Unified ILP Formulation of Protection in Mesh Networks

Christian Mauz
Communication Technology Laboratory
Swiss Federal Institute of Technology (ETH) Zurich
Switzerland
mauz@nari.ee.ethz.ch

***Abstract*— An unified formulation for planning of protection resources in mesh-networks as an Integer Linear Program (ILP) is developed. Various protection schemes have been considered and the similarities among them are worked out. The objective is to minimize the required total capacity. The ILP formulation turns out to be especially useful for shared protection, whereas for dedicated protection simple exact algorithms do exist. Case studies for Pan-European networks of the COST action 266 have been performed and the results are presented.**

***Index Terms*— resilience, shared protection, optical network planing, integer linear programming**

I. Introduction

WAVELENGTH-DIVISION multiplexing (WDM) is used to an increasing extent for point-to-point connections in wide area networks to satisfy the demand for transport capacity and to cope with the tremendous demand in bandwidth. SDH (Synchronous Digital Hierarchy) or SONET (Synchronous Optical Network) represent the classic transport protocols. In addition, ATM (Asynchronous Transfer Mode), IP (Internet Protocol) or PDH (Plesiochronous Digital Hierarchy) can also be found in a transport network.

To simplify routing, light-paths are directly switched in the optical path layer (OPL) [1]. One has to distinguish wavelength path (WP) networks, where the light path does not change its wavelength through its way through the network, and virtual wavelength path (VWP) networks, where the network nodes have wavelength changing capabilities.

The transmission capacity of each optical fiber is steadily extended due to the utilization of WDM and high-speed time division multiplexing (TDM) techniques. In case of a failure, a huge amount of traffic is hit. Therefore, high availability of the connections is mandatory and protection is an important design issue.

To guarantee low reaction times and to have simple recovery schemes, it is beneficial to use protection not only in the transport protocol layer but also directly in the OPL.

Here, different protection schemes do exist [2]. Dedicated protection, where the protection resources are exclusively devoted to an entity, is in general simple to realize but needs significant more transport capacity then shared protection schemes, where several entities use a common pool of protection resources.

In case of dedicated protection, it is almost always possible to divide the network dimension problem into simple sub-problems, for which exact algorithms with low complexity exist. For shared protection schemes, this is in general not feasible. Beside heuristics, which tries to find good solutions with low computational effort and which have to be specifically tailored to the problem, often integer linear programming (ILP) formulation is being used.

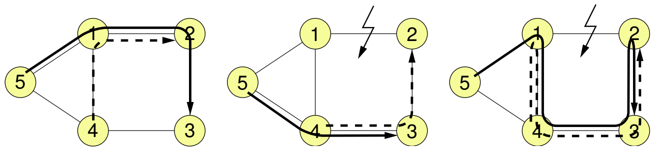

Fig. 1. Differences between path and span protection (from left to right: working state, path protection, and span protection).

The purpose of this work is to derive an unified formulation for different shared protection schemes in mesh networks. This enables some insight in the common structure of these problems. In principle, the formulation can also be applied to describe dedicated protection schemes, but as explained earlier, this problem may be solved with simpler methods.

The structure of the article is as follows. In the first section, the different investigated protection schemes are shortly introduced. Then, the unified ILP formulation is presented and discussed. It follows a case study for Pan-European networks. A short summary at the end includes an outlook for further work.

II. Protection schemes in the optical layer

There exist different possibilities to perform protection in the optical layer in mesh networks. A distinction can be made with respect to the entity which will be protected namely path protection and span protection. In Fig. 1, a network with two light-paths is shown in the working and the protection state.

Normally, path protection is more difficult to realize because in case of a failure, a reconfiguration of all optical cross connects (OXC) of affected work paths has to be performed. When the span (or link) is protected, a reconfiguration of only neighboring OXC is necessary. In the latter case, it is also much easier to detect the occurrence of an fault. It is even possible, to reroute the complete optical multiplex section signal containing several wavelengths by simple fiber switching. On the other hand, the length of the light-path may

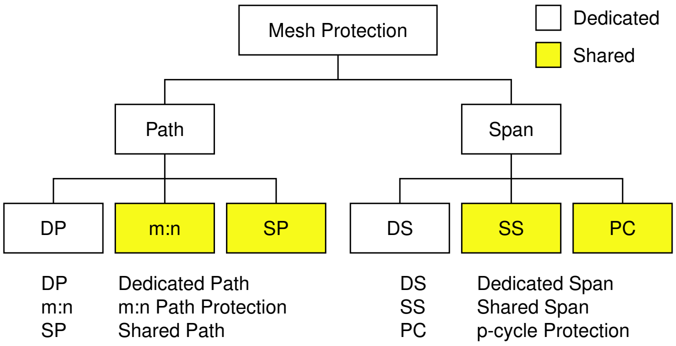

Fig. 2. Different protection schemes in mesh networks.

