Generalized MMSE Detection Techniques for Multipoint-to-Point Systems

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***Abstract*— We propose a receiver for multipoint-to-point systems based on the *minimum mean square error* (MMSE) criterion, where the symbols are *detected in groups* and already detected symbols are fed back for interference subtraction, as known for *decision feedback equalization* (DFE). The proposed *scaled DFE* (SDFE) has two special cases: 1) DFE for a group size of one, i.e., for symbol-by-symbol detection. 2) *Maximum likelihood detection* (MLD), if the group comprises all transmitted symbols. The diversity order of SDFE lies between the poor diversity order of DFE and the full diversity order of MLD. Therefore, SDFE offers a trade-off between complexity due to the group-wise symbol detection and the increased diversity order compared to DFE. We also present an algorithm to compute the SDFE filters with an order of complexity which is the same as that to compute linear equalization filters. Motivated by the promising results of detectors based on *lattice reduction* (LR), we combine SDFE with LR. The resulting detector exhibits full diversity order and improved performance compared to LR-DFE.**

The simulations show that SDFE is an interesting generalization of DFE for detectors with zero-forcing constraint, since SDFE even outperforms LR-DFE for realistic *signal-to-noise-ratio* (SNR). However, LR-DFE exhibits the best results for an affordable complexity, when dropping the zero-forcing constraint.

I. Introduction

In a system with a *multiple-input-multiple-output* (MIMO) channel [1], the received signal is not only perturbed by noise but also by the interference between the transmitted data streams. MLD leads to full order of diversity as it optimally takes into account the properties of the noise and the interference. However, MLD has prohibitive (non-polynomial, e.g., [2]) complexity due to the necessary search over all possible data vectors. Contrary to MLD, suboptimal detection schemes such as linear equalization [3] and DFE (V-BLAST, e.g., [4], [5], [6]) exhibit a complexity for detection that is quadratic in the number of data streams. Linear equalization and DFE first suppress interference by some linear transformation of the received signal before the data are detected symbol-wise. For linear equalization, this linear transformation has to combat the interference of all other data streams. Therefore, the equalizing filter is highly constrained and the resulting diversity order is poor. Although DFE feeds back already detected symbols to suppress the interference induced by them, DFE has the same diversity order as linear equalization, because the data stream detected first is equalized by a linear filter which has to suppress the interference caused by all other data streams.

In [7], a theoretical framework was proposed with the special cases *zero-forcing* (ZF) DFE and MLD, that enables alternative feedback detectors with diversity orders between that of DFE and MLD. The scheme of [7] offers a good trade-off between error performance (different possible diversity orders) and complexity for detection. Heuristically, the detection order was computed following the principle of V-BLAST [4], i.e., the stream with the best SNR is detected first. A similar ZF scheme was also presented in [8].

The *scaled DFE* (SDFE) proposed in this paper follows the same idea as in [7], that is, the symbols are detected group-wise, where the group size is a design parameter. Contrary to [7] and [8], we do not restrict to ZF approaches and our SDFE design is based on the minimization of the *mean square error* (MSE) without ZF constraint. We formulate the SDFE filter design as an optimization problem similar to the approach to DFE in [6] and find an optimal rule to compute the detection order. Since the computation of the optimal detection order has high complexity, we present an alternative implementation (V-BLAST detection order) of the SDFE filter computation algorithm whose complexity is cubic in the number of data streams, i.e., the algorithm has the same order of complexity as that for the computation of linear equalization filters.

For QAM alphabets, MLD can be interpreted as a closest point search in a lattice (e.g., [9]). As suggested by Babai [10], a very powerful approximation for this closest point search is the *nearest-plane algorithm*, which can be interpreted as a DFE for a *Lenstra-Lenstra-Lovász* (LLL, [11]) reduced channel matrix. Babai's approximation has been applied to ZF detection in [12], [13] and to MMSE detection in [14]. The resulting LR-DFEs show very good performance and have full diversity order [15]. We apply LR to the proposed SDFE and end up with LR-SDFE which has full diversity order.

After reviewing MLD and DFE in Sections III and IV, respectively, we introduce SDFE together with LR-SDFE and give an efficient algorithm to compute the SDFE filters in Section V. The simulation results are presented in Section VI.

II. System Description

We consider a MIMO system with B transmit and N_R receive antennas. The B transmit antennas are not allowed to cooperate and can be treated as B distinct single-antenna users which communicate with a N_R receive-antenna base station. The input signals s_i, $i = 1, \ldots, B$ are independent complex

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baseband taking values from a constellation alphabet $\mathbb{A}$. The signal vector $\boldsymbol{s} = [s_1, \ldots, s_B]^{\mathrm{T}}$,[1] with covariance matrix $\boldsymbol{\Phi}_{ss} = \sigma_s^2 \boldsymbol{I}_B$, is transmitted over a flat fading MIMO channel $\boldsymbol{H} \in \mathbb{C}^{N_{\mathrm{R}} \times B}$ with tap-gain $[\boldsymbol{H}]_{j,i}$ from transmit antenna i to receive antenna j. The signal of interest is perturbed at the receiver by the additive zero-mean complex Gaussian noise vector $\boldsymbol{\eta}$ with covariance matrix $\boldsymbol{\Phi}_{\eta\eta}$, yielding the observation $\boldsymbol{x} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{\eta}$. We assume the receiver has perfect knowledge of the channel matrix $\boldsymbol{H}$.

