{C}_{\rm BRC}(\alpha^\star)$. The idea of the algorithm is shown in Fig. 5. For each value of the TD factor α, we can determine the point $(p(\alpha), t)$ on the boundary of $\mathcal{C}_{\rm BRC}(\alpha)$ for

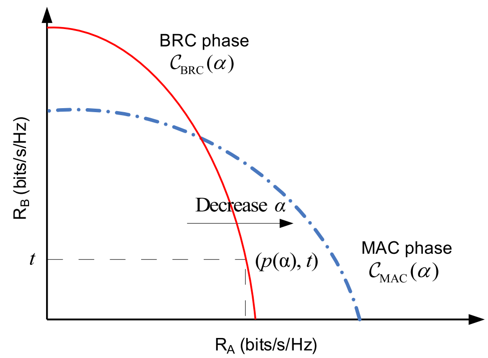

Fig. 5. For a given value t and TD factor α, calculate the value $p(\alpha)$ such that $(p(\alpha), t)$ is on the boundary of $\mathcal{C}_{\rm MAC}(\alpha)$. Decreasing α expands $\mathcal{C}_{\rm BRC}(\alpha)$ and shrinks $\mathcal{C}_{\rm MAC}(\alpha)$.

the given value t. Since $(p(\alpha), t) \in \mathcal{C}_{\rm BRC}(\alpha)$ is satisfied, $p^\star$ is the largest *achievable* value of $p(\alpha)$, i.e.,

$$p^\star = \max_{0\le\alpha\le 1} \{p(\alpha)|(p(\alpha), t) \in \mathcal{C}_{\rm MAC}(\alpha)\}. \quad (17)$$

For the given value of α, we can determine whether $(p(\alpha), t) \in \mathcal{C}_{\rm MAC}(\alpha)$. If $(p(\alpha), t) \in \mathcal{C}_{\rm MAC}(\alpha)$, a smaller TD factor α' can be chosen with increased $p(\alpha')$ while still keeping the point $(p(\alpha'), t)$ to be inside $\mathcal{C}_{\rm MAC}(\alpha')$. Otherwise, the present value of α is too small and should be increased. The fact that decreasing the TD factor α expands $\mathcal{C}_{\rm BRC}(\alpha)$ and shrinks $\mathcal{C}_{\rm MAC}(\alpha)$ is due to Lemma 4.

The details of the algorithm work as follows: For a given value t, where $0 \le t \le t_{\max}$, we first choose an initial value of α, where $0 \le \alpha \le 1$. For the given value t and the TD factor α, the point $(p(\alpha), t)$ on the boundary of $\mathcal{C}_{\rm BRC}(\alpha)$ under average power constraint can be calculated by solving the following convex optimization problem (**Q3**):

$$\begin{array}{ll} \text{maximize} & (1-\alpha)\log_2\det\left(\mathbf{I}_{N_{\sf B}} + \frac{P_{\sf R}^*}{N_{\sf R}\sigma_{\sf B}^2}\mathbf{H}_{\sf B}\mathbf{\Omega}\mathbf{H}_{\sf B}^H\right) \\ \text{subject to} & (1-\alpha)\log_2\det\left(\mathbf{I}_{N_{\sf A}} + \frac{P_{\sf R}^*}{N_{\sf R}\sigma_{\sf A}^2}\mathbf{H}_{\sf A}\mathbf{\Omega}\mathbf{H}_{\sf A}^H\right) \ge t \\ & {\rm tr}(\mathbf{\Omega}) \le N_{\sf R}, \quad \mathbf{\Omega} \succeq 0 \\ \text{variable} & \mathbf{\Omega} \end{array}$$

where $P_{\sf R}^* = P_{\sf R}/(1-\alpha)$. The maximum value of the objective function in **Q3** is $p(\alpha)$. Secondly, we can check whether $(p(\alpha), t) \in \mathcal{C}_{\rm MAC}(\alpha)$ with the TD factor α under average power constraint. That is, we solve the following convex feasibility problem (**Q4**):

