On Phase-space Equalization of CPM in Severe Intersymbol Interference

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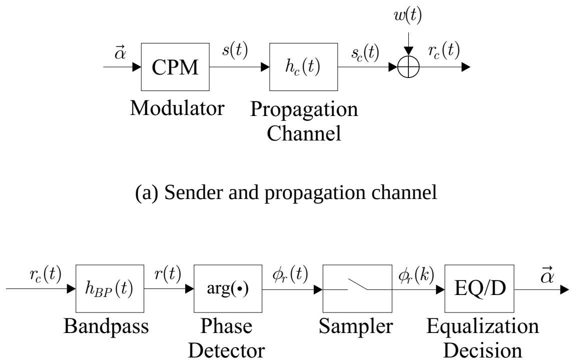
Abstract— **Future wireless personal and sensor networks will be established by a huge number of mobile nodes with transceiver capabilities. In this context, cost aware and power efficient technologies have attracted considerable attention. A proven technology for physical layer cost reduction is the use of Continuous Phase Modulation (CPM) and hard (amplitude) limiting receiver structures that substantially reduce the cost for the analogue domain of the receivers. The major penalty of those systems is their failure in severe intersymbol-interference (ISI) environments even in combination with Maximum Likelihood Sequence Estimation (MLSE) decoders.**
In this paper we present a family of new MLSE based decoders that improve the ISI performance to such an extent that receiver structures with hard limiter become an attractive alternative to the expensive linear receivers. The most powerful equalizers for such nonlinear receivers are phase-space equalizers: MLSE decoders for phase detection generally use a Look-Up-Table (LUT) consisting of the phase values of the desired signal [1]. The key idea of the new decoder is to use a LUT that additionally includes amplitude values associated with the phase values. Based on this information metrics with different and feasible complexity are derived. By means of simulations the performance of a GMSK transmission system is examined for a set of representative channels. Resulting bit error ratios (BERs) are compared with BERs for state-of-the-art decoders.

I. Introduction

Future wireless networks will provide high data rates, multiple services, high flexibility and will be heterogenous. The networks will be ad hoc organized and a huge number of nodes can participate in an entire network. For personal, pervasive and in particular for sensor networks low cost and low power technologies will play an essential role [2] [3] [4]. Hard amplitude limiting receivers are known as very low cost receiver types. Compared to linear receivers power consumption and hardware effort can be reduced. Due to the amplitude limitation the dynamic range of those systems is minimized and no adaptive gain control (AGC) circuit is required.
However, these receivers fail in multipath propagation where severe intersymbol interference (ISI) is present. State-of-the-art receivers use phase values to perform a Maximum Likelihood Sequence Estimation (MLSE)[1]. Reference phase values are stored in a Look-Up-Table (LUT) which is the basis for the computing of a decision cost function. These phase values (also denoted desired phases) are approximately a priori known – at least for ideal propagation channels.
The key idea of this paper is to extend the LUT by amplitude values which means that the LUT is complex. Therefore, an estimation of the channel impulse response (CIR) is performed using an appropriate training sequence. Note, that the estimation is only based on phase samples which can be provided by a hard limiter receiver – no additional envelope detection is required. The estimation of the complex CIR on basis of phase samples is shown



(a) Sender and propagation channel

(b) Phase detection receiver

Fig. 1. System model

in [5]. The complex LUT allows to deduce new Maximum Likelihood (ML) based decoders that outperform today's state-of-the-art receivers.
In Section II the considered system model with the examined model channels is introduced. Section III describes the new ML decoder and shows bit error ratio (BER) simulation results compared to a "State-of-the-art" decoder. In Section IV some approximations of the metric are given which establish a family of ML based decoders with scalable complexity and performance.