III. Maximum Likelihood Detection

The MLD decides for the signal $\hat{\boldsymbol{s}}_{\mathrm{ML}}$ which maximizes the likelihood of the received signal $\boldsymbol{x} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{\eta}$:

$$\begin{aligned} \hat{\boldsymbol{s}}_{\mathrm{ML}} &= \underset{\boldsymbol{s} \in \mathbb{A}^B}{\operatorname{argmax}} (\ln p(\boldsymbol{x}|\boldsymbol{s})) \\ &= \underset{\boldsymbol{s} \in \mathbb{A}^B}{\operatorname{argmin}} (\boldsymbol{x} - \boldsymbol{H}\boldsymbol{s})^{\mathrm{H}} \boldsymbol{\Phi}_{\eta\eta}^{-1} (\boldsymbol{x} - \boldsymbol{H}\boldsymbol{s}) \\ &= \underset{\boldsymbol{s} \in \mathbb{A}^B}{\operatorname{argmin}} \|\boldsymbol{\Phi}_{\eta\eta}^{-1/2} (\boldsymbol{x} - \boldsymbol{H}\boldsymbol{s})\|_2^2. \end{aligned} \tag{1}$$

This minimization is a closest-point search problem in a B-dimensional lattice. The search can be accelerated by the sphere decoder [9]. Although MLD is the best detection method for equiprobable input symbols, the complexity of the subspace search in (1) is prohibitive for practical systems.

IV. Decision Feedback Equalization

The DFE scheme described here follows [6]. Fig. 1 depicts the reference model for DFE detection. The received signal $\boldsymbol{x}$ is first processed by the linear filter $\boldsymbol{G}$ which includes a proper reordering $\mathcal{O}$ of the data streams. The ML quantizer $\mathrm{Q}(\bullet)$ detects the entries of $\hat{\boldsymbol{s}}_{\mathrm{p}}$ element-wise starting from the first entry. The feedback filter $\boldsymbol{F}$ removes the interference of the already detected symbols to the not yet detected, and has a lower triangular structure with zero main diagonal to assure realizability. After the loop is completed, the symbols attain the original ordering through multiplication with the transpose of the permutation matrix $\boldsymbol{\Pi} = \sum_{i=1}^{B} \boldsymbol{e}_i \boldsymbol{e}_{k_i}^{\mathrm{T}}$.

For the filter design, we assume that the output of the quantizer is error-free. This model is depicted in Fig. 2 (without the dashed box), where the input to the feedback filter is the actual signal $\boldsymbol{s}$ after permutation with $\boldsymbol{\Pi}$. The MMSE-DFE tries to minimize the MSE of

$$\hat{\boldsymbol{s}}_{\mathrm{p}} = \boldsymbol{G}\boldsymbol{x} + \boldsymbol{F}\boldsymbol{\Pi}\boldsymbol{s} \tag{2}$$

with respect to the permuted data signal $\boldsymbol{\Pi}\boldsymbol{s}$. We define the error vector $\boldsymbol{\varepsilon}_{\mathrm{p}} = \boldsymbol{\Pi}\boldsymbol{s} - \hat{\boldsymbol{s}}_{\mathrm{p}}$ and the MSE ϕ reads as

$$\phi = \mathrm{E}\left[\|\boldsymbol{\varepsilon}_{\mathrm{p}}\|_2^2\right] = \mathrm{tr}(\boldsymbol{\Phi}_{\varepsilon\varepsilon,\mathrm{p}}), \tag{3}$$

where $\boldsymbol{\Phi}_{\varepsilon\varepsilon,\mathrm{p}}$ is the covariance matrix of $\boldsymbol{\varepsilon}_{\mathrm{p}}$:

[1] *Notation:* Throughout the paper, we denote vectors and matrices with lower and upper case bold letters, respectively. We use $\mathrm{E}[\bullet]$, $(\bullet)^{\mathrm{T}}$, $(\bullet)^{\mathrm{H}}$, $\mathrm{tr}(\bullet)$, $\otimes$, and $\Re(\bullet)$ for expectation, transposition, conjugate transposition, the trace of a matrix, the Kronecker product, and the real part, respectively. The $N \times N$ identity matrix is $\boldsymbol{I}_N$ and its i-th column $\boldsymbol{e}_i$. The B-dimensional all-ones and all-zeros vector are $\boldsymbol{1}_B$ and $\boldsymbol{0}_B$, respectively. The $N \times M$ all-zeros matrix is denoted by $\boldsymbol{0}_{N \times M}$.

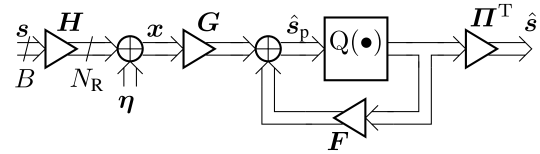

Fig. 1. Decision-feedback equalizer reference model

$$\begin{aligned} \boldsymbol{\Phi}_{\varepsilon\varepsilon,\mathrm{p}} = (\boldsymbol{I}_B - \boldsymbol{F})\boldsymbol{\Pi}\boldsymbol{\Phi}_{ss}\boldsymbol{\Pi}^{\mathrm{T}}(\boldsymbol{I}_B - \boldsymbol{F}^{\mathrm{H}}) + \boldsymbol{G}\boldsymbol{\Phi}_{xx}\boldsymbol{G}^{\mathrm{H}} \\ -\boldsymbol{G}\boldsymbol{\Phi}_{sx}^{\mathrm{H}}\boldsymbol{\Pi}^{\mathrm{T}}(\boldsymbol{I}_B - \boldsymbol{F}^{\mathrm{H}}) - (\boldsymbol{I}_B - \boldsymbol{F})\boldsymbol{\Pi}\boldsymbol{\Phi}_{sx}\boldsymbol{G}^{\mathrm{H}}. \end{aligned} \tag{4}$$