$$\begin{array}{ll} \text{find} & \mathbf{\Omega}_{\sf A}, \mathbf{\Omega}_{\sf B} \\ \text{subject to} & \alpha\log_2\det\left(\mathbf{I}_{N_{\sf R}} + \frac{P_{\sf A}^*}{N_{\sf A}\sigma^2}\mathbf{H}_{\sf A}\mathbf{\Omega}_{\sf A}\mathbf{H}_{\sf A}^H\right) \ge p(\alpha) \\ & \alpha\log_2\det\left(\mathbf{I}_{N_{\sf R}} + \frac{P_{\sf B}^*}{N_{\sf B}\sigma^2}\mathbf{H}_{\sf B}\mathbf{\Omega}_{\sf B}\mathbf{H}_{\sf B}^H\right) \ge t \\ & \alpha\log_2\det\left(\mathbf{I}_{N_{\sf R}} + \sum_{k\in\{{\sf A},{\sf B}\}} \frac{P_k^*}{N_k\sigma^2}\mathbf{H}_k\mathbf{\Omega}_k\mathbf{H}_k^H\right) \\ & \quad \ge p(\alpha) + t \\ & {\rm tr}(\mathbf{\Omega}_{\sf A}) \le N_{\sf A}, \quad \mathbf{\Omega}_{\sf A} \succeq 0 \\ & {\rm tr}(\mathbf{\Omega}_{\sf B}) \le N_{\sf B}, \quad \mathbf{\Omega}_{\sf B} \succeq 0. \end{array}$$

where $P_{\sf A}^* = P_{\sf A}/\alpha$ and $P_{\sf B}^* = P_{\sf B}/\alpha$. By solving the problem **Q4**, we get a *feasibility certificate* to show whether suitable

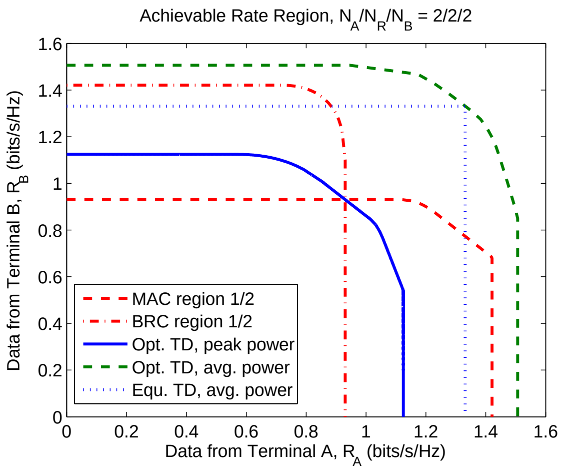

Fig. 6. Achievable rate regions under peak and average power constraints. "MAC region 1/2" and "BRC region 1/2" correspond to $0.5\mathcal{C}_{\text{MAC}}(1)$ and $0.5\mathcal{C}_{\text{BRC}}(0)$, respectively. Their intersection region is the achievable rate region with equal TD under peak power constraint.

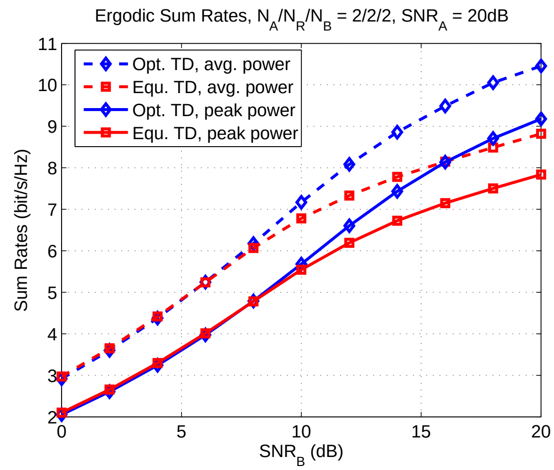

Fig. 7. Ergodic sum rate under peak and average power constraints, $N_{\mathsf{A}}/N_{\mathsf{R}}/N_{\mathsf{B}} = 2/2/2$, $\text{SNR}_{\mathsf{A}} = 20\text{dB}$