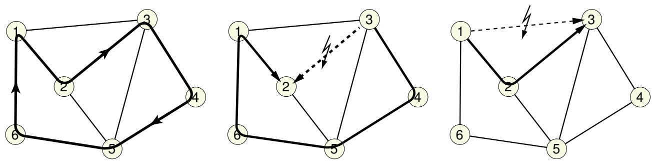

Fig. 3. p-cycle protection. From left to right: network with established p-cycle, failure of an on-cycle and of a straddling span.

get significantly enlarged. This results in a degradation of the signals and may require additional regeneration of them.

In Fig 2, a classification of common protection schemes in mesh networks is shown. A very simple approach is dedicated path protection (DP). In DP, for every working path a protection path is reserved. This scheme is also denoted as 1+1 protection. When n working paths share one protection path, this is named $1 : n$ protection or, more general, when m protection paths are available, $m : n$ protection. When all protection paths can use a common pool of protection resources, this is called shared path protection (SP). In general, this is the most capacity-efficient protection scheme.

Similar to that, dedicated span protection (DS) and shared span protection (SP) exist.

A special case of span protection is the p-cycle protection scheme (see Fig. 3), introduced by Grover and Stamatelakis [3]. The protection capacity is organized in cycles. Similar to a self-healing ring, when a failure of an on-cycle link occurs, the signal passes the cycle in the counter-propagating direction. The difference to an SDH shared protection ring (SPRing) is that also the straddling links may benefit from the protection by the cycle. Therefore, p-cycle protection is much more capacity-efficient then a protection schemes based on rings.

These schemes have in common, that the protection resources are planned in advance and that for the occurrence of a failure a specific reaction is foreseen. In contrast to that resilience procedures must start from the actual network state and the available resources, in order to a reestablish broken connections.

III. ILP Formulation

The network is modeled by the graph $G = G(V, E)$. V and E denote the node and edge set, respectively. The objective is to minimize the total capacity

$$\min \sum_{i=1}^{|E|} (w_i + s_i) \tag{1}$$

where w_i is the work load and s_i the spare capacity on link i. The load and the capacity are measured in number of wavelength-paths. Here, a simple hop-metric is used.

The minimization should be done subject to the following boundary conditions. First, the work-load is related to the chosen routing by

$$\sum_{j=1}^{|W|} n_j \cdot \Gamma^{\mathrm{p}}_{i,j} = w_i, \qquad \forall i \in E. \tag{2}$$

The set W contains all candidates for working paths. The indicator function (or coefficient)

$$\Gamma^{\mathrm{p}}_{i,j} = \begin{cases} 1, & \text{path } j \text{ uses link } i, \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

determines if a working-path requires resources on a specific link. There are n_j connections using path j as the working path. When W contains just one possible path for every node pair $\alpha = (\nu_1, \nu_2)$ of the network, with $\nu_1, \nu_2 \in N$, e.g. the shortest path between ν_1 and ν_2, then $n_j = T_\alpha$. T_α denotes the traffic demand between the two nodes.

Next, enough spare capacity to protect the working traffic should be installed

$$\sum_{j=1}^{|P_{\mathrm{s}}|} m_j \cdot \Gamma^{\mathrm{s}}_{i,j} \cdot \Delta^{\mathrm{s}}_{j,k} \le s_i \qquad \forall i \in E, \quad \forall k \in F. \tag{4}$$

All possible protection structures, like protection paths or p-cycles, are contained in the set P_{s}. Similar to (3), the indicator function

$$\Gamma^{\mathrm{s}}_{i,j} = \begin{cases} 1, & \text{protection structure } j \text{ uses link } i \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

is defined. It is possible to consider different failure scenarios F. For example, only single link failures or node and link failures may be regarded. In every failure scenario, there might be a simultaneous break-down of several entities. This correlation is reflected by the indicator function

$$\Delta^{\mathrm{s}}_{j,k} = \begin{cases} 1, & \text{protection structure } j \text{ is used in case} \\ & \text{of failure } k, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

For $\Delta_{j,k} = 1, \forall k \in F$, this results in dedicated protection.

