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Optimizing Zero-Forcing Based Gain Allocation for Wireless Multiuser Networks

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***Abstract*— We consider a wireless multiuser network where a number of source/destination pairs communicate concurrently over the same physical channel. A set of amplify-and-forward relay nodes assist the communication by multiplying their received signal with a complex gain factor before retransmission. We propose and compare different strategies for finding the gain factor for every relay terminal, motivated by the chosen figures of merit: average sum rate, diversity, and fairness. A new and practical definition of diversity is provided in the context of our system model. For each strategy we investigate the provided gains in terms of the figures of merit, and the trade-off between them. The existing schemes in the literature act as references to the proposed schemes.**
Keywords – cooperative relaying, ad-hoc networks, multiuser zero forcing relaying, diversity, fairness.

I. Introduction

In cooperative relaying, multiple relays jointly assist the communication between sources and destinations. This approach is promising to supersede conventional multiple-input multiple-output (MIMO) systems with respect to diversity and spatial multiplexing gain, especially in unfavourable channel conditions. In the area of cooperative wireless relaying, the notion of (user) cooperative diversity is introduced for example in [3], [4]. Spatial diversity is achieved by multiple antennas belonging to different users. They form a virtual array by distributed transmission and signal processing. The maximum achievable diversity in a wireless network featuring single-antenna terminals is in the order of cooperating nodes [3]. In order to provide a spatial multiplexing gain, independent data streams are transmitted in the same frequency band and at the same time.

We focus on coherent relaying where all relays are assumed to have the same phase reference. Consequenlty, the relays can act as a distributed antenna array and thus achieve a distributed array gain [2]. In contrast to that, noncoherent forwarding schemes cannot achieve any distributed array gain. In a relay selection scheme, a single terminal is chosen from the set of all possible relays to forward its received signal.

Decode-and-forward (DF) relays completely decode their received signal before forwarding it. As a consequence, the retransmitted signal is assumed to be noiseless. In contrast to that, amplify-and-forward (AF) relays simply forward a scaled and possibly rotated version of their received signal. Inevitably, AF relays forward not only the desired signal but also noise. Thus, AF schemes might not be a appropriate solution in multihop networks where the noise is accumulated with increasing number of hops [5]. However, in contrast to DF schemes, they are transparent to coding or modulation. Furthermore, time synchronization is only required on frame level. In heterogeneous wireless ad-hoc networks consisting of terminals operating in different standards, this will be a big advantage.

In [6], the authors show that the capacity of a wireless relay network can scale linearly with the total number of transmit antennas. They present a zero-forcing (ZF) based scheme for the case of infinite number of relays which exhibits this gain. For finite number of single-antenna relays, a scheme which is based on ZF by performing a nullspace projection is presented in [2]. Multiple AF relays assist the communication between single-antenna source/destination (S/D) pairs. Choosing the gain factors of the relays accordingly, the S/D links are orthogonalized. This allows them to simultaneously transmit over the same physical channel without causing interference to each other. A minimum number of relays is needed to achieve this. When there are more relays, the gain factors can be further optimized [7]. In this paper, we propose several optimization criteria to determine the gain factors in the excess relay case. We derive their solutions and investigate the trade-offs between sum rate, diversity and fairness.

Notation: We use boldface lowercase and capital letters to indicate vectors and matrices, respectively. The superscripts $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ stand for conjugate complex, matrix transpose and conjugate complex transpose, respectively. The operators $\odot$, $\mathrm{E}_{\{x\}}[\cdot]$, $\mathrm{diag}\{\mathbf{x}\}$, $\lambda_{\max}\{\mathbf{X}\}$ denote the elementwise product (i.e. $\mathbf{c} = \mathbf{a} \odot \mathbf{b} \rightarrow c_i = a_i \cdot b_i \forall i$), expectation with respect to x, a diagonal matrix with $\mathbf{x}$ on its diagonal, and the maximum eigenvalue of $\mathbf{X}$, respectively. $\mathbf{I}_N$ is a $N \times N$ identity matrix. A vector $\mathbf{x}$ whose entries are taken from a complex normal distribution with mean μ and variance σ^2 is denoted by $\mathbf{x} \sim \mathcal{CN}(\mu, \sigma^2\mathbf{I})$.

II. System Model

We consider a wireless network where N_{SD} S/D pairs communicate concurrently on the same physical channel. N_{R} amplify-and-forward relay nodes assist the communication in a half-duplex scheme. They coherently amplify the signals they receive from the sources by multiplying them with complex gain factors prior to retransmission. All nodes in the network employ only one antenna. For the sake of a simple notation, we assume that the number of sources equals the number of destinations. This can be done without loss of generality as inactive sources or destinations can always be omitted in the consideration. The communication follows a two-hop relay traffic pattern, i.e., each transmission cycle includes two channel uses: one for the *uplink* transmission from the sources to all relays, and one for the *downlink* transmission from the relays to the destinations. The direct link is not taken into account here. **Figs. 1** and **2** show the system configuration and the compound block diagram, respectively.

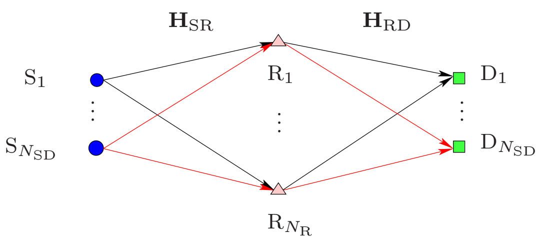

Fig. 1. Two hop system configuration with half-duplex relays

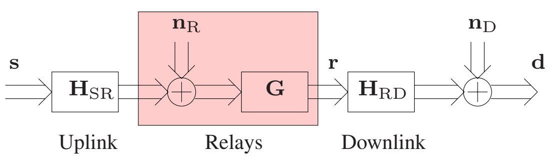

Fig. 2. Block diagram with AWGN at the relays and destinations

The scalar transmit symbols are stacked in the vector $\mathbf{s} \in \mathbb{C}^{N_{\mathrm{SD}}}$. They are first transmitted over the uplink matrix channel $\mathbf{H}_{\mathrm{SR}} \in \mathbb{C}^{N_{\mathrm{R}} \times N_{\mathrm{SD}}}$ to the relays. As the sources do not have any channel state information (CSI), they transmit i.i.d. symbols with the same transmit power each, i.e., $\mathrm{E}_{\{\mathbf{s}\}}\left[\mathbf{s}\mathbf{s}^{\mathrm{H}}\right] = \sigma_{\mathbf{s}}^2 \mathbf{I}_{N_{\mathrm{SD}}}$. The vector $\mathbf{n}_{\mathrm{R}} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_{n_{\mathrm{R}}}^2 \mathbf{I}_{N_{\mathrm{R}}}\right)$ comprises the additive, white, Gaussian noise (AWGN) contributions at the relay nodes. After multiplication with the gain matrix $\mathbf{G} \in \mathbb{C}^{N_{\mathrm{R}} \times N_{\mathrm{R}}}$, the signals stacked in $\mathbf{r}$ are passed through the downlink matrix channel $\mathbf{H}_{\mathrm{RD}} \in \mathbb{C}^{N_{\mathrm{SD}} \times N_{\mathrm{R}}}$ to the N_{SD} destination nodes. The sum transmit power of all relay terminals is equal to the total transmit power of all sources, i.e., $P_{\mathrm{R}} = \mathrm{E}_{\{\mathbf{r}\}}\left[\mathbf{r}^{\mathrm{H}}\mathbf{r}\right] = N_{\mathrm{SD}}\sigma_{\mathbf{s}}^2$. We use this constraint to keep the total power consumption of the network constant when utilizing additional relays. The vector $\mathbf{n}_{\mathrm{D}} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_{n_{\mathrm{D}}}^2 \mathbf{I}_{N_{\mathrm{SD}}}\right)$ comprises the AWGN contribution at the destinations. Finally, the signals at the destinations are stacked in the vector

$$\mathbf{d} = \mathbf{H}_{\mathrm{RD}}\mathbf{G}\mathbf{H}_{\mathrm{SR}} \cdot \mathbf{s} + \mathbf{H}_{\mathrm{RD}}\mathbf{G} \cdot \mathbf{n}_{\mathrm{R}} + \mathbf{n}_{\mathrm{D}} := \mathbf{H}_{\mathrm{SD}}\mathbf{s} + \mathbf{n}, \quad (1)$$

where $\mathbf{H}_{\mathrm{SD}}$ is called the *equivalent channel matrix* and $\mathbf{n}$ the *equivalent noise vector*. The components of $\mathbf{n}$ are spatially no longer white because the multiplication with $\mathbf{G}$ and transmission over the downlink channel introduces correlation between the relay noise samples.

In the following we let $\sigma_{\mathrm{nR}}^2 = \sigma_{\mathrm{nD}}^2 := \sigma_{\mathrm{n}}^2$. The covariance matrix of the compound noise vector is

$$\mathbf{R}_{\mathrm{n}} = \mathrm{E}_{\{\mathbf{s},\mathbf{n}_{\mathrm{R}}\}}\left[\mathbf{n}\mathbf{n}^{\mathrm{H}}\right] = \sigma_{\mathrm{n}}^2 \left(\mathbf{H}_{\mathrm{RD}}\mathbf{G}\mathbf{G}^{\mathrm{H}}\mathbf{H}_{\mathrm{RD}}^{\mathrm{H}} + \mathbf{I}_{N_{\mathrm{SD}}}\right). \quad (2)$$

All channel coefficients are assumed to be independent, complex Gaussian random variables with circular symmetric probability density function (frequency flat Rayleigh fading): $\mathbf{H}_{\mathrm{SR}}[l,k] \sim \mathcal{CN}\left(0, \sigma_{\mathrm{h}}^2\right)$ and $\mathbf{H}_{\mathrm{RD}}[m,l] \sim \mathcal{CN}\left(0, \sigma_{\mathrm{h}}^2\right)$ for all $k, m \in \{1, \ldots, N_{\mathrm{SD}}\}$, $l \in \{1, \ldots, N_{\mathrm{R}}\}$. Furthermore we assume a constant channel during at least one transmission cycle while different channel realisations are assumed to be temporally uncorrelated (block fading).

III. Figures of Merit

The coherent gain allocation scheme derived in [2] needs at least $N_{\mathrm{R}} = N_{\mathrm{SD}}\left(N_{\mathrm{SD}} - 1\right) + 1$ relay nodes to find the ZF gain factors with a nullspace projection approach. For a *minimum relay configuration*, which comprises the minimum number of relay nodes, the nullspace has dimension one and thus the gain factors are uniquely defined. The nullspace grows accordingly for larger numbers of relays. In this case, the gain vector can be optimized within the nullspace. We want to optimize the gain vector with respect to sum rate, diversity gain, or fairness.

Sum rate: We investigate gain allocation strategies that completely orthogonalize all S/D pairs such that they can communicate concurrently over the same physical channel without causing interference to each other. Consequently, multiple parallel channels are created over which the sources transmit independent data streams. When there are more than necessary relays to orthogonalize all S/D pairs, the additional degrees of freedom can be used to weight the links and thus increase the sum rate of the system.

Effective diversity at outage probability: We use a new, practical definition for the diversity which was first introduced in [7] as *effective diversity gain at outage probability* P_{outage}. Consider S/D pair k. The idea is to fit a chi-square distribution with $2N$ degress of freedom to the empirical cdf of the signal-to-noise ratio (SNR). We use two fitting points, namely the mean and 1% outage SNR (see **Fig. 3**). With this definition,

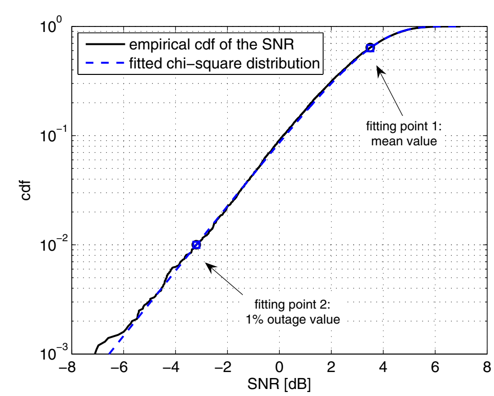

Fig. 3. Fitting a chi-square distribution to the empirical cdf of the SNR

the effective diversity is equal to half the number of degrees of freedom of this fitted chi-square distribution:

$$r_{\mathrm{effective}} = N. \quad (3)$$

The intuition behind this is as follows: The SNR of a MISO (SIMO) link with M source (destination) antennas and frequency flat, complex i.i.d. channel coefficients that fully exploits transmit (receive) diversity is chi-square distributed with $2M$ degrees of freedom. This corresponds to a diversity value equal to the number of source (destination) antennas M. Consequently, fitting a chi-square distribution to the mean and outage value of the empirical distribution of the SNR indicates how many source (destination) antennas a corresponding MISO (SIMO) system would need to experience the same outage behavior. This relates the diversity to the number of independent channel realizations that can be exploited for the transmission of one data stream. Here and in the sequel, the terms *diversity* and *effective diversity at outage probability* are used interchangebly.

Fairness: There exist several approaches to provide fairness among different users such as *max-min* or *proportional fairness* [8]. Although these approaches aim to be fair, they do not provide a measure of fairness. We use a fairness measure that was introduced in [9] for a cellular system with K users that want to communicate with a single base station. It is motivated by the formulation of Shannon's *self-information*. The so-called *self-fairness* is defined as

$$F_k = \frac{\log p^{(k)}}{\log \frac{1}{K}} \tag{4}$$

and relates the actual amount of resources allocated to source/destination pair k to the fair share of resource it is entitled to. $p^{(k)}$ is the proportion of resources allocated to source/destination pair k and $\log \frac{1}{K}$ a normalization factor representing the fair share of resources. This formulation considers equal priorities for all users. The self-fairness of a user is equal to 1 when it gets its fair share. The less resources it consumes, the more fair it behaves to other users which in turn gives rise to an increase in its self-fairness. On the contrary, the more resources it consumes, the less fair it is to others and its self-fairness decreases. Thus, self-fairness ranges between 0 and ∞, where 1 is the optimum value. Accordingly, the *self-unfairness* for source/destination pair k is defined as $U_k = 1/F_k$.

The *average fairness* and the *average unfairness* are defined as

$$\overline{F} = \sum_{k=1}^{K} p^{(k)} F_k \quad \text{and} \quad \overline{U} = \sum_{k=1}^{K} p^{(k)} U_k, \tag{5}$$

respectively. They range from 0 to 1. The maximum value 1 is achieved when all users get their fair share of resources, and the minimum of 0 when all resources are occupied by a single user.

IV. Optimization of Relay Gains

In [2], the authors derive a coherent gain allocation scheme where the gain matrix $\mathbf{G} = \mathrm{diag}\{\mathbf{g}_{\text{ZF}}\}$ is chosen such that it orthogonalizes the S/D links. Defining the column vectors $\mathbf{h}_{\text{RD}}^{(m)}$ and $\mathbf{h}_{\text{SR}}^{(k)}$ as the transpose of the mth row vector of $\mathbf{H}_{\text{RD}}$ and the kth column vector of $\mathbf{H}_{\text{SR}}$, respectively; let $\mathbf{H}_{\text{I}} \in \mathbb{C}^{N_{\text{SD}}-1 \times N_{\text{SD}}-1}$ be a matrix comprising the columns $\left(\mathbf{h}_{\text{RD}}^{(m)} \odot \mathbf{h}_{\text{SR}}^{(k)}\right)$ for all $k, m \in \{1, \ldots, N_{\text{SD}}\}$ and $k \neq m$.

Then $\mathbf{g}^{\text{T}}\mathbf{H}_{\text{I}}$ delivers a vector containing all $N_{\text{SD}}(N_{\text{SD}}-1)$ off-diagonal elements of the equivalent channel matrix $\mathbf{H}_{\text{SD}}$. In order to cancel the interstream interference and thus orthogonalizing all S/D pairs, the ZF gain vector $\mathbf{g}_{\text{ZF}}$ has to fullfill

$$\mathbf{g}_{\text{ZF}}^{\text{T}}\mathbf{H}_{\text{I}} = \mathbf{0}^{\text{T}}. \tag{6}$$

Any vector $\mathbf{g}_{\text{ZF}}$ that lies in the nullspace $\mathbf{Z} = \mathrm{null}\{\mathbf{H}_{\text{I}}\}$ of $\mathbf{H}_{\text{I}}$ fulfills (6). Thus, the ZF gain vector can be expressed as a linear combination of the columns of $\mathbf{Z}$ which form a basis of the nullspace

$$\mathbf{g}_{\text{ZF}} = \mathbf{Z}\mathbf{x}, \tag{7}$$

where $\mathbf{Z} \in \mathbb{C}^{N_{\text{R}} \times N_{\text{R}}-(N_{\text{SD}}^2-N_{\text{SD}})}$ and $\mathbf{x} \in \mathbb{C}^{N_{\text{R}}-(N_{\text{SD}}^2-N_{\text{SD}})}$. Finally, with (7) the equivalent channel coefficient between the kth S/D pair can be written as

$$\mathbf{H}_{\text{SD}}[k,k] = \mathbf{x}^{\text{T}}\mathbf{Z}^{\text{T}}\mathbf{h}_{\text{S}}^{(k)}, \tag{8}$$

where $\mathbf{h}_{\text{S}}^{(k)} = \mathbf{h}_{\text{RD}}^{(k)} \odot \mathbf{h}_{\text{SR}}^{(k)}$ for all $k \in \{1, \ldots, N_{\text{SD}}\}$.

In the following our goal is to optimize $\mathbf{x}$ with respect to *average sum rate*, *fairness* or *diversity*. The first two schemes aim to obtain maximum achievable sum rate and fairness. Next, two more schemes are proposed to approach these maximal performances with rather simple schemes and to provide a trade-off between the different figures of merit.

A. Maximization of the Sum Rate (MaxSumRate)

The instantaneous rate of the ith S/D link is

$$\mathrm{R}_i = \frac{1}{2}\log_2(1 + \mathrm{SNR}_i),$$

where the instantenous signal-to-noise ratio SNR_i is given by

$$\mathrm{SNR}_i = \frac{\sigma_s^2 |\mathbf{H}_{\text{SD}}[i,i]|^2}{\sigma_n^2\left(1 + (\mathbf{g}_{\text{ZF}} \odot \mathbf{h}_{\text{RD}}^{(i)})^{\text{H}}(\mathbf{g}_{\text{ZF}} \odot \mathbf{h}_{\text{RD}}^{(i)})\right)}.$$

The maximum achievable sum rate can be found by solving the optimization problem

$$\max_{\mathbf{x}} \sum_{i=1}^{N_{\text{SD}}} \mathrm{R}_i \qquad \text{subject to} \quad \mathbf{x}^{\text{T}}\mathbf{Q}_{\text{Z}}^{*}\mathbf{x}^{*} = P_{\text{S}}, \tag{9}$$

where $\mathbf{Q}_{\text{Z}} = \mathbf{Z}^{\text{H}}(\sigma_s^2\mathbf{H}_{\text{SR}}\mathbf{H}_{\text{SR}}^{\text{H}} \odot \mathbf{I}_{N_R} + \sigma_n^2\mathbf{I}_{N_R})\mathbf{Z}$. The constraint assures that the instantenous relay power constraint is met.

The optimization problem (9) is non-linear and non-convex. Thus, unfortunately, we cannot provide a closed-form solution. Instead, we solve (9) with a combined numerical optimization technique employing mainly *the exterior penalty method* and *the conjugate gradient method* [10]. The exterior penalty method converts nonlinear constrained problem to an unconstrained one by penalizing the problem when constraints are violated. Moreover, to accelerate the convergence, the conjugate gradient method is implemented in which the gradient directions are used instead of local gradients.

B. Maximization of the Minimum Link Rate (MaxMinRate)

Maximizing the sum rate favours the strong links while punishing the weak ones. This kind of gain allocation is known to be rather unfair as there are users that are only provided with a small transmission rate. To avoid this, the optimization problem

$$\max_{\mathbf{x}} \min_{1\le i\le N_{\mathrm{SD}}} \mathrm{R}_i \qquad \text{subject to} \quad \mathbf{x}^{\mathrm{T}}\mathbf{Q}_{\mathrm{Z}}^{*}\mathbf{x}^{*} = P_{\mathrm{S}} \tag{10}$$

aims at maximizing the minimum rate between any S/D pair with the constraint that an increment in a S/D link's rate does not lead to reduction of some other link's rate, which was already smaller than the same link [8]. It therefore provides the maximum achievable system fairness.

As in the case of *MaxSumRate*, we cannot provide a closed-form solution to (10) as the problem is nonconvex. Instead we use the same numerical optimizaton technique mentioned in the previous section to obtain the solution.

There is an oblique relation between the fairness and the diversity concepts introduced in Section III. The solution of (10) compels all S/D pairs to have similar rates, which in return shrinks the range of the SNR values that a link can have. The probability of the occurence of a very low SNR value is reduced with the sacrifice of strong links. On the contrary, the chance of having a very high SNR is relinquished to support weak links. Thus, aiming at fairness leads to a narrow range of SNRs, which means a steeper SNR cdf curve giving rise to a higher diversity. We can conclude that the approaches achieving fairness, also provide high diversity. This observation will be further explored in the simulations.

C. Maximization of the Minimum Link Signal Power (MMHsd)

In [7], a max-min fairness approach is motivated to determine $\mathbf{x}$, such that the link with the minimum signal power (i.e. $\sigma_s^2|\mathbf{H}_{\mathrm{SD}}[i,i]|^2, i = 1,\ldots,N_{\mathrm{SD}}$) is maximized. A closed form solution of this approach could only be derived for $N_{\mathrm{SD}} = 2$, and the authors resorted to numerical optimization schemes for $N_{\mathrm{SD}} \ge 3$. In the following, we reformulate the same optimization problem and present a generic solution.

Let $\mathbf{H}_{\mathrm{Z}}^{(i)}$ be the matrix

$$\mathbf{H}_{\mathrm{Z}}^{(i)} = \mathbf{Z}^{\mathrm{H}}\Big((\mathbf{h}_{\mathrm{S}}^{(i)})^{*}(\mathbf{h}_{\mathrm{S}}^{(i)})^{\mathrm{T}}\Big)\mathbf{Z}, \tag{11}$$

for $i = 0,\ldots,N_{\mathrm{SD}}$. Then, with (11) the max-min fairness approach can be written as

$$\max_{\mathbf{x}} \min_{1\le i\le N_{\mathrm{SD}}} \mathbf{x}^{\mathrm{T}}\mathbf{H}_{\mathrm{Z}}^{(i)*}\mathbf{x}^{*} \qquad \text{subject to} \quad \mathbf{x}^{\mathrm{T}}\mathbf{x}^{*} = c, \tag{12}$$

where c is assumed to be 1 throughout this derivation so that the constraint imposes an average rather than an instantenous relay sum power consumption [7]. Introducing a dummy variable t, the original max-min problem (12) can be transformed to a simple constrainted minization problem:

$$\begin{aligned} \min_{t,\mathbf{x}} \ -t \qquad \text{subject to} \quad & \mathbf{x}^{\mathrm{T}}\mathbf{H}_{\mathrm{Z}}^{(i)*}\mathbf{x}^{*} \ge t, \ \forall i \\ & \mathbf{x}^{\mathrm{T}}\mathbf{x}^{*} = 1. \end{aligned} \tag{13}$$

To form the Lagrangian we introduce $\gamma_i \in \mathcal{R}$ for the N_{SD} inequality constraints and $\mu \in \mathcal{R}$ for the equality contraint, and obtain

$$L(\mathbf{x},\boldsymbol{\gamma},\mu) = -t + \sum_{i=1}^{N_{\mathrm{SD}}} \gamma_i(t - \mathbf{x}^{\mathrm{T}}\mathbf{H}_{\mathrm{Z}}^{(i)*}\mathbf{x}^{*}) + \mu(\mathbf{x}^{\mathrm{T}}\mathbf{x}^{*} - 1). \tag{14}$$

So, the Lagrangian dual function is

$$g(\boldsymbol{\gamma},\mu) = \inf_{\mathbf{x}} L(\mathbf{x},\boldsymbol{\gamma},\mu) = f + \inf_{\mathbf{x}} \mathbf{x}^{\mathrm{T}}\Big(-\sum_{i=1}^{N_{\mathrm{SD}}} \gamma_i \mathbf{H}_{\mathrm{Z}}^{(i)*} + \mu\mathbf{I}_{\mathrm{SD}}\Big)\mathbf{x}^{*},$$

where $f = t(\sum_{i=1}^{N_{\mathrm{SD}}}\gamma_i - 1) - \mu$. If a quadratic form is positive semidefinite, the infimum of it is either 0 or $-\infty$. Hence, the dual function can be represented as

$$g(\boldsymbol{\gamma},\mu) = \begin{cases} f & -\sum_{i=1}^{N_{\mathrm{SD}}}\gamma_i\mathbf{H}_{\mathrm{Z}}^{(i)*} + \mu\mathbf{I}_{\mathrm{SD}} \succeq 0 \\ -\infty & \text{otherwise.} \end{cases} \tag{15}$$

and with (15) the Lagrange dual problem is