II. System Model

Representative for Continuous Phase Modulation (CPM) a GMSK transmission system is considered. The block diagram is shown in Fig.1. The system is described in its equivalent baseband representation. The transmit signal is

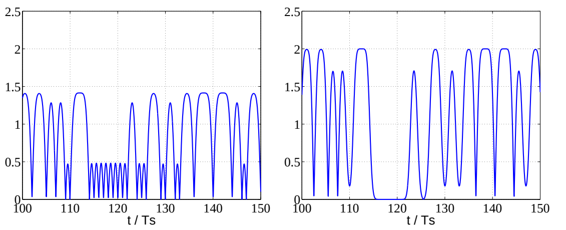
$$\begin{aligned} s(t) &= \exp\left(j\phi_S(t)\right) \\ &= \exp\left(j\pi h \cdot \sum_k \left(\alpha(k)\cdot q\left(t-kT_S\right)\right)\right), \end{aligned} \quad (1)$$

where $\alpha(k) \in \{-1,1\}$ are binary symbols and $q(t)$ is the phase pulse.

$$q(t) = \int_{-\infty}^{t} g(\tau)d\tau \quad (2)$$

$g(t)$ is the GMSK frequency pulse as described in [6] ($BT = 0.3$). With the impulse response of the propagation channel $h_c(t)$ the input signal of the receiver is:

$$r_c(t) = s(t) * h_c(t) + w(t) \quad (3)$$



(a) $1-D$ channel (b) $1-D^2$ channel

Fig. 2. Amplitude of model channels

$w(t)$ is complex symmetrical additive white Gaussian noise (AWGN) with the two sided spectral noise power density $N_0/2$. With

$$\begin{aligned} r(t) &= r_c(t) * h_{BP}(t) \\ &= |r(t)| \cdot \exp\left(j\phi_r(t)\right) \end{aligned} \tag{4}$$

the output of the sampler is the sum of a desired (reference) phase $\phi_d(k)$ value and a noise component $\Delta\phi(k)$:

$$\phi_r(k) = \phi_d(k) + \Delta\phi(k), \tag{5}$$

phases distributed within $[-\pi, \pi]$.

The simulations in this paper are based on exemplary model channels representing typical propagation environments. An ideal propagation channel

$$h_c(t) = \delta(t) \tag{6}$$

represents good and moderate conditions. The second model channel introduces large and critical ISI. The impulse response is given by:

$$h_c(t) = \delta(t) - \delta(t - T_S) \tag{7}$$

It is denoted as $1-D$-channel, where D represents the delay operator with a delay of one symbol duration $1 \cdot T_S$. This channel introduces amplitude zeros as exemplarily shown in Fig.2(a). The third channel is an example for an extremely critical ISI channel.

$$h_c(t) = \delta(t) - \delta(t - 2T_S) \tag{8}$$

This $1-D^2$-channel introduces amplitude zeros over several symbol periods which theoretically can result in a full zero-amplitude burst. The amplitude is shown in Fig.2(b).

The bandpass is assumed to have Nyquist characteristic.

III. New Maximum Likelihood Decoder with known CIR

Generally, the Maximum Likelihood approach for phase detection of a symbol sequence $\vec{\alpha}$ is given by the maximum of the probability density function:

$$\hat{\vec{\alpha}} = \max_{\vec{\alpha}^{(l)}} p\left(\vec{\phi}_r \middle| \vec{\alpha}^{(l)}\right) \tag{9}$$

Up to now, since only phase samples are available at the receiver, the state-of-the-art decoders use a phase LUT. Therefore, this approach could be written as

$$\max_l p\left(\vec{\phi}_r \middle| \vec{\phi}_d^{(l)}\right), \tag{10}$$

where $\vec{\phi}_d^{(l)}$ is the noiseless ("desired") phase sequence associated with the lth symbol sequence $\vec{\alpha}^{(l)}$. This probability density actually depends on the instantaneous signal-to-noise ratio (SNR). Since the SNR is not known it is assumed to be constant in literature and the density is approximated: In [7] a Gaussian approximation is used and in [1] the density is computed for a constant SNR. This method yields best results to the author's knowledge and is called "State-of-the-art" decoder.