matrices $\mathbf{\Omega}_{\mathsf{A}}$ and $\mathbf{\Omega}_{\mathsf{B}}$ can be found satisfying the constraints. If suitable covariance matrices $\mathbf{\Omega}_{\mathsf{A}}$ and $\mathbf{\Omega}_{\mathsf{B}}$ can be found satisfying the constraints of **Q4**, then $(p(\alpha), t) \in \mathcal{C}_{\text{MAC}}(\alpha)$. This indicates that $\alpha \geq \alpha^\star$ and the present choice of TD factor should be decreased; if no suitable covariance matrices $\mathbf{\Omega}_{\mathsf{A}}$ and $\mathbf{\Omega}_{\mathsf{B}}$ can be found, then $(p(\alpha), t) \notin \mathcal{C}_{\text{MAC}}(\alpha)$, which indicates that $\alpha < \alpha^\star$ and the present choice of TD factor should be increased. Here we utilized Lemma 4. This feasibility certificate can be considered as a subgradient for finding the optimum factor $\alpha^\star$. This process repeats until α converges. This algorithm is summarized in Algorithm 3:

Algorithm 3 Bisection method to determine the boundary point $(p^\star, t)$ on $\mathcal{R}_{\text{OPT}}$ and the optimum TD factor $\alpha^\star$ for given QoS requirement $R_{\mathsf{B}} = t$

```
Initialize α^min = 0, α^max = 1 and α = (α^min + α^max)/2.
while α_max − α_min > ε do
    Solve (p(α), t) for the convex optimization problem Q3;
    Check the feasibility of (p(α), t) in the feasibility check problem Q4;
    if (p(α), t) is feasible then
        α^max = α; α^min unchanged; α = (α^max + α^min)/2
    else
        α^min = α; α^max unchanged; α = (α^max + α^min)/2
    end if
end while
return p* = p(α), α* = α.
```

It is worth mentioning that we get the corresponding $\mathbf{\Omega}$, $\mathbf{\Omega}_{\mathsf{A}}$ and $\mathbf{\Omega}_{\mathsf{B}}$ when we solve **Q3** and **Q4**. Evaluation for every point of $0 \leq t \leq t_{\max}$ yields the boundary of the whole achievable rate region $\mathcal{R}_{\text{OPT}}$ under average power constraint.

V. Simulation Results

A. Achievable Rate Region $\mathcal{R}_{\text{OPT}}$ for Static Channels

The achievable rate regions $\mathcal{R}_{\text{OPT}}$ with optimum TD strategies for two exemplary channel matrices $\mathbf{H}_{\mathsf{A}}$ and $\mathbf{H}_{\mathsf{B}}$ are shown in Fig. 6. Both peak and average power constraints are considered. The channel matrices are randomly generated as follows

$$\mathbf{H}_{\mathsf{A}} = \begin{bmatrix} -1.30 - 0.45i & -0.60 + 0.13i \\ -1.88 + 0.33i & 0.34 + 0.66i \end{bmatrix}$$
$$\mathbf{H}_{\mathsf{B}} = \begin{bmatrix} -0.05 - 0.45i & -0.55 + 0.19i \\ 0.60 - 0.99i & 1.13 - 0.01i \end{bmatrix}$$
$$\mathbf{G}_{\mathsf{A}} = \mathbf{H}_{\mathsf{A}}^T, \quad \mathbf{G}_{\mathsf{B}} = \mathbf{H}_{\mathsf{B}}^T.$$

The channels remain constant during their corresponding transmission phase. Furthermore, we have $P_k/\sigma^2 = 1$, where $k \in \{\mathsf{A}, \mathsf{B}, \mathsf{R}\}$. Given $R_{\mathsf{B}} = 0.5$ bits/s/Hz, the maximum achievable rate of R_{A} is shown in Table I. The gain of the achievable rate R_{A} in optimum TD case is significant compared to equal TD case. For average power constraint, the actual transmit power P_k^* is larger than P_k. Due to power scaling, the achievable rate region under average power constraint is also larger than that under peak power constraint.