Here, $\boldsymbol{\Phi}_{xx}$ and $\boldsymbol{\Phi}_{sx}$ are the following covariance matrices

$$\boldsymbol{\Phi}_{xx} = \mathrm{E}\left[\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}\right] = \boldsymbol{H}\boldsymbol{\Phi}_{ss}\boldsymbol{H}^{\mathrm{H}} + \boldsymbol{\Phi}_{\eta\eta}, \tag{5}$$

$$\boldsymbol{\Phi}_{sx} = \mathrm{E}\left[\boldsymbol{s}\boldsymbol{x}^{\mathrm{H}}\right] = \boldsymbol{\Phi}_{ss}\boldsymbol{H}^{\mathrm{H}}. \tag{6}$$

The optimization problem can be formulated as

$$\begin{aligned} \{\boldsymbol{G}_{\mathrm{opt}}, \boldsymbol{F}_{\mathrm{opt}}, \mathcal{O}_{\mathrm{opt}}\} &= \underset{\{\boldsymbol{G},\boldsymbol{F},\mathcal{O}\}}{\operatorname{argmin}}\ \phi \\ \text{s.t.} \quad \boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{F}\boldsymbol{S}_i^{\mathrm{T}} &= \boldsymbol{0}_{B-i+1}^{\mathrm{T}} \quad \text{for} \quad i = 1 \ldots, B. \end{aligned} \tag{7}$$

The selection matrix

$$\boldsymbol{S}_i = [\boldsymbol{0}_{B-i+1,i-1}, \boldsymbol{I}_{B-i+1}] \in \{0,1\}^{B-i+1 \times B} \tag{8}$$

cuts out the last $B-i+1$ elements of a B-dimensional vector. Using Lagrangian multipliers we obtain the filter solution

$$\boldsymbol{F}_{\mathrm{opt}} = \boldsymbol{I}_B - \boldsymbol{L}^{-1}, \quad \boldsymbol{G}_{\mathrm{opt}} = \boldsymbol{D}\boldsymbol{L}^{\mathrm{H}}\boldsymbol{\Pi}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{\Phi}_{\eta\eta}^{-1}, \tag{9}$$

for a given ordering $\boldsymbol{\Pi}$, where $\boldsymbol{L}$ and $\boldsymbol{D}$ are given by the *Cholesky factorization with symmetric permutation*

$$\boldsymbol{\Pi}\boldsymbol{\Phi}\boldsymbol{\Pi}^{\mathrm{T}} = \boldsymbol{L}\boldsymbol{D}\boldsymbol{L}^{\mathrm{H}}, \tag{10}$$

$$\boldsymbol{\Phi} = (\boldsymbol{\Phi}_{ss}^{-1} + \boldsymbol{H}^{\mathrm{H}}\boldsymbol{\Phi}_{\eta\eta}^{-1}\boldsymbol{H})^{-1}. \tag{11}$$

$\boldsymbol{\Phi}$ is the *error covariance matrix* [6]. $\boldsymbol{L}$ is unit lower triangular and $\boldsymbol{D}$ is diagonal. From (9) and (3), we get for the MSE

$$\phi = \mathrm{tr}(\boldsymbol{D}) = \sum_{i=1}^{B} d_i. \tag{12}$$

Minimizing each summand in (12) separately yields the MMSE V-BLAST detection order

$$k_{i,\mathrm{subopt}} = \underset{k \notin \{k_1,\ldots,k_{i-1}\}_{\mathrm{subopt}}}{\operatorname{argmin}} d_i. \tag{13}$$

An iterative algorithm that calculates the Cholesky factorization in (10) by minimizing the MSE at every step using (13) is presented in detail in [6].

V. Scaled Decision Feedback Equalization

SDFE combines the architectures of MLD and DFE. The reference model is identical to Fig. 1, with $\mathrm{Q}_p(\bullet)$ instead of $\mathrm{Q}(\bullet)$. DFE operates in a symbol-wise manner, while SDFE operates in a p-wise fashion. The modified quantizer $\mathrm{Q}_p(\bullet)$ decides on p *symbols at a time*, by performing a ML-like search. The group size p is a system parameter and takes values

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$1 \le p \le B$ and $B/p \in \mathbb{N}$.[2] The detected symbols are then fed back and the interference caused to the not yet detected symbols is removed via $\boldsymbol{F}$. Due to the p-wise operation, the backward filter $\boldsymbol{F}$ has a *block lower triangular* structure, where its $p \times p$ diagonal blocks are zero. The feedforward filter $\boldsymbol{G}$ suppresses the interference caused by the not yet detected symbols to the block of symbols to be quantized, but *ignores the interference inside the latter block of* p *symbols*. This interference will be resolved by the quantizer. Hence, we linearly suppress $B-p$ instead of $B-1$ interferers and gain this way $p-1$ degrees of freedom compared to DFE (cf. [8]). We can enforce the special function of $\boldsymbol{G}$ by artificially extracting the interference inside these blocks using an auxiliary matrix $\boldsymbol{C}$. This modification can be seen inside the dashed box in Fig. 2. The signal $\boldsymbol{s}$ is first permuted, then processed with the filter $\boldsymbol{C}$ and added to the loopback. $\boldsymbol{C}$ has a $p \times p$ block diagonal structure with zero main diagonal, and perfectly extracts the *intra-block* interference. Thus, $\boldsymbol{G}$ will only suppress the remaining *inter-block* interference. The auxiliary matrix $\boldsymbol{C}$ will be used only for the calculation of the filters $\boldsymbol{G}$ and $\boldsymbol{F}$, and not during the operation of the receiver. For the filter design, we assume that the modified quantizer $\mathrm{Q}_p(\bullet)$ takes always correct decisions, like for DFE (cf. Fig. 2).

