On the Optimal Power Allocation for Nonregenerative OFDM Relay Links

Ingmar Hammerström and Armin Wittneben
Communication Technology Laboratory, ETH Zurich, 8092 Zurich, Switzerland
{hammerstroem, wittneben}@nari.ee.ethz.ch

Abstract— **We consider a two-hop communication scheme using OFDM modulation. The relay is assumed to be nonregenerative (or amplify-and-forward). We examine the possibilities of power allocation (PA) over the frequency subchannels at source and relay, with respect to separate transmit power constraints, if channel state information at the transmitter (CSIT) is given.**

We provide the optimal PA at the relay (source) that maximizes the instantaneous rate for a given source (relay) PA. Furthermore, we show that alternate, separate optimization of source and relay PA converges to the solution of the joint optimization of source and relay PA. To further enhance the rate of the considered scheme, the OFDM subchannels of the source to relay and relay to destination channel can be paired according to their actual magnitude.

In the case of a joint sum power constraint, we propose to allocate the transmit power between source and relay with respect to average channel attenuation of first and second hop. It is shown that this leads to a information rate that is near to the optimal rate achieved by a joint optimization of source and relay PA with joint transmit power constraint.

I. Introduction

Cooperative relaying strategies have become a major topic in the wireless research community. First research results on relay channels were obtained in the seventies in [1]–[3]. The interest in this topic was re-initiated recently by the seminal papers [4]–[6] and triggered a large amount of work in this area.

Most of the literature available today consider frequency-flat fading. In [7] cooperative diversity protocols are analyzed for combating multi-path fading and shadowing effects in a wireless network and thereby increasing the robustness of the wireless connection between source and destination. In [8] a form of spatial diversity is investigated, in which diversity gains are achieved via the cooperation of two mobile users that communicate with a base station. It is shown that cooperation leads not only to an increase in uplink rate for both users but also to a more robust system, where user rates are less sensitive to channel variations. In [9], [10] and [11] optimal PAs between source and relay (regenerative and nonregenerative) are discussed for the case that both share a total amount of transmit power over the two time-slots required for relaying. In [12] the optimal gain allocation is presented which maximizes the instantaneous rate for multiple nonregenerative coherent relays, retransmitting in the same bandwidth. This gain allocation can be interpreted as a distributed maximum-ratio combiner.

The case of cooperative relaying in frequency-selective fading channels is much less examined so far. In [13], the authors determine PAs for multiple orthogonal nonregenerative relays (which is the same as having one relay using OFDM) maximizing the average SNR of the maximum-ratio combiner at the destination node. In [14] the information rate of OFDM and OFDMA networks consisting of one source/destination pair and multiple relays is examined. In the case of OFDM only one amplification gain is used for all subcarriers at the nonregenerative relay. Therefore, the rate is not optimized with respect to the frequency-selective channel. In the case of OFDMA only one nonregenerative relay is assigned to one subcarrier, which results in an optimization problem that can be solved by integer programming. In [15] distributed Alamouti coding [16] for OFDM relaying links is proposed. Furthermore, a closed form expression for the bit error ratio (BER) assuming BPSK modulation is presented.

In this paper we focus on a two-hop AF relay link using OFDM modulation. The transmitted signals are subject to frequency-selective fading channels. We examine the possibilities of PA over the frequency subchannels at source and relay to maximize the instantaneous rate of this link. It is assumed that source and relay have their own separate transmit power constraint. We give the optimal PA at the relay (or source) that maximizes the instantaneous rate for a given source (or relay) PA. Furthermore, we show that an alternate, separate optimization of source and relay PA converges to the solution of the joint optimization of source and relay PA. To further enhance the achievable rate, the OFDM subchannels of the source to relay and relay to destination channel are paired according to their actual magnitude.

In the case of a joint sum power constraint we propose to allocate the transmit power between source and relay with respect to average channel attenuation of first and second hop. It is shown that this leads to a rate that is near to the optimal rate achieved by the joint optimization of source and relay PA with joint transmit power constraint.

The remainder of the paper is organized as follows. In the next section the system model is introduced. In section III we present our PAs. Performance results are presented in section IV. Conclusions are given in the last section.

II. System Model

We consider a two-hop relay link consisting of one source/destination pair and one nonregenerative relay. For broadband communication between the nodes OFDM is used, i.e., the available bandwidth is divided into N_{fft} subcarriers in which the channel is assumed to be frequency-flat. The channel coefficient of the k-th subcarrier between source and destination, source and relay, and relay and destination is denoted by $h_{0,k}$, $h_{1,k}$, and $h_{2,k}$, respectively.