TABLE I
Maximum Achievable Rate of R_{A} for Given QoS Requirement $R_{\mathsf{B}} = 0.5$ bits/s/Hz

	Equal TD	Optimum TD	Increase
Peak Power Constraint	0.93 bits/s/Hz	1.12 bits/s/Hz	20.43%
Average Power Constraint	1.33 bits/s/Hz	1.51 bits/s/Hz	13.53%

B. Ergodic Sum Rate in Rayleigh Fading Channels

Fig. 7 shows the ergodic sum rate of a two-way relaying system in Rayleigh fading channels. The number of antennas at terminal A, the relay and terminal B are $N_{\mathsf{A}} = N_{\mathsf{R}} = N_{\mathsf{B}} = 2$. Each entry in $\mathbf{H}_{\mathsf{A}}$, $\mathbf{H}_{\mathsf{B}}$, $\mathbf{G}_{\mathsf{A}}$ and $\mathbf{G}_{\mathsf{B}}$ are $\mathcal{CN}(0, 1)$ random

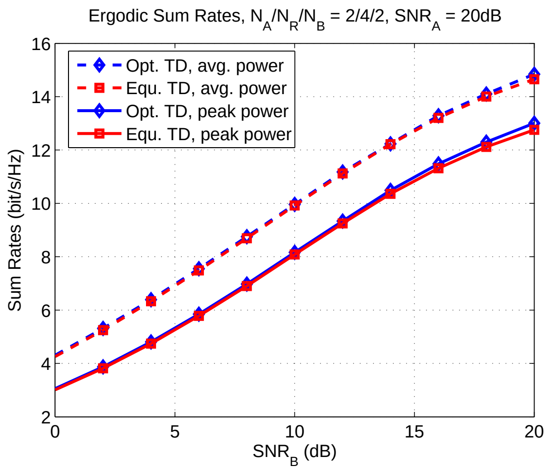

Fig. 8. Ergodic sum rate under peak and average power constraints, $N_{\mathsf{A}}/N_{\mathsf{R}}/N_{\mathsf{B}} = 2/4/2$, $\mathrm{SNR}_{\mathsf{A}} = 20\mathrm{dB}$

variables. We have $\mathrm{SNR}_{\mathsf{A}} = P_{\mathsf{A}}/\sigma^2 = P_{\mathsf{R}}/\sigma^2_{\mathsf{A}} = 20\mathrm{dB}$, which means that the distance between terminal A and the relay is fixed. We also define $\mathrm{SNR}_{\mathsf{B}} = P_{\mathsf{B}}/\sigma^2 = P_{\mathsf{R}}/\sigma^2_{\mathsf{B}}$, which is shown as the x-axis in the figure. Fig. 7 shows that the optimum TD strategies do not gain in the ergodic sum rate when $\mathrm{SNR}_{\mathsf{B}}$ is below 8dB. Since $\mathrm{SNR}_{\mathsf{A}}$ is high, the weak link, i.e., the link between the relay and terminal B, determines the achievable rate region when $\mathrm{SNR}_{\mathsf{B}}$ is low. By choosing different TD factors, we can increase either R_{A} or R_{B}, but at the price of the other. In this case, optimum TD strategies do not increase the sum rate of the system. However, when $\mathrm{SNR}_{\mathsf{B}} = 20\mathrm{dB}$, optimum TD strategies increase the sum rate by 1.2 bits/s/Hz and 1.5 bits/s/Hz compared to equal TD case under peak power constraint and average power constraint, respectively. This is because the relay has only two antennas. When $\mathrm{SNR}_{\mathsf{B}}$ is high, the MAC phase becomes the bottleneck of the system. By increasing the duration of the MAC phase, more data from terminal A and B can be decoded at the relay. Those data can still be retransmitted back to the two terminals in the BRC phase even though the duration of the BRC phase is shorter now. By choosing optimum TD strategies, the sum rate can be increased. On the other hand, optimum TD strategies do not increase the sum rate much when the number of antennas is increased to $N_{\mathsf{R}} = 4$ as shown in Fig. 8. In this case, the MAC phase is not a constraint of the system any more. By increasing the duration of the MAC phase, more data from terminal A and B can be decoded at the relay. However, those data cannot be transmitted to the two terminals in the BRC phase due to its shorter duration. By comparing Fig. 8 and Fig. 7, we can observe that by increasing the number of antennas N_{R} at the relay, the sum rates have been improved by about 1 bits/s/Hz both under peak and average power constraints.