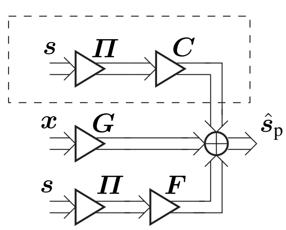

Fig. 2. Equivalent model without the quantizer for SDFE and DFE, with and without the dashed box, respectively.

A. MMSE Scaled Decision Feedback Equalizer

In the MMSE approach, we try to minimize the MSE of

$$\hat{\boldsymbol{s}}_{\mathrm{p}} = \boldsymbol{G}\boldsymbol{x} + \boldsymbol{C}\boldsymbol{\Pi}\boldsymbol{s} + \boldsymbol{F}\boldsymbol{\Pi}\boldsymbol{s} = \boldsymbol{G}\boldsymbol{x} + (\boldsymbol{C}+\boldsymbol{F})\boldsymbol{\Pi}\boldsymbol{s} \qquad (14)$$

with respect to the permuted data signal $\boldsymbol{\Pi}\boldsymbol{s}$. Recall that $\boldsymbol{F}$ has a block lower triangular structure while $\boldsymbol{C}$ has a block diagonal structure. Thus, we can use the auxiliary matrix $\boldsymbol{K} = \boldsymbol{F} + \boldsymbol{C}$ for our derivation, and extract $\boldsymbol{F}$ and $\boldsymbol{C}$ at the end. The desired signal for $\hat{\boldsymbol{s}}_{\mathrm{p}}$ is the permuted signal $\boldsymbol{\Pi}\boldsymbol{s}$, and the error vector is

$$\boldsymbol{\varepsilon}_{\mathrm{p}} = \boldsymbol{\Pi}\boldsymbol{s} - \hat{\boldsymbol{s}}_{\mathrm{p}} = (\boldsymbol{I}_B - \boldsymbol{K})\boldsymbol{\Pi}\boldsymbol{s} - \boldsymbol{G}\boldsymbol{x}. \qquad (15)$$

The covariance matrix of $\boldsymbol{\varepsilon}_{\mathrm{p}}$ is given by

$$\begin{aligned}\boldsymbol{\Phi}_{\varepsilon\varepsilon,\mathrm{p}} = &(\boldsymbol{I}_B - \boldsymbol{K})\boldsymbol{\Pi}\boldsymbol{\Phi}_{ss}\boldsymbol{\Pi}^{\mathrm{T}}(\boldsymbol{I}_B - \boldsymbol{K}^{\mathrm{H}}) + \boldsymbol{G}\boldsymbol{\Phi}_{xx}\boldsymbol{G}^{\mathrm{H}} \\ &-\boldsymbol{G}\boldsymbol{\Phi}_{sx}^{\mathrm{H}}\boldsymbol{\Pi}^{\mathrm{T}}(\boldsymbol{I}_B - \boldsymbol{K}^{\mathrm{H}}) - (\boldsymbol{I}_B - \boldsymbol{K})\boldsymbol{\Pi}\boldsymbol{\Phi}_{sx}\boldsymbol{G}^{\mathrm{H}}, \qquad (16)\end{aligned}$$

whose trace is the MSE ϕ (cf. Eq. 3) and $\boldsymbol{\Phi}_{xx}$ and $\boldsymbol{\Phi}_{sx}$ are defined in (5) and (6), respectively. The optimization problem takes the following form

$$\{\boldsymbol{G}_{\mathrm{opt}}, \boldsymbol{K}_{\mathrm{opt}}, \mathcal{O}_{\mathrm{opt}}\} = \underset{\{\boldsymbol{G},\boldsymbol{K},\mathcal{O}\}}{\operatorname{argmin}}\ \operatorname{tr}(\Phi_{\varepsilon\varepsilon,\mathrm{p}}) \qquad (17)$$

$$\text{s.t. } (\boldsymbol{e}_i^{\mathrm{T}} \otimes \boldsymbol{I}_p)\boldsymbol{K}\boldsymbol{S}_{pi+1}^{\mathrm{T}} = \boldsymbol{0}_{p\times B-pi},\ \ i = 1\ldots, B/p-1$$

$$\bar{\boldsymbol{e}}_i^{\mathrm{T}}\boldsymbol{K}\bar{\boldsymbol{e}}_i = 0 \quad \text{for} \quad i = 1,\ldots,B \qquad (18)$$

where $\boldsymbol{e}_i$ is the i-th column of the $B/p \times B/p$ identity matrix and $\bar{\boldsymbol{e}}_i$ is the i-th column of the $B \times B$ identity matrix. $\boldsymbol{S}_i$ is given in (8). The constraints (18) impose the special structure on $\boldsymbol{K}$. We solve the optimization problem using Lagrangian multipliers and obtain the filter solution

$$\boldsymbol{G}_{\mathrm{opt}} = \sum_{i=1}^{B} \frac{1}{\bar{\boldsymbol{e}}_i^{\mathrm{T}}\boldsymbol{D}^{-1}\bar{\boldsymbol{e}}_i}\bar{\boldsymbol{e}}_i\bar{\boldsymbol{e}}_i^{\mathrm{T}}\boldsymbol{L}^{\mathrm{H}}\boldsymbol{\Pi}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{\Phi}_{\eta\eta}^{-1}, \qquad (19)$$