We assume that the source sends data with power $P_{\mathrm{s},k}$ on the k-th subcarrier to relay and destination in a first time slot, while the relay retransmits an amplified version of the received data to the destination in a second time slot. The relay multiplies the received signal by the factor

$$g_k = \sqrt{\frac{P_{\mathrm{r},k}}{P_{\mathrm{s},k}\left|h_{1,k}\right|^2 + \sigma_{\mathrm{r}}^2}} \tag{1}$$

to ensure a relay transmit power on the k-th subcarrier of $P_{\mathrm{r},k}$. The noise variance at the relay within one OFDM subchannel is denoted by σ_{r}^2. The signal to noise ratio (SNR) at the output of an temporal maximum-ratio combiner, which combines the signal contributions of both time slot at the destination is then given by

$$\begin{aligned}\rho_k &= \frac{P_{\mathrm{s},k}\left|h_{2,k}g_k h_{1,k}\right|^2}{\sigma_{\mathrm{d}}^2 + \sigma_{\mathrm{r}}^2\left|g_k h_{2,k}\right|^2} + \frac{P_{\mathrm{s},k}\left|h_{0,k}\right|^2}{\sigma_{\mathrm{d}}^2} \\ &= \frac{P_{\mathrm{s},k}a_k \cdot P_{\mathrm{r},k}b_k}{1 + P_{\mathrm{s},k}a_k + P_{\mathrm{r},k}b_k} + P_{\mathrm{s},k}c_k,\end{aligned} \tag{2}$$

where $a_k = \frac{|h_{1,k}|^2}{\sigma_{\mathrm{r}}^2}$, $b_k = \frac{|h_{2,k}|^2}{\sigma_{\mathrm{r}}^2}$, and $c_k = \frac{|h_{0,k}|^2}{\sigma_{\mathrm{d}}^2}$. The noise variance at the destination within one OFDM subchannel is denoted by σ_{d}^2. If the destination only receives the signal from the relay in the second time slot the SNR is

$$\rho_k = \frac{P_{\mathrm{s},k}a_k \cdot P_{\mathrm{r},k}b_k}{1 + P_{\mathrm{s},k}a_k + P_{\mathrm{r},k}b_k}. \tag{3}$$

The instantaneous rate of the communication between source and destination with the half-duplex relay on the k-th subcarrier is therefore given by

$$C_{\mathrm{I},k} = \frac{1}{2}\log_2\left(1 + \rho_k\right), \tag{4}$$

where the factor $1/2$ is due to the two time slots (channel uses) which are needed in this traffic pattern. Since different codebooks can be used for each subcarrier, the instantaneous rate over all subcarriers is

$$C_{\mathrm{I}} = \sum_{k=1}^{N_{\mathrm{fft}}} C_{\mathrm{I},k}. \tag{5}$$

III. Optimization without Direct Link

In the following we assume that the destination is not able to receive the signal from the source directly, which may result from high shadowing between both nodes. The considered SNR at the destination is therefore given by (3).

A. Separate power constraints

We want to optimize the transmit PA of the relay and/or source over the N_{fft} subcarriers with respect to separated sum power constraints at both nodes, i.e.,

$$\sum_{k=1}^{N_{\mathrm{fft}}} P_{\mathrm{s},k} = \mathbf{1}^T\mathbf{p}_{\mathrm{S}} = P_{\mathrm{S}}, \tag{6}$$

$$\sum_{k=1}^{N_{\mathrm{fft}}} P_{\mathrm{r},k} = \mathbf{1}^T\mathbf{p}_{\mathrm{R}} = P_{\mathrm{R}}. \tag{7}$$

The values of the transmit power over the subcarriers are thereby stacked in the vectors $\mathbf{p}_{\mathrm{S}} = [P_{\mathrm{s},1}, P_{\mathrm{s},2}, \ldots, P_{\mathrm{s},N_{\mathrm{fft}}}]^T$ and $\mathbf{p}_{\mathrm{R}} = [P_{\mathrm{r},1}, P_{\mathrm{r},2}, \ldots, P_{\mathrm{r},N_{\mathrm{fft}}}]^T$, respectively. We furthermore assume that all parameters which are not subject of the current optimization problem are known to the optimizing node. This includes channel coefficients and noise variances.

Firstly, we derive the optimal transmit PA over the subcarriers at the relay assuming a given (e.g. uniform) transmit PA at the source. Secondly, we derive the optimal transmit PA over the subcarriers at the source assuming a given transmit PA at the relay. Thirdly, we propose an alternating separate optimization of relay and source transmit PA. In the section IV we show that this alternating optimization converges to the rate of a joint optimization of the PA at source and relay with separate power constraints (6) and (7).