Every entity of the working traffic should be protected against loss. One has to distinguish between path and span protection:

$$\sum_{j=1}^{|P_{\mathrm{s}}|} m_j \cdot \Psi^{\mathrm{s}}_{i,j} \ge \begin{cases} n_i, & \forall i \in W, \quad \text{for path protection,} \\ w_i, & \forall i \in E, \quad \text{for span protection.} \end{cases} \tag{7}$$

The indicator function $\Psi^{\mathrm{s}}_{i,j}$ determines, if entity i (path or span) may be protected by the protection structure j.

Finally, the optimization variables m_j have to be integers

$$m_j \in \mathbb{N}_0, \qquad \forall j \in P_{\mathrm{s}}. \tag{8}$$

TABLE I
COMPLEXITY OF THE ILP PROBLEM.

Problem	Complexity
Routing	$O(N \cdot (N-1) \cdot k_\mathrm{w})$
SP	$O(N \cdot (N-1) \cdot k_\mathrm{p})$
SS	$O(\lvert E\rvert \cdot k_\mathrm{p})$
PC	$O(\#\ \text{cycles})$

The variables w_i and s_i are only used to clarify the description, but are in principle not needed for the optimization process (and may probably be eliminated by the ILP solver).

Often the capacity requirements for a survivable network may be significantly reduced, when the routing is included in the optimization process. This can easily be accomplished, when the n_j are optimization variables, too:

$$n_j \in \mathbb{N}_0, \qquad \forall j \in W. \tag{9}$$

In total, for every request of the traffic matrix there should a working path exist

$$\sum_{j=1}^{|W|} n_j \cdot \Phi^{\mathrm{p}}_{\alpha,j} = T_\alpha, \qquad \forall \alpha \in V \times V. \tag{10}$$

The function

$$\Phi^{\mathrm{p}}_{\alpha,j} = \begin{cases} 1, & \text{path } j \text{ is valid working path for node pair } \alpha \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

indicates, if a path may be used as working path for a node pair. Usually, the k_w shortest paths for every node pair are chosen for W.

Often, it is worth while in order to reduce the complexity of the ILP problem, to chose those candidates for the set P_s of possible protection resources, that are the most promising. This is done for example by considering the k_p shortest possible protection paths for every working path or span. In case of p-cycle protection, the topological score may be used [4].

This representation of the protection design problem in mesh networks allows to plan even a network which makes use of different protection schemes at the same time.

In Tab I the complexity in terms of the number of variables of the different ILP problems is displayed. Here, k_w denotes the number of working path candidates for each node-pair, and k_p the number of protection candidates for each entity considered. The effort to determine the working path and the protection structure candidates is not included. This is normally done in a pre-processing step.

The complexity of the problem may be reduced by requiring a symmetric routing. This is possible when the traffic demands are also symmetric, and does normally not result in any penalty.

For $m : n$ protection, the problem may be divided into $N \cdot (N-1)$ independent sub-problems, one for each node-pair. Each of these problems has a complexity of $O(k_\mathrm{p})$.

In typical transport networks, the number of edges is proportional to N. Therefore, the path protection schemes are with $O(N^2)$ much harder to tackle then the span protection scheme with $O(\lvert E\rvert) \sim O(N)$. The number of cycles in a network

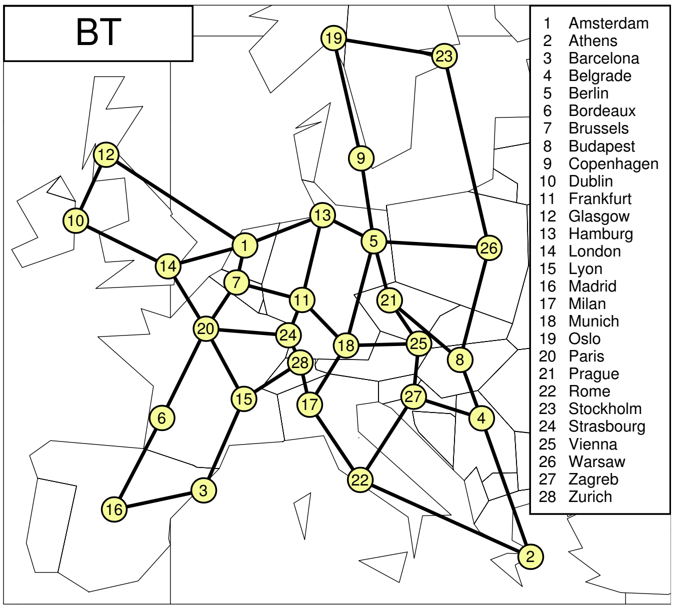

Fig. 4. Topology of the BT network.

for $\lvert E\rvert \geq N$ is bound by $2^{\lvert E\rvert - N + 1}$ [5]. Thus, for large and dense networks, one has to limit the maximal circumference of a cycle, otherwise the number of cycles will explode. The cycles themselves may be found by a depth-first search [6].