$$\boldsymbol{K}_{\mathrm{opt}} = \boldsymbol{I}_B - \sum_{i=1}^{B} \frac{1}{\bar{\boldsymbol{e}}_i^{\mathrm{T}}\boldsymbol{D}^{-1}\bar{\boldsymbol{e}}_i}\bar{\boldsymbol{e}}_i\bar{\boldsymbol{e}}_i^{\mathrm{T}}\boldsymbol{D}^{-1}\boldsymbol{L}^{-1}, \qquad (20)$$

for a given detection order $\boldsymbol{\Pi}$. The matrices $\boldsymbol{L}$ and $\boldsymbol{D}$ are given by the *block Cholesky factorization* of $\boldsymbol{\Phi}$

$$\boldsymbol{\Pi}\boldsymbol{\Phi}\boldsymbol{\Pi}^{\mathrm{T}} = \boldsymbol{L}\boldsymbol{D}\boldsymbol{L}^{\mathrm{H}}, \qquad (21)$$

where $\boldsymbol{L}$ is *block unit lower triangular* with $p \times p$ identity submatrices on the main diagonal and $\boldsymbol{D}$ is *block diagonal* with $p \times p$ submatrices on the main diagonal. From (16), (19) and (20), the MSE ϕ for the given ordering $\boldsymbol{\Pi}$ can be written as

$$\phi = \operatorname{tr}(\boldsymbol{\Phi}_{\varepsilon\varepsilon,\mathrm{p}}) = \sum_{i=1}^{B}\frac{1}{\bar{\boldsymbol{e}}_i^{\mathrm{T}}\boldsymbol{D}^{-1}\bar{\boldsymbol{e}}_i}. \qquad (22)$$

Similar to (12), ϕ depends on the block diagonal entries of $\boldsymbol{D}$. The values of $\boldsymbol{D}$ depend in turn on the block Cholesky factorization (21) which depends on the ordering $\boldsymbol{\Pi}$. A successive algorithm computes the factorization trying to minimize the respective MSE summand in (22) at every iteration. The following table summarizes the pseudocode for the proposed block Cholesky factorization. The factorization algorithm in [6] is a special case of this algorithm for $p = 1$. However, the complexity of the proposed algorithm increases for high values of p due to the combinatorial search in line 5. A low complexity suboptimum version of the algorithm uses the ordering for $p = 1$ for all values of p (lines 5 to 12 are replaced with $q = \operatorname{argmin}_{q'=i,\ldots,B/p}\boldsymbol{\Phi}(q', q')$).

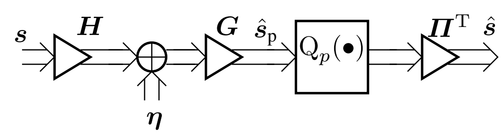

Fig. 3. Reference model for the derivation of the modified quantizer $\mathrm{Q}_p(\bullet)$.

We conclude with the derivation of the decision rule of the modified quantizer $\mathrm{Q}_p(\bullet)$. Fig. 3 depicts the reference model.

[2]In principle, B/p need not be integer. However, we make this restriction for the sake of notational simplicity.

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Algorithm: Calculation of the Block Cholesky Factorization

factorize: $\boldsymbol{\Pi}\boldsymbol{\Phi}\boldsymbol{\Pi}^{\mathrm{T}} = \boldsymbol{L}\boldsymbol{D}\boldsymbol{L}^{\mathrm{H}}$ (find: $\boldsymbol{\Pi}, \boldsymbol{L}, \boldsymbol{D}$)

$\boldsymbol{\Pi} = \boldsymbol{I}_B,\ \boldsymbol{D} = \boldsymbol{0}_{B\times B}$
for $i = 1, \dots, B/p$
  $\boldsymbol{k} = [pi-p+1, pi-p+2, \dots, pi]$
  $\boldsymbol{m} = [pi+1, pi+2, \dots, B]$
  find all combinations of the set of indices $\{pi-p+1, pi-p+2, \dots, B\}$ taken p at a time and place them in $\boldsymbol{q}$ row wise.
  for all rows in $\boldsymbol{q}$
    $\boldsymbol{\Pi}_i = \boldsymbol{I}_B$ whose rows $\boldsymbol{k}$ are exchanged with the rows with indices $\boldsymbol{q}$
    $\boldsymbol{\Pi}_{\text{temp}} = \boldsymbol{\Pi}_i\boldsymbol{\Pi}$
    $\boldsymbol{\Phi}_{\text{temp}} = \boldsymbol{\Pi}_i\boldsymbol{\Phi}\boldsymbol{\Pi}_i$
    $\boldsymbol{D}(\boldsymbol{k},\boldsymbol{k})_{\text{temp}} = \boldsymbol{\Phi}(\boldsymbol{k},\boldsymbol{k})$
    $m = \sum_{i=1}^{p} \frac{1}{\bar{\boldsymbol{e}}_i^{\mathrm{T}}\boldsymbol{D}_{\text{temp}}^{-1}\bar{\boldsymbol{e}}_i}$
  Keep the indices $\boldsymbol{q}$ that give the lowest m
  $\boldsymbol{\Pi}_i = \boldsymbol{I}_B$ whose rows $\boldsymbol{k}$ are exchanged with the rows with indices $\boldsymbol{q}$
  $\boldsymbol{\Pi} = \boldsymbol{\Pi}_i\boldsymbol{\Pi}$
  $\boldsymbol{\Phi} = \boldsymbol{\Pi}_i\boldsymbol{\Phi}\boldsymbol{\Pi}_i^{\mathrm{T}}$
  $\boldsymbol{D}(\boldsymbol{k},\boldsymbol{k}) = \boldsymbol{\Phi}(\boldsymbol{k},\boldsymbol{k})$
  $\boldsymbol{\Phi}(pi-p+1:B,\boldsymbol{k}) = \boldsymbol{\Phi}(pi-p+1:B,\boldsymbol{k})(\boldsymbol{D}(\boldsymbol{k},\boldsymbol{k}))^{-1}$
  $\boldsymbol{\Phi}(\boldsymbol{m},\boldsymbol{m}) = \boldsymbol{\Phi}(\boldsymbol{m},\boldsymbol{m}) - \boldsymbol{\Phi}(\boldsymbol{m},\boldsymbol{k})\boldsymbol{D}(\boldsymbol{k},\boldsymbol{k})(\boldsymbol{\Phi}(\boldsymbol{m},\boldsymbol{k}))^{\mathrm{H}}$
$\boldsymbol{L}^{\mathrm{H}}$ = upper triangular part of $\boldsymbol{\Phi}$