1) Optimization of Relay PA: We assume that the vector $\mathbf{p}_{\mathrm{S}}$ is given. Hence, we only want to optimize $\mathbf{p}_{\mathrm{R}}$ such that the instantaneous rate C_{I} in (5) is maximized. Mathematically spoken, this is

$$\begin{aligned}\mathbf{p}_{\mathrm{R}}^{\star} = \arg\max_{\mathbf{p}_{\mathrm{R}}} \quad & \sum_{k=1}^{N_{\mathrm{fft}}} \frac{1}{2}\log_2\left(1 + \rho_k\right) \\ \text{subject to} \quad & \mathbf{1}^T\mathbf{p}_{\mathrm{R}} = P_{\mathrm{R}} \\ & \mathbf{p}_{\mathrm{R}} \succeq 0.\end{aligned} \tag{8}$$

Using the Karush-Kuhn-Tucker (KKT) conditions [17] we get the solution of the optimization problem stated in (8) as

$$P_{\mathrm{r},k} = \frac{1}{b_k}\left[\frac{P_{\mathrm{s},k}a_k}{2}\left(\sqrt{1 + \frac{4b_k}{P_{\mathrm{s},k}a_k\nu}} - 1\right) - 1\right]^+, \tag{9}$$

where $[x]^+ = \max\{0, x\}$. The constant ν has to be chosen such that the sum power constraint $\mathbf{1}^T\mathbf{p}_{\mathrm{R}} = P_{\mathrm{R}}$ is fulfilled.

A detailed derivation of this solution can be found in the appendix.

2) Optimization of Source PA: Now we assume that the vector $\mathbf{p}_{\mathrm{R}}$ is given. Thus, we optimize $\mathbf{p}_{\mathrm{S}}$ such that the instantaneous rate C_{I} in (5) is maximized:

$$\begin{aligned}\mathbf{p}_{\mathrm{S}}^{\star} = \arg\max_{\mathbf{p}_{\mathrm{S}}} \quad & \sum_{k=1}^{N_{\mathrm{fft}}} \frac{1}{2}\log_2\left(1 + \rho_k\right) \\ \text{subject to} \quad & \mathbf{1}^T\mathbf{p}_{\mathrm{S}} = P_{\mathrm{S}} \\ & \mathbf{p}_{\mathrm{S}} \succeq 0.\end{aligned} \tag{10}$$

The expression of the SNR ρ_k at the destination is symmetric with respect to the transmit power of the source or the relay. Therefore, we obtain as solution to this problem (10) similar to (9),

$$P_{\mathrm{s},k} = \frac{1}{a_k}\left[\frac{P_{\mathrm{r},k} b_k}{2}\left(\sqrt{1+\frac{4a_k}{P_{\mathrm{r},k} b_k \nu}}-1\right)-1\right]^+ . \quad (11)$$

The parameter ν is chosen such that the sum power constraint $\mathbf{1}^T\mathbf{p}_{\mathrm{S}} = P_{\mathrm{S}}$ is fulfilled. The derivation of this solution follows the same steps as for the solution of (8) in the appendix.

3) Alternate Optimization of Source and Relay PA: Both optimizations (8) and (10) can be repeated alternately such that the output of the previous optimization is the input of the other. In the performance section IV we will show that this alternate, separate optimization of source and relay transmit PA over the subcarriers converges and achieves higher rates. As a requirement for the convergence of the alternate optimization scheme the elements of the starting vectors $\mathbf{p}_{\mathrm{S}}$ (or $\mathbf{p}_{\mathrm{R}}$) should be uniformly distributed. Thus, no subchannel is preferred in the beginning by allocating more transmit power to it.

B. Power Allocation over Time-Slots

Up to now, we have assumed that both nodes, source and relay, optimize their power distribution over the subcarriers according to their own sum power constraints. The advantage of this approach is, that both optimizations (8) and (10) can be calculated in both nodes separately. The disadvantage of separate power constraints at source and relay is that a joint optimization of the source and relay PA with joint transmit power constraint would certainly provide a higher rate. This joint optimization problem can be stated as

$$\begin{aligned}(\mathbf{p}_{\mathrm{S}}^{\star}, \mathbf{p}_{\mathrm{R}}^{\star}) = \arg\max_{\mathbf{p}_{\mathrm{S}},\mathbf{p}_{\mathrm{R}}} \quad & \sum_{k=1}^{N_{\mathrm{fft}}} \frac{1}{2}\log_2(1+\rho_k) \quad (12)\\ \text{subject to} \quad & \mathbf{1}^T\mathbf{p}_{\mathrm{S}} + \mathbf{1}^T\mathbf{p}_{\mathrm{R}} = P_{\Sigma}\\ & \mathbf{p}_{\mathrm{S}} \succeq 0\\ & \mathbf{p}_{\mathrm{R}} \succeq 0.\end{aligned}$$

By means of the joint power constraint this optimization is capable of responding more efficiently to the relative path losses between source and relay and between relay and destination. If, e.g., the attenuation between source and relay is much smaller than between relay and destination, this optimization would give a higher fraction of the overall transmit power P_{Σ} to the relay.