In principle, a p-cycle is an enhanced SPRing, where also the straddling links are protected. Therefore, SPRing networks may be designed with this method as well. In this case, two aspects have to be considered. First, a ring provides normally a certain amount of wavelength channels. Second, it is favorable to minimize the required ring changes of a connection, because this is an expensive functionality and increases the complexity of the network management.

So far, only VWP-networks have been investigated, where one has only to deal with capacities. When the nodes have no wavelength conversion capabilities, every path and protection structure gets colored. Therefore, the number of variables are multiplied by a factor of $\lvert\Lambda\rvert$, when Λ is the set of available wavelengths. This makes the ILP approach even for small networks infeasible, and one has to rely on heuristics with lower complexity.

IV. CASE-STUDY

A case study for a Pan-European network using the reference scenario of the COST action 266 has been performed, where a network with 28 nodes is considered. From a basic topology (BT) with 41 edges, a sparser network with only 34 links, a ring topology (RT), and a denser topology with 60 edges, called triangular topology (TT), are derived. The BT topology is shown in Fig. 4. Details for the reference scenario may be found in [7].

The same traffic matrix (total traffic for 2002) is used throughout this case study. A granularity of STM-16 ($2.5\,\mathrm{Gbit/s}$) per channel has been assumed. To compare the results, the required capacity for the different protection schemes has been normalized to the necessary working capacity for the

TABLE II
KEY FIGURES OF THE INVESTIGATED NETWORKS, WORK CAPACITY FOR SHORTEST PATH ROUTING.

	N	$\lvert E\rvert$	δ	**Work capacity**
BT	28	41	2.93	3940
RT	28	34	2.43	4592
TT	28	60	4.29	3310

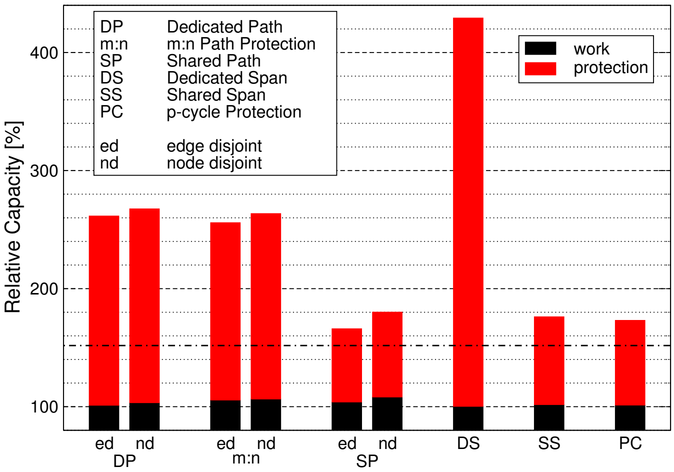

Fig. 5. Capacity requirements for the BT network of the COST 266 action.

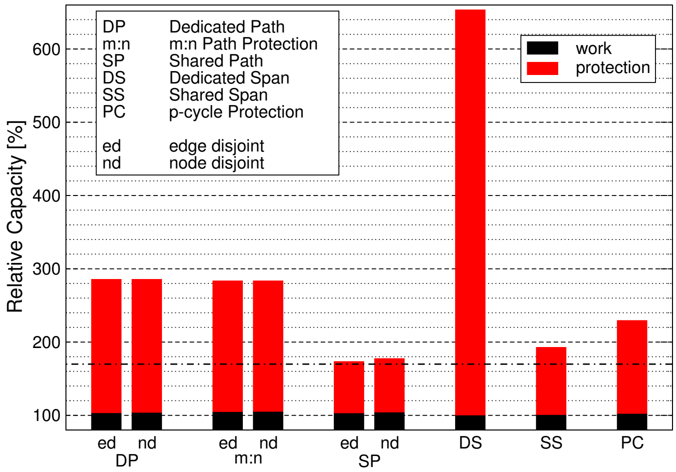

Fig. 6. Capacity requirements for the RT network of the COST 266 action.