We start from the general ML rule applied to the whole filtered vector $\hat{\boldsymbol{s}}_{\mathrm{p}}$[3]. Similar to (1), we have

$$\begin{aligned}\hat{\boldsymbol{s}} &= \underset{\boldsymbol{s}\,\in\,\mathbb{A}^B}{\operatorname{argmin}}(\hat{\boldsymbol{s}}_{\mathrm{p}} - \boldsymbol{G}\boldsymbol{H}\boldsymbol{s})^{\mathrm{H}}(\boldsymbol{G}\boldsymbol{\Phi}_{\eta\eta}\boldsymbol{G}^{\mathrm{H}})^{-1}(\hat{\boldsymbol{s}}_{\mathrm{p}} - \boldsymbol{G}\boldsymbol{H}\boldsymbol{s})\\ &= \underset{\boldsymbol{s}\,\in\,\mathbb{A}^B}{\operatorname{argmin}}\, \boldsymbol{s}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{G}^{\mathrm{H}}(\boldsymbol{G}\boldsymbol{\Phi}_{\eta\eta}\boldsymbol{G}^{\mathrm{H}})^{-1}\boldsymbol{G}\boldsymbol{H}\boldsymbol{s}\\ &\qquad - 2\Re(\boldsymbol{s}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{G}^{\mathrm{H}}(\boldsymbol{G}\boldsymbol{\Phi}_{\eta\eta}\boldsymbol{G}^{\mathrm{H}})^{-1}\hat{\boldsymbol{s}}_{\mathrm{p}})\\ &= \underset{\boldsymbol{s}\,\in\,\mathbb{A}^B}{\operatorname{argmin}}\, \boldsymbol{s}_{\mathrm{p}}^{\mathrm{H}}(\boldsymbol{L}^{-\mathrm{H}}\boldsymbol{D}^{-1}\boldsymbol{L}^{-1} - \boldsymbol{\Pi}\boldsymbol{\Phi}_{ss}^{-1}\boldsymbol{\Pi}^{\mathrm{T}})\boldsymbol{s}_{\mathrm{p}}\\ &\qquad - 2\Re(\boldsymbol{s}^{\mathrm{H}}\boldsymbol{\Pi}^{\mathrm{T}}\boldsymbol{L}^{-\mathrm{H}}\boldsymbol{N}^{-1}\hat{\boldsymbol{s}}_{\mathrm{p}}),\end{aligned} \tag{23}$$

where $\boldsymbol{s}_{\mathrm{p}} = \boldsymbol{\Pi}\boldsymbol{s}$ and $\boldsymbol{N} = \sum_{i=1}^{B}\frac{1}{\bar{\boldsymbol{e}}_i^{\mathrm{T}}\boldsymbol{D}^{-1}\bar{\boldsymbol{e}}_i}\bar{\boldsymbol{e}}_i\bar{\boldsymbol{e}}_i^{\mathrm{T}}$.[4] Note that the above rule is just another expression for the ML rule in (1). At this point, we define the projector matrix

$$\boldsymbol{\Pi}_i = \sum_{j=1}^{i}\bar{\boldsymbol{e}}_j\bar{\boldsymbol{e}}_j^{\mathrm{T}} \tag{24}$$

and introduce the heuristic rule

$$\begin{aligned}\tilde{\boldsymbol{s}}_{\mathrm{p},i} \;=\; &\underset{\boldsymbol{s}_{\mathrm{p},i}\,\in\,\mathbb{A}^p}{\operatorname{argmin}}\, \boldsymbol{s}_{\mathrm{p}}^{\mathrm{H}}(\boldsymbol{\Lambda}_{i,p}^{\mathrm{H}}\boldsymbol{D}^{-1}\boldsymbol{\Lambda}_{i,p} - \boldsymbol{\Pi}\boldsymbol{\Phi}_{ss}^{-1}\boldsymbol{\Pi}^{\mathrm{T}})\boldsymbol{s}_{\mathrm{p}}\\ &-2\Re(\boldsymbol{s}_{\mathrm{p}}^{\mathrm{H}}\boldsymbol{\Lambda}_{i,p}^{\mathrm{H}}\boldsymbol{N}^{-1}\hat{\boldsymbol{s}}_{\mathrm{p}}),\end{aligned} \tag{25}$$

for $i = 1, \dots, B/p$, where $\boldsymbol{\Lambda}_{i,p} = \boldsymbol{\Pi}_{pi}\boldsymbol{L}^{-1}\boldsymbol{\Pi}_{pi}^{\mathrm{T}}$. The heuristic rule computes the estimates for $\hat{\boldsymbol{s}}$ successively in B/p iterations. $\boldsymbol{s}_{\mathrm{p}} = [\tilde{\boldsymbol{s}}_{\mathrm{p},1}, \dots, \tilde{\boldsymbol{s}}_{\mathrm{p},i-1}, \boldsymbol{s}_{\mathrm{p},i}, \boldsymbol{0}, \dots, \boldsymbol{0}]^{\mathrm{T}}$ consists of the already computed parts $\tilde{\boldsymbol{s}}_{\mathrm{p},j},\ j = 1, \dots, i-1$ and the not yet detected part $\boldsymbol{s}_{\mathrm{p},i}$. Due to the projector matrix $\boldsymbol{\Pi}_i$, only the first i groups of p symbols of $\hat{\boldsymbol{s}}_{\mathrm{p}}$ are used at the i-th step.