On the other side, such joint optimization is only a reasonable approach in low mobility wireless networks, where the channel does not vary fast over time. In other practical systems the signaling overhead due to joint optimization seems to be prohibitive. Therefore, schemes which allow separate power constraints are favorable. Thus, we suggest to adjust the fractions of the overall transmit power P_{Σ} at source and relay according to their average channel attenuation.

In the case of a frequency-flat Rayleigh fading it has been shown in [11] that the fractional PA

$$P_{\mathrm{S}} = \frac{\sqrt{\bar{B}}}{\sqrt{\bar{A}}+\sqrt{\bar{B}}}P_{\Sigma}, \quad (13)$$

$$P_{\mathrm{R}} = \frac{\sqrt{\bar{A}}}{\sqrt{\bar{A}}+\sqrt{\bar{B}}}P_{\Sigma}, \quad (14)$$

with $\bar{A} = \mathrm{E}\{a_1\}$ and $\bar{B} = \mathrm{E}\{b_1\}$, is a good approximation for the optimal average PA among source and relay in order to minimize the outage probability. Motivated by this result, we propose to define $\bar{A}$ and $\bar{B}$ as the average single hop channel power to noise ratio at relay and destination, given by

$$\bar{A} = \mathrm{E}\left\{\frac{1}{N_{\mathrm{fft}}}\sum_{k=1}^{N_{\mathrm{fft}}} a_k\right\},$$

$$\bar{B} = \mathrm{E}\left\{\frac{1}{N_{\mathrm{fft}}}\sum_{k=1}^{N_{\mathrm{fft}}} b_k\right\}.$$

We then adjust the transmit powers P_{S} and P_{R} according to (13) and (14), respectively.

C. Pairing of Subcarriers

In the previous sections, we have assumed that the signals of the source transmitted over the k-th subcarrier are amplified by the relay and also retransmitted on the k-th subcarrier. A higher performance in terms of rate can be achieved if the subcarrier of both channels, source to relay and relay to destination, are paired according to their actual strength. I.e, the best source to relay channel is paired with the best relay to destination channel. This operation only requires a subcarrier permutation matrix. This permutation matrix has only to be known for decoding at the destination, if the source has encoded the signals over the subcarriers.

IV. Performance

In this section we present the performance of our proposed PA schemes for OFDM nonregenerative relay links by means of Monte-Carlo simulations. In our simulations we assume that all three nodes, source relay and destination, are located on a line. The distance between source and destination and source and relay is denoted by d_0 and d_1, respectively.

We consider frequency-selective channels, defined in the time domain by

$$h(t) = \sum_{n=0}^{L-1} h_n \cdot \delta(t-nT), \quad (15)$$

where h_n is the complex amplitude of path n and L the number of channel taps. We assume a uniform power delay profile, where all taps are subject to Rayleigh fading and path-loss, with a path loss exponent α. Therefore, the n-th channel coefficient between two nodes with distance d meters is distributed as

$$h_n \sim \mathcal{CN}\left(0, \frac{1}{L(1+d)^{\alpha}}\right). \quad (16)$$

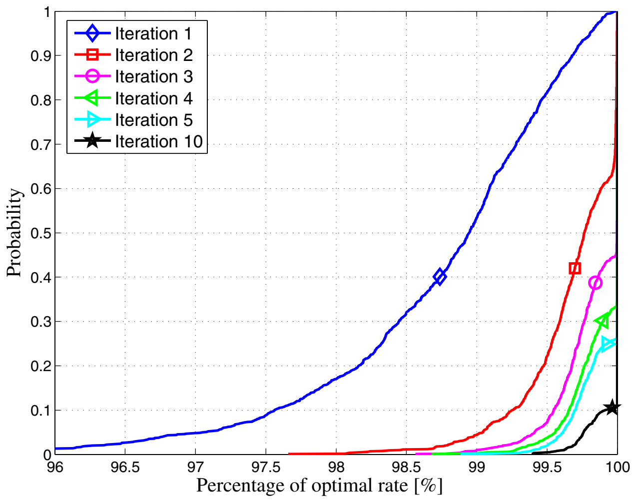


Fig. 1. CDFs of percentage of optimal rate obtained by iterative optimization for different number of iterations; equal transmit power, i.e., $P_{\mathrm{S}} = P_{\mathrm{R}}$; uniform power delay profile; $L = 4$; $N_{\mathrm{fft}} = 16$; $\rho_0 = 0$dB; $d_r = d_1/d_0 = 0.5$