In [5], for the first time, a method is proposed to estimate the complex desired values $d^{(l)}(k)$ on basis of phase samples.

$$d^{(l)}(k) = |d^{(l)}(k)| \cdot \exp\left(\phi_d^{(l)}(k)\right) \tag{11}$$

Neglecting the impact of the bandpass filter, the complex desired values $d^{(l)}(k)$ correspond to samples of the channel's output signal $s_c(t)$. It is shown that $d^{(l)}(k)$ can be estimated nearly perfectly even for critical channels [5] [8]. With it, a new ML decoder with known CIR can be deduced:

$$\max_l p\left(\vec{\phi}_r \middle| \vec{d}^{(l)}\right) \tag{12}$$

In contrast to standard MLSE approaches the dimension of the received signal (real) and the desired signal (complex) are different. It is assumed that $d(k)$ is perturbed by complex white Gaussian noise. Therefore, the phase errors are statistically independent and (12) can be factorized:

$$\begin{aligned} &\max_l p\left(\vec{\phi}_r \middle| \vec{d}^{(l)}\right) \\ &= \max_l \prod_{k=1}^{K} p\left(\phi_r(k) \middle| d^{(l)}(k)\right) \\ &= \max_l \prod_{k=1}^{K} p\left(\Delta\phi^{(l)}(k) \middle| |d^{(l)}(k)|\right) \end{aligned} \tag{13}$$

The probability density $p\left(\Delta\phi^{(l)}(k) \middle| |d^{(l)}(k)|\right)$ is given by Pawula in [9]:

$$\begin{aligned} &p_{\Delta\phi|\rho}(\Delta\phi(k)|\rho(k)) \\ &= \frac{\exp(-\rho(k))}{2\pi} + \sqrt{\frac{\rho(k)}{4\pi}} \cdot \exp\left(-\rho(k)\sin^2(\Delta\phi(k))\right) \\ &\cdot \cos(\Delta\phi(k)) \cdot \operatorname{erfc}\left(-\sqrt{\rho(k)}\cos(\Delta\phi(k))\right) \end{aligned} \tag{14}$$

$$\rho(k) = \frac{|d(k)|^2}{\sigma_w^2} \tag{15}$$

The signal-to-noise ratio $\rho(k)$ is also known from the channel estimation. So, the new decoder can be implemented. σ_w^2 is a system parameter which is determined by the quality of the amplifier stages of the receiver. For practical reasons the Log-Likelihood function is implemented and the resulting metric is:

$$\min_l \left\{ -\sum_{k=1}^{K} \ln p_{\Delta\phi|\rho}\left(\Delta\phi^{(l)}(k) \middle| \rho^{(l)}(k)\right) \right\} \tag{16}$$

a) Simulation results: Fig.3 shows simulation results for the $1-D$-channel. At a BER of 10^{-2} the performance of the State-of-the-art decoder is about 6dB worse than the performance

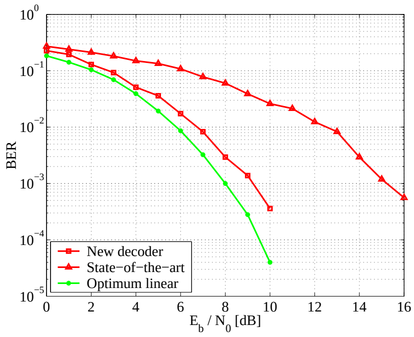


Fig. 3. BER for $1 - D$-channel

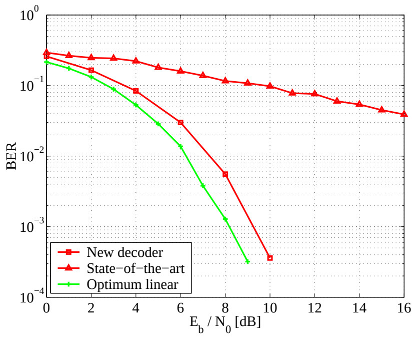


Fig. 4. BER for $1 - D^2$-channel

of the new decoder. Its loss compared to the optimum linear receiver is only about 1dB. The same simulations have been done for the $1 - D^2$-channel. In this case the State-of-the-art decoder fails more or less whereas the new decoder keeps unaffected (Fig.4).

IV. Metric Approximations

The complexity of the receiver is essentially determined by the metric of the decoder. With the density in (14) the metric of the new phase decoder (16) is very complex. Therefore, approximated metrics with lower complexity are derived, here. All metrics assume statistically independent noise samples. For a linear receiver the Euclidean metric is optimal:

$$\min_l \sum_k \left| r(k) - d^{(l)}(k) \right|^2 \tag{17}$$

The phase detector is not able to detect $|r(k)|$. The idea is to replace $|r(k)|$ by its hypothesis $|d^{(l)}(k)|$:

$$\begin{aligned} &\min_l \sum_k \left| |r(k)| \exp(j\phi_r(k)) - |d^{(l)}(k)| \exp(j\phi_d^{(l)}(k)) \right|^2 \\ &\approx \min_l \sum_k \left| |d^{(l)}(k)| \exp(j\phi_r(k)) - |d^{(l)}(k)| \exp(j\phi_d^{(l)}(k)) \right|^2 \\ &= \min_l \sum_k |d^{(l)}(k)|^2 \cdot \left| \exp(j\phi_r(k)) - \exp(j\phi_d^{(l)}(k)) \right|^2 \end{aligned} \tag{18}$$

The result is a new metric that weights the phase difference by the amplitude. Since large phase errors occur usually at low amplitudes their relevance decreases through this metric.

The weighting of the phase difference by the amplitude seems reasonable whereas the usage of the squared of the amplitude seems to be arbitrary. The metric can be modified by the exponent of the amplitude:

$$\min_l \sum_k |d^{(l)}(k)|^\kappa \cdot \left| \exp(j\phi_r(k)) - \exp(j\phi_d^{(l)}(k)) \right|^2 \tag{19}$$

Simulations have shown that a linear amplitude weighting yields best performance:

$$\kappa_{opt} = 1 \tag{20}$$

A further simplification is to approximate the difference of the phasors from (19) by the phase difference.

$$\begin{aligned} &\min_l \sum_k |d^{(l)}(k)| \cdot \left| \phi_r(k) - \phi_d^{(l)}(k) \right|^2 \\ &= \min_l \sum_k |d^{(l)}(k)| \cdot \left| \Delta\phi^{(l)}(k) \right|^2 \end{aligned} \tag{21}$$

with $|\Delta\phi^{(l)}(k)| \in [0, \pi]$. This metric is the simplest way to use the weighting function.

From (14) can be seen that not the amplitude but the instantaneous SNR $\rho(k)$ determines the probability density function of the phase. (Using $\rho(k)$ instead of $|d(k)|^2$ in the metrics does not change the decoder's performance because the minimum search is independent of a constant factor.)

Equation (19) looks similar to Ariyavisitakul's approximation of $p_{\Delta\phi|\rho}(\Delta\phi(k)|\rho(k))$ in [10] which results in the metric:

$$\begin{aligned} \min_l = \sum_k \Bigg(&-\frac{1}{2} \ln \frac{\rho^{(l)}(k)}{\pi} + \\ &\rho^{(l)}(k) \left| \exp(j\phi_r(k)) - \exp(j\phi_d^{(l)}(k)) \right|^2 \Bigg) \end{aligned} \tag{22}$$

This metric has a major advantage compared to the simpler weighting metrics. For the metrics in (19) and (21) zero amplitudes yield zero costs: For propagation channels that introduce zeros over several symbol periods, like the $1 - D^2$-channel, a useful decision is not possible, because the transitions associated with the zeros have minimum costs anyway. However, the first term of the sum in (22) prevents this effect. For $\rho(k) \ll 1$ this term determines the sum and causes higher costs for zero amplitudes.

A simpler Gaussian approximation

$$p(\Delta\phi(k)) \approx \frac{1}{\sqrt{2\pi}\sigma_{\Delta\phi}^{(l)}(k)} \exp\left(-\frac{(\Delta\phi(k))^2}{2(\sigma_{\Delta\phi}^{(l)}(k))^2} \right) \tag{23}$$

with

$$\sigma_{\Delta\phi}^{(l)}(k) = \frac{\sigma_w}{\sqrt{2} \cdot |d^{(l)}(k)|} \tag{24}$$

results in the metric:

$$\min_l \sum_k \left(-\sigma_w^2 \ln |d^{(l)}(k)| + |d^{(l)}(k)|^2 (\Delta\phi^{(l)}(k))^2 \right) \tag{25}$$

This metric looks similar to (21) and again promises to outperform (21) for the $1 - D^2$-channel. For low SNR this approximation deteriorates because the mod 2π distribution of the phase is