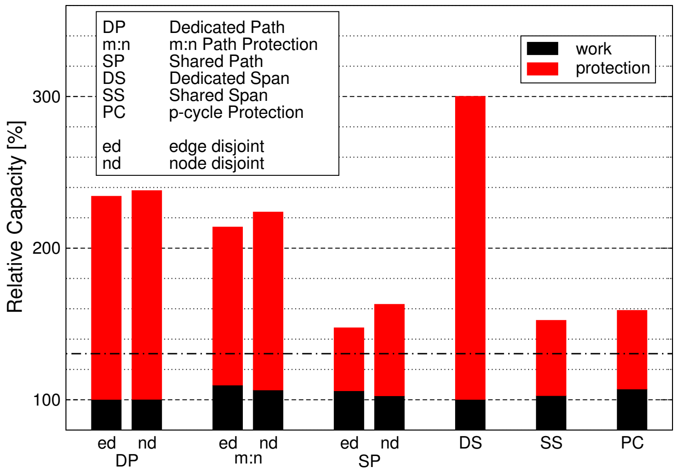

Fig. 7. Capacity requirements for the TT network of the COST 266 action.

networks when using shortest path routing. In Tab. II, the key figures of the networks are summarized.

For path protection, two failure scenarios are investigated, namely a link failure for which an edge disjoint (ed) protection path is required, and the occurrence of link or node failures, which may be counteracted by a node disjoint (nd) protection path. Note, that node disjoint implies edge disjoint.

The corresponding ILP problems where solved with CPLEX 7.0 [8]. Here, the maximal computation has been restricted to $30\,\mathrm{min}$. For the working path, up to 8 shortest paths between source and destination node have been considered. For the protection paths, up to 16 candidates for every entity have been considered in the set P_s. For the p-cycle scheme all available cycles of the topology have been used except for the TT, where due to the huge amount of cycles, only those consisting of maximal 11 nodes have been allowed. In general, it is more favorable to have a large k_p than a large k_w, because more candidates for the working paths improve the solution only slighly. Good results are often found with $k_\mathrm{w} = 2 \ldots 4$.

The results for the different protection schemes and the BT of the COST action 266 are shown in Fig. 5. For dedicated path protection, more then $160\,\%$ additional capacity are required for dedicated path protection. To avoid trap-constellations, a slightly longer working-path then the shortest path has to be used. For node disjoint protection paths, $7\,\%$ more capacity compared to only edge disjoint paths are necessary.

These figures are only slightly improved by $m : n$ protection. In contrast to that, for shared path protection only $165\,\%$ capacity have to be installed for working and protection traffic.

Dedicated span protection needs over $329\,\%$ protection capacity. Here, sharing among the spans reduces this to $74\,\%$. Therefore, slightly more capacity than for shared path protection is required. For a p-cycle scheme, this accounts to $71\,\%$. This is due to the better implicit pre-selection of possible protection candidates. Thus, a better solution in the restricted search space may be found.

For the RT network, the corresponding results are presented in Fig. 6. Because this topology has a lower mean nodal degree, one needs significant more protection capacity then for the BT network. Especially dedicated span protection performs badly. Here, a resilient network requires at least $285\,\%$ total capacity when using shared path protection.

Just the other way round the situation for the TT network appears as shown in Fig. 7. Due to the high meshing degree of the topology, only $47\,\%$ capacity is required to make the network survivable. In this case, shared path and shared span protection perform equally well.

For shared span protection, a lower bound of the minimal necessary protection capacity of $1/(\bar{\delta} - 1)$ has been derived [9], where $\bar{\delta}$ is the mean node degree. This bound is shown in the figures as a dot-dashed line. It has been derived for a well-balanced traffic pattern with equal traffic load on all links. Due to the inhomogeneous traffic distribution, the span protection schemes approach only to about $20\,\%$ of the bound for the investigated networks. With shared path protection it is possible to perform even better.

V. Conclusions

An unified ILP formulation for different mesh-protection schemes has been presented. This allows insight in the common structure and to observe the differences and similarities among the schemes. The objective has been to minimize the required capacity to realize a network resilient to single link or single link and single node failures. This ILP formulation is especially useful for planning networks with shared protection. It would even enable to combine several schemes and to performe the planning and optimization jointly.

For dedicated protection schemes it is possible to divide the problem into less complex sub-problems, for which simple exact algorithms exist.

A case study for different Pan-European networks of the COST action 266 has been performed. The higher the mean degree of the network, the less capacity is required for work and protection.

For WP networks the complexity of the resulting ILP problems is too high to solve them directly. Therefore especially tailored heuristics are required. Here it would be desirable to have a unified approach, too.

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