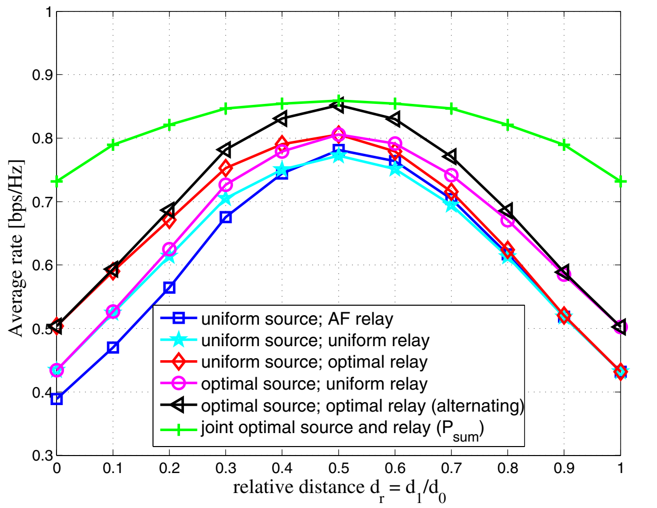


Fig. 2. Normalized average rate vs. relative distance with equal transmit power, i.e., $P_{\mathrm{S}} = P_{\mathrm{R}}$; uniform power delay profile; $L = 4$; $N_{\mathrm{fft}} = 16$; $\rho_0 = 0$dB;

The frequency domain channel is given by Fourier Transformation with N_{fft} subcarriers.

We define the SNR between source and destination as ρ_0 given by

$$\rho_0 = \frac{P_{\mathrm{S}}}{N_{\mathrm{fft}}\sigma_{\mathrm{d}}^2 \left(1 + d_0\right)^{\alpha}}$$

which is a more or less *virtual* SNR because we assume that the destination does not receive the signals directly from the source, but it is used to determine P_{S}, P_{R}, σ_{d}^2, and σ_{r}^2. If not stated otherwise in the simulations we have chosen $L = 4$, $N_{\mathrm{fft}} = 16$, $\alpha = 3$, $d_0 = 1000$ m, $\rho_0 = 0$ dB, and $\sigma_{\mathrm{d}}^2 = \sigma_{\mathrm{r}}^2$. Note, that the values of the information rate (5) are normalized to the number of OFDM subchannels.

1) Convergence of Iterative Optimization: First, we examine the convergence of iterative optimization of the relay and source PA with separate sum power constraints as discussed in section III-A.3.

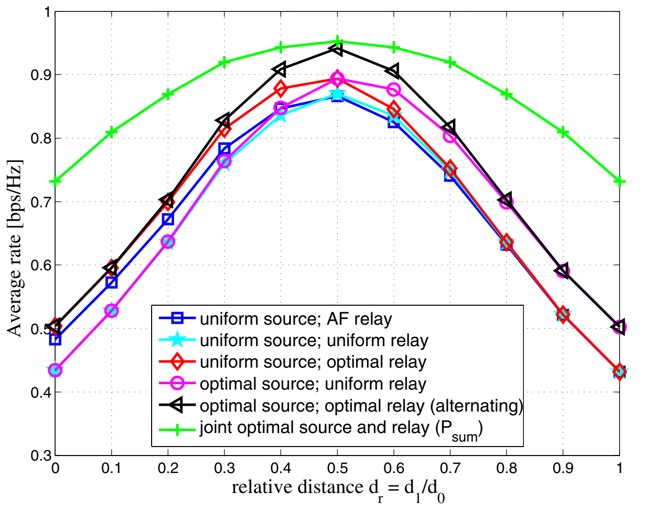
In Fig. 1 the CDF of the rate of the iterative optimization which has been obtained relatively to the optimal rate is shown. The optimal rate value, which is the reference for this consideration, is obtained by numerical joint PA optimization at source and relay with separate sum transmit power constraints. The parameter of the curves is the number of iterations. We define the procedure of one optimization of the relay PA (8) plus one optimization of the source PA (10) as one iteration. It can be seen that with increasing number of iterations the relative error between optimal rate and iteratively obtained rate decreases. After two iterations already in nearly 40% of all cases the optimal value can be achieved, and after ten iterations this value is achieved by 90%. Furthermore, all CDFs are bounded and the minimum values increase with the number of iterations. From this, we can conclude that the iterative optimization always converges.

2) Rate vs. Relative Distance: In the following we show the performance of our proposed PA schemes by varying the relative distance between source and relay compared to the distance of source and destination, i.e., $d_r = d_1/d_0$. We compare the optimized PA schemes to two reference cases. First reference case is a uniform PA over the subcarriers at source and relay. Second is a uniform PA at source and single amplification gain for all subcarrier (AF) at the relay. This single amplification factor is chosen such that the sum power constraint at the relay is guaranteed. As consequence, the relay allocates less transmit power to subcarriers where the source to relay channel is weak, without regarding the strength of the relay to destination channel in this subcarrier.