VI. Conclusions

We proposed two methods for characterizing the achievable rate regions with optimum TD strategies for two-way DF relaying systems with multiple antennas. Both peak power constraint and average power constraint were considered. Simulation results showed that the achievable rate region can be further increased by choosing optimum TD strategies. At high SNR, the optimum TD strategies improve the ergodic sum rate when the MAC phase is the bottleneck of the system, e.g., when $N_{\mathsf{A}} + N_{\mathsf{B}} > N_{\mathsf{R}}$. The gain in ergodic sum rate by using the optimum TD strategies is small when $N_{\mathsf{A}} + N_{\mathsf{B}} \leq N_{\mathsf{R}}$.

References

[1] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Select. Areas Commun.*, vol. 25, pp. 379–389, Feb. 2007.

[2] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE J. Select. Areas Commun.*, vol. 21, pp. 684–702, June 2003.

[3] Y. Wu, P. Chou, and S.-Y. Kung, "Information exchange in wireless networks with network coding and physical-layer broadcast," in *Proc. of 39th Annual Conf. on Information Sciences and Systems (CISS)*, Mar. 2005.

[4] R. F. Wyrembelski, T. J. Oechtering, I. Bjelakovic, C. Schnurr, and H. Boche, "Capacity of Gaussian MIMO bidirectional broadcast channels," in *IEEE Int. Symposium on Inf. Theory*, (Toronto, Canada), pp. 584–588, July 2008.

[5] R. Vaze and R. W. Heath, "Capacity scaling for MIMO two-way relaying," in *Proc. IEEE Int. Symposium on Inf. Theory*, (Nice, France), June 24 – 29, 2007.

[6] T. J. Oechtering and H. Boche, "Optimal time-division for bidirectional relaying using superposition encoding," *IEEE Commun. Lett.*, vol. 12, Apr. 2008.

[7] T. Oechtering, *Spectrally Efficient Bidirectional Decode-and-Forward Relaying for Wireless Networks*. PhD thesis, Technical University Berlin, Oct. 2007.

[8] D. P. Bertsekas, *Nonlinear Programming*. Athena Scientific, 2nd edition ed., 1999.

[9] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[10] R. Tütüncü, K. Toh, and M. Todd, "Solving semidefinite-quadratic-linear programs using SDPT3," *Mathematical Programming*, no. 95, pp. 189–217, 2003.

[11] H. Viswanathan, S. Venkatesan, and H. Huang, "Downlink capacity evaluation of cellular networks with known-interference cancellation," *IEEE J. Select. Areas Commun.*, vol. 21, pp. 802–811, June 2003.

[12] J. Liu and Y. T. Hou, "Maximum weighted sum rate of multi-antenna broadcast channels," in *Proc. IEEE Int. Conf. on Communications*, (Beijing, China), May 19–23, 2008.

[13] J. Lee and N. Jindal, "Symmetric capacity of MIMO downlink channels," in *Proc. IEEE Int. Symposium on Inf. Theory*, (Seattle, WA), pp. 1031–1035, July 2006.

[14] K. Fan, "Minimax theorems," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 39, pp. 42–47, Jan. 15 1953.