[3] Note that in the following, $\hat{\boldsymbol{s}}_{\mathrm{p}}$ does not include the feedback like in Fig. 1.
[4] In (23), we substituted $\boldsymbol{G}$ from (19).

Using properties of the matrices $\boldsymbol{L}, \boldsymbol{D}$, and $\boldsymbol{\Pi}_i$, we can further simplify (25)

$$\begin{aligned}\tilde{\boldsymbol{s}}_{\mathrm{p},i} = \underset{\boldsymbol{s}_{\mathrm{p},i}\,\in\,\mathbb{A}^p}{\operatorname{argmin}}\Bigg\|&\left([\boldsymbol{D}^{-1}]_i - \frac{1}{\sigma_s^2}\boldsymbol{I}_p\right)^{1/2}\boldsymbol{s}_{\mathrm{p},i}\\ &-\left([\boldsymbol{D}^{-1}]_i - \frac{1}{\sigma_s^2}\boldsymbol{I}_p\right)^{-1/2}[\boldsymbol{N}^{-1}]_i\hat{\boldsymbol{s}}_{l,i}\Bigg\|_2^2.\end{aligned} \tag{26}$$

Here, $\hat{\boldsymbol{s}}_{l,i}$ is the signal $\hat{\boldsymbol{s}}_{\mathrm{p}}$ after the loopback filter $\boldsymbol{F}$, as computed during the operation of the loopback of Fig. 1. $[\bullet]_i$ denotes the i-th diagonal $p\times p$ block of a matrix. Rule (26) describes the ML-like operation of the modified quantizer $\mathrm{Q}_p(\bullet)$. As we can see, the calculation of $\tilde{\boldsymbol{s}}_{\mathrm{p},i}$ results from a closest point search in a p-dimensional subspace, which defines the complexity of the detector.

For $p = B$, the projector matrix in (24) is $\boldsymbol{\Pi}_B = \boldsymbol{I}_B$ and we have only one iteration in (25) with the whole vector $\hat{\boldsymbol{s}}_{\mathrm{p}}$. Thus, the heuristic rule of (25) is identical to the ML rule in (23) and SDFE converges to MLD for $p = B$.

For $p = 1$, the block Cholesky factorization in (21) is equal to the factorization in (10). $\boldsymbol{D}$ is diagonal, $\boldsymbol{N} = \boldsymbol{D}$ and the filter solutions (20) and (19) are equal to (9). It can be shown that the modified quantizer $\mathrm{Q}_p(\bullet)$ using (26), takes the same decisions as the ML quantizer $\mathrm{Q}(\bullet)$ in DFE. Thus, for $p = 1$, SDFE converges to DFE.

For values of p between 1 and B, we get a scaled system whose performance lies between DFE and MLD. The closest-point search in (25) is performed in a p-dimensional subspace instead of a B-dimensional subspace as for MLD. The resulting diversity order is equal to $N_{\mathrm{R}} - B + p$, while the complexity, which is a function of p, scales between the quadratic complexity of DFE and the exponential complexity of MLD (cf. [8]).

B. Lattice Reduction Aided Scaled DFE

We can combine SDFE with LR aided detection. This requires the use of the equivalent real-valued model, for which $p = 1, \dots, 2B$ (cf. [16]). For MMSE-LR-SDFE, we use the extended channel $\boldsymbol{H}_{\text{ext}} = [\boldsymbol{H}_{\mathrm{r}}^{\mathrm{T}}, \sigma_\eta/\sigma_s\boldsymbol{I}_{2B}^{\mathrm{T}}]^{\mathrm{T}}$ (cf. [14])[5] followed by ZF-SDFE. For the ZF variant of SDFE we use $\boldsymbol{\Phi} = (\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}})^{-1}$ in (11) and omit the terms $\frac{1}{\sigma_s^2}\boldsymbol{I}_p$ in (26). Let $\boldsymbol{s}_{\mathrm{r}} = \alpha(\bar{\boldsymbol{s}} + \frac{1}{2}\boldsymbol{1}_{2B})$, where $\bar{\boldsymbol{s}} \in \{-\sqrt{M}, \dots, \sqrt{M}-1\}^{2B}$. α depends on the modulation alphabet (cf. [14]). The basis reduction is performed on $\boldsymbol{H}_{\text{ext}}$, i.e. $\boldsymbol{H}_{\text{red}} = \boldsymbol{H}_{\text{ext}}\boldsymbol{T}$, where $\boldsymbol{T}$ is a unimodular integer matrix. For LR-SDFE, the ZF-SDFE filters are computed using $\boldsymbol{H}_{\text{red}}$. Before entering the SDFE loop we perform the normalization step

$$\boldsymbol{z} = \frac{1}{\alpha}\boldsymbol{G}_{\text{ZF}}\boldsymbol{x}_{\mathrm{r}} - \frac{1}{2}\boldsymbol{G}_{\text{ZF}}\boldsymbol{H}_{\text{ext}}\boldsymbol{1}_{2B}, \tag{27}$$