In Fig. 2 this is shown for $P_{\mathrm{S}} = P_{\mathrm{R}}$ and no pairing of subcarriers. As an upper bound the optimal rate obtained by (12) with $P_{\Sigma} = P_{\mathrm{S}} + P_{\mathrm{R}}$ is also plotted.

It can be seen that all PA schemes achieve their maximum average rate for the case, where the relay is located in the middle between source and destination. For $d_r = 0.5$, the path losses of both channels are perfectly balanced. This means, that because of the path loss exponent of $\alpha = 3$ the average attenuation of both channels is decreased by the factor $2^{\alpha} = 8$, compared to a distance between two nodes which is twice as large. Moving the relay away from the middle, i.e., $d_r \neq 0.5$, either source or relay has to cope with a higher path loss. For values below and above $d_r = 0.5$ the PA schemes with separate power constraint degrade more than the reference optimization (12) with joint power constraint.

Furthermore it can be seen, that for relative distances $d_r < 0.5$ the AF relay PA with only one amplification value for all subcarriers achieves only a lower average rate than the uniform PA at the relay. This is due to the fact, that in this area the attenuation between relay and destination is larger than the attenuation between source and relay. Therefore, it is more important to allocate more power to the good subchannels of



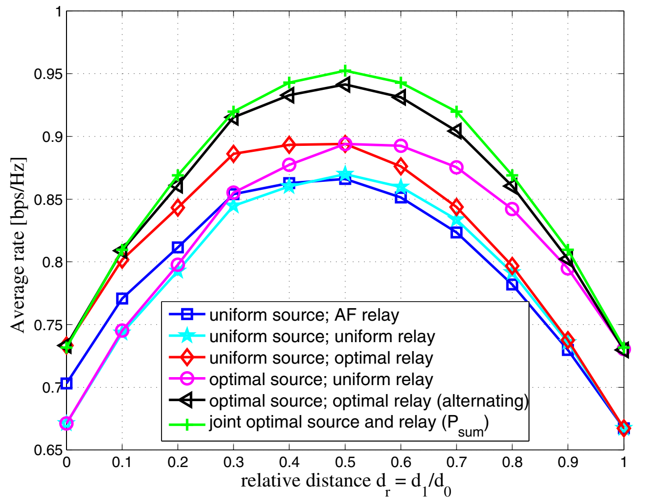


Fig. 3. Normalized average rate vs. relative distance with pairing of subcarrier and equal transmit power, i.e., $P_{\mathrm{S}} = P_{\mathrm{R}}$; uniform power delay profile; $L = 4$; $N_{\mathrm{fft}} = 16$; $\rho_0 = 0$dB;

Fig. 4. Normalized average rate vs. relative distance with pairing of subcarrier and joint average transmit power according to (13) and (14) ; uniform power delay profile; $L = 4$; $N_{\mathrm{fft}} = 16$; $\rho_0 = 0$dB;

the second hop. But if the channel is unknown, the best thing to do is to allocate the same power to all subchannels, as it is done by the uniform PA at the relay.

This can also be observed for the optimized PA schemes. For relative distances $d_r < 0.5$ an optimal relay PA with an uniform source achieves a higher average rate than an optimized source with an uniform relay. For $d_r > 0.5$ this tendency changes. Now the path loss from source to relay is higher than from relay to destination. Therefore, allocating more power to good subchannels of the first hop is essential for a high information rate. The alternating optimization of both, source and relay PA, achieves the best performance of all PAs with separated power constraints, and nearly the same average rate as the joint optimization (12) for $d_r = 0.5$.

In Fig. 3 the performance is shown for $P_{\mathrm{S}} = P_{\mathrm{R}}$ but now with pairing of subcarrier as discussed in section III-C. An increase in average rate for all schemes can be observed. Furthermore it can be seen, that the AF gain allocation now performs better than the uniform PA at the relay for $d_r < 0.5$. This is due to the fact that now the relay to destination channel is known and more transmit power is allocated to strong pairs of first and second hop channel, whereas less power is allocated to weak pairs. This can be interpreted as a soft form of waterfilling without closing some subchannels.