TABLE I
COMPLEXITY OF METRICS

Label	**Equation**	+	$\cdot$	ln
simpl. AWP	(21)	1	2	0
AWP-1	(19)	3	3	0
Gauss	(25)	2	4	1
Ariya	(22)	4	5	1
CGU	(28)	2	4	1Q(!)

ignored. Therefore, the integral from $-\pi$ to π over this density is not 1. A normalized Gaussian distribution where the integral is 1 is given in (26) where the index k is neglected for a better visibility:

$$p\left(\Delta\phi\right) \approx \begin{cases} \frac{1}{Q\left(-\frac{\pi}{\sigma_{\Delta\phi}}\right)-Q\left(\frac{\pi}{\sigma_{\Delta\phi}}\right)} \cdot \frac{1}{\sqrt{2\pi}\sigma_{\Delta\phi}} \cdot \exp\left(-\frac{\Delta\phi^2}{2\sigma^2_{\Delta\phi}}\right), \left|\Delta\phi\right| \leqslant \pi \\ 0, \text{ else} \end{cases} \tag{26}$$

$Q(x)$ is the well known function:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_x^\infty \exp\left(\frac{\zeta^2}{2}\right) d\zeta \tag{27}$$

A better way to handle the phases being outside of $[-\pi,\pi]$ is to approximate their distribution by a uniform distribution. Here, a compound Gaussian and uniform approximation is proposed:

$$p\left(\Delta\phi\right) \approx \begin{cases} a_n \cdot \frac{1}{\sqrt{2\pi}\sigma_{\Delta\phi}} \cdot \exp\left(-\frac{\Delta\phi^2}{2\sigma^2_{\Delta\phi}}\right) + p_{uni}\text{ , for } \left|\Delta\phi\right| \leqslant \pi \\ 0, \text{ else} \end{cases} \tag{28}$$

a_n is determined in a way that the integral is 1.
An estimation of the complexity is done by means of the number of required multiplications, additions, logarithms. The computing of a_n from (28) is difficult. Therefore, a_n is assumed to be quantized and stored in additional memory. This reduces the number of multiplications and additions but increases the hardware effort. So, it must not be compared with the other metrics except if the logarithms are also realized this way. As shown in Table I the simplified amplitude-weighted phase metric (AWP) needs the fewest number of operations. The metrics are sorted according to its complexity.

b) Simulation results: The BER performance of the metrics is shown in Fig.5 over the SNR (behind the bandpass). Surprisingly, the performance of the AWP-2 metric is much worse than the performance of the simplified AWP. AWP-1 is not shown because it is quite similar to its simplified version. For SNR $<$ 2dB the compound Gaussian and uniform distribution ($CGU : p_{uni} = 1/(20\pi)$) yields best results. For SNR $\geq$ 4dB Ariyavisitakul's approximation achieves the best performance. The Gaussian approximation is quite good between -4dB and 0dB.

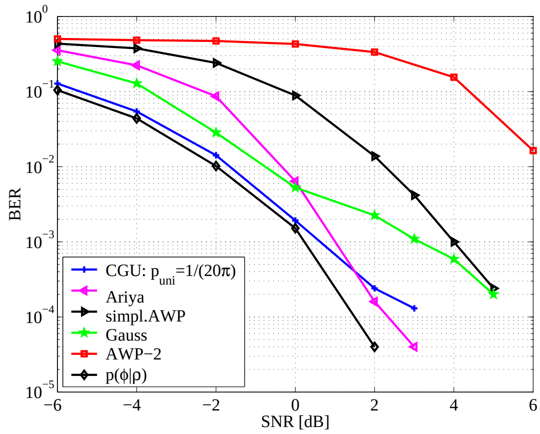


Fig. 5. BER for metric approximations , $1-D$-channel

V. CONCLUSION

A new MLSE decoder for phase detection with hard limiting receiver was presented. In severe ISI channels it outperforms state-of-the-art decoders to a very high extent. Even in critical ISI this receiver structure achieves a performance which is competitive to that of expensive linear structures. With a family of simpler approximated metrics the employment of the new decoder becomes scalable in complexity and performance.

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