where $\boldsymbol{z} = \boldsymbol{T}^{-1}\boldsymbol{s}_{\mathrm{r}}$ and $\boldsymbol{x}_{\mathrm{r}} = [(\boldsymbol{H}_{\mathrm{r}}\boldsymbol{s}_{\mathrm{r}}+\boldsymbol{\eta}_{\mathrm{r}})^{\mathrm{T}}; \boldsymbol{0}^{\mathrm{T}}]^{\mathrm{T}}$. The estimate of $\boldsymbol{z}$ is found by the modified quantizer

$$\tilde{\boldsymbol{z}}_{\mathrm{p},i} = \underset{\boldsymbol{z}_{\mathrm{p},i}\,\in\,\mathbb{Z}^p}{\operatorname{argmin}}\left\|\left([\boldsymbol{D}^{-1}]_i\right)^{\frac{1}{2}}\boldsymbol{z}_{\mathrm{p},i} - \left([\boldsymbol{D}^{-1}]_i\right)^{-\frac{1}{2}}[\boldsymbol{N}^{-1}]_i\hat{\boldsymbol{z}}_{l,i}\right\|_2^2.$$

[5] The index ’r’ denotes the real-valued model.

Note that the above closest-point search is performed in $\mathbb{Z}^p$. Finally, the estimate of $\boldsymbol{s}_\mathrm{r}$ is

$$\tilde{\boldsymbol{s}}_\mathrm{r} = \alpha \boldsymbol{T} \tilde{\boldsymbol{z}} + \frac{1}{2}\alpha \mathbf{1}_{2B}. \tag{28}$$

VI. Simulation Results and Conclusions

The channel used for our simulations has i.i.d. unit variance Rayleigh fading coefficients and emulates an idealized rich-scattering environment. The noise $\boldsymbol{\eta}$ is white with covariance matrix $\boldsymbol{\Phi}_{\eta\eta} = \sigma_\eta^2 \boldsymbol{I}_B$. Figs. 4 and 5 show the average uncoded *bit error rate* (BER) over $E_\mathrm{b}/N_0 = \frac{N_\mathrm{R}\sigma_s^2}{R_\mathrm{b}\sigma_\eta^2}$, where R_b is the number of information bits per symbol.

Fig. 4 compares ZF-SDFE with ZF-LR aided detection, linear equalization and DFE [13]. The variant with the suboptimum block Cholesky factorization is also depicted. Although the LR curves exploit the full channel diversity, SDFE clearly outperforms them in the high-BER-low-SNR region. The shortcoming of ZF-LR in this region is due to a quantization loss at the translation of the result to the initial basis (cf. [13]). We do not show the results for ZF-LR-SDFE, since it provided insignificant gains with respect to ZF-LR detection.

Fig. 5 compares MMSE-DFE with MMSE-LR-SDFE.[6] We see that MMSE-LR-SDFE ($p = 1$, [14]) is the favorable technique due to its low complexity and superb performance. The margin between MMSE-LR-SDFE and MLD is now much smaller than in Fig. 4, and for $p > 1$, the gains are insignificant given the much higher computational complexity.

From the above results, we draw a two-fold conclusion. First, ZF-SDFE is an interesting alternative to standard DFE (in accordance to [7]) and also LR aided ZF detection (not considered in [7]). Second, one has to resort to the simple MMSE-LR-DFE of [14], when using MMSE detectors (which clearly outperform ZF detectors).

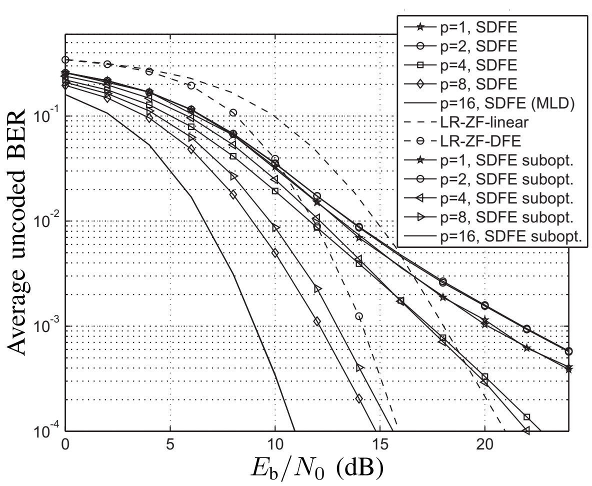

Fig. 4. $B = N_\mathrm{R} = 8$, ZF detection, QPSK modulation.

[6]We do not show the results for MMSE-SDFE, since they are very similar to the results for MMSE-DFE.

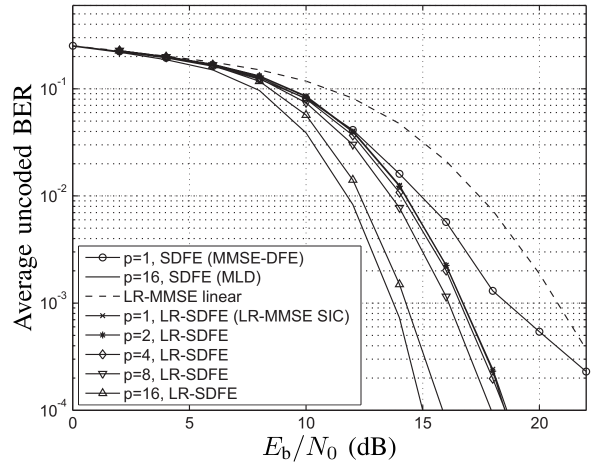

Fig. 5. $B = N_\mathrm{R} = 8$, MMSE detection, 16-QAM modulation, LR-SDFE uses the suboptimum block Cholesky factorization.

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