In the case of a joint sum power constraint of source and relay, we proposed in section III-B to allocate the fractions of the power according to the average channel attenuation. In a multi-hop system average channel attenuation is more or less related to path loss due to different path length, rather than related to small-scale fading effects. In Fig. 4 the performance results for our proposed average joint power constraint are shown for the case of $P_\Sigma = P_{\mathrm{S}} + P_{\mathrm{R}}$ and pairing of subchannels. It can be seen that the rate of all schemes (except for the reference curve of joint optimization with instantaneous joint power constraint (12)) is increased for

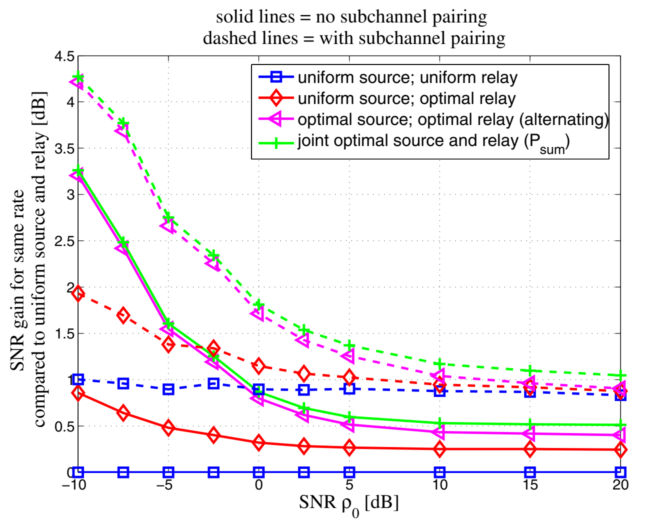


Fig. 5. SNR gain [dB] to achieve same average rate as uniform PA at source and relay without pairing of subchannels ; uniform power delay profile; $L = 4$; $N_{\mathrm{fft}} = 16$; $d_r = 0.5$

values of $d_r \neq 0.5$. For the $d_r = 0.5$ the capacities are equal to the values of Fig. 3 which is due to the fact that for this case the average PA between source and relay is $P_{\mathrm{S}} = P_{\mathrm{R}}$. Note that the alternate optimization of source and relay PA over the subchannels nearly achieves the same rate as the joint optimization with instantaneous joint transmit power constraint (12).

Next, we examine the influence of the SNR ρ_0 on the achieved rate gains of the optimized PA schemes (with and without subchannel pairing) compared to a uniform PA at source and relay (without subchannel pairing). Fig. 5 shows the possible savings in transmit power (in dB) for the optimized PA schemes compared to a uniform PA which is needed to achieve the same rate. The relative distance is chosen as $d_r = 0.5$. The figure shows that the lower the reference SNR ρ_0 the higher the gains. For increasing reference SNR these gains nearly diminish. Finally, it can be seen that the gain obtained from the pairing of OFDM subchannels of first and second hop is independent of the reference SNR ρ_0.

V. Conclusions

We presented the optimal PA at the relay (source) for a given source (relay) PA that maximizes the instantaneous rate of the considered scheme. The achievable gains in terms of average rate for this two optimization schemes are small compared to a uniform source and relay PA over the OFDM subchannels. We showed by means of simulation that an alternate, separate optimization of source and relay PA converges to the solution of the joint optimization of source and relay PA, which shows larger rate gains. Pairing of first and second hop OFDM subchannels further enhances the rate. It has been shown that the SNR gain due to pairing is independent of the SNR regime, which makes it to an efficient means for nonregenerative multihop communications.

Appendix I
Derivation of Solution of (8)

We set up the Lagrangian function as

$$L\left(\mathbf{p}_{\mathrm{R}}, \boldsymbol{\lambda}, \nu\right) = \sum_{k=1}^{N_{\mathrm{fft}}} \log\left(1+\rho_k\right) + \boldsymbol{\lambda}^T \mathbf{p}_{\mathrm{R}} - \nu\left(\mathbf{1}^T \mathbf{p}_{\mathrm{R}} - P_{\mathrm{R}}\right). \tag{17}$$

The derivative of the Lagrangian with respect to $P_{\mathrm{r},k}$ is given by

$$\frac{\partial L\left(\mathbf{p}_{\mathrm{R}}, \boldsymbol{\lambda}, \nu\right)}{\partial P_{\mathrm{r},k}} = \frac{P_{\mathrm{s},k} a_k b_k}{\left(1+P_{\mathrm{r},k} b_k\right)\left(1+P_{\mathrm{s},k} a_k + P_{\mathrm{r},k} b_k\right)} + \lambda_k + \nu \tag{18}$$

Setting (18) to zero, we get

$$\lambda_k = \nu - \frac{P_{\mathrm{s},k} a_k b_k}{\left(1+P_{\mathrm{r},k} b_k\right)\left(1+P_{\mathrm{s},k} a_k + P_{\mathrm{r},k} b_k\right)} \tag{19}$$

From the KKT conditions [17] we know that $\lambda_k \geq 0$. Thus, we get

$$\nu \geq \frac{P_{\mathrm{s},k} a_k b_k}{\left(1+P_{\mathrm{r},k} b_k\right)\left(1+P_{\mathrm{s},k} a_k + P_{\mathrm{r},k} b_k\right)}. \tag{20}$$

Another KKT condition is that $\lambda_k P_{\mathrm{r},k} = 0$, i.e.,

$$\left(\nu - \frac{P_{\mathrm{s},k} a_k b_k}{\left(1+P_{\mathrm{r},k} b_k\right)\left(1+P_{\mathrm{s},k} a_k + P_{\mathrm{r},k} b_k\right)}\right) P_{\mathrm{r},k} = 0. \tag{21}$$

If $\nu < \frac{P_{\mathrm{s},k} a_k b_k}{1+P_{\mathrm{s},k} a_k}$, condition (20) can only be fulfilled if $P_{\mathrm{r},k} > 0$. In this case (21) implies that

$$\nu = \frac{P_{\mathrm{s},k} a_k b_k}{\left(1+P_{\mathrm{r},k} b_k\right)\left(1+P_{\mathrm{s},k} a_k + P_{\mathrm{r},k} b_k\right)}. \tag{22}$$

If $\nu \geq \frac{P_{\mathrm{s},k} a_k b_k}{1+P_{\mathrm{s},k} a_k}$, with $P_{\mathrm{r},k} > 0$ it is impossible to meet (21). Therefore, (21) implies that $P_{\mathrm{r},k} = 0$.

Solving (22) with respect to $P_{\mathrm{r},k}$, after some algebraic manipulations one gets

$$P_{\mathrm{r},k} = \frac{1}{b_k}\left[\frac{P_{\mathrm{s},k} a_k}{2}\left(\sqrt{1+\frac{4 b_k}{P_{\mathrm{s},k} a_k \nu}} - 1\right) - 1\right]^+ \tag{23}$$

which is the solution of (8).

References

[1] E. C. van der Meulen, "Three-terminal communication channels," *Adv. Appl. Prob.*, vol. 3, pp. 120–154, 1971.

[2] H. Sato, "Information transmission through a channel with relay," Tech. Rep. B76-7, The Aloha System, University of Hawai, Honolulu, Mar. 1976.

[3] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, vol. 25, pp. 572–584, Sept. 1979.

[4] A. Sendonaris, E. Erkip, and B. Aazhang, "Increasing uplink capacity via user cooperation diversity," in *Proc. IEEE Int. Symposium on Inf. Theory*, (Cambridge, MA), p. 156, Aug. 16–21, 1998.

[5] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. 46, pp. 388–404, Mar. 2000.

[6] J. N. Laneman, D. N. Tse, and G. W. Wornell, "An efficient protocol for realizing cooperative diversity in wireless networks," in *Proc. IEEE Int. Symposium on Inf. Theory*, (Washington DC), p. 294, June 24–29, 2000.

[7] N. J. Laneman, D. N. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[8] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-Part I: System description," *IEEE Trans. Commun.*, vol. 51, pp. 1927–1938, Nov. 2003.

[9] J. Zhang, Q. Zhang, C. Shao, Y. Wang, P. Zhang, and Z. Zhang, "Adaptive optimal transmit power allocation for two-hop non-regenerative wireless relaying system," in *Proc. 60th IEEE Veh. Tech. Conf.*, (Los Angeles, CA), Sep. 26–29, 2004.

[10] Q. Zhang, J. Zhang, C. Shao, Y. Wang, P. Zhang, and R. Hu, "Power allocation for regenerative relay channels with rayleigh fading," in *Proc. 59th IEEE Veh. Tech. Conf.*, (Los Angeles, CA), Sep. 26–29, 2004.

[11] M. O. Hasna and M.-S. Alouini, "Optimal power allocation for relayed transmissions over rayleigh-fading channels," *IEEE Trans. Wirel. Comm.*, vol. 3, pp. 1999–2004, Nov. 2004.

[12] I. Hammerstroem, M. Kuhn, and A. Wittneben, "Impact of relay gain allocation on the performance of cooperative diversity networks," in *Proc. 60th IEEE Veh. Tech. Conf.*, (Los Angeles, CA), Sep. 26–29, 2004.

[13] I. Maric and R. Yates, "Bandwidth and power allocation for cooperative strategies in gaussian relay networks," in *Proc. Asilomar Conf. Signals, Syst., Comput.*, (Pacific Grove, CA), Nov. 2004.

[14] G. Li and H. Liu, "On the capacity of the broadband relay networks," in *Proc. Asilomar Conf. Signals, Syst., Comput.*, (Pacific Grove, CA), Nov. 7–10, 2004.

[15] S. Barbarossa and G. Scutari, "Distributed space-time coding for multi-hop networks," in *Proc. ICC*, vol. 2, (Paris, France), pp. 916–920, Jun. 20–24, 2004.

[16] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.

[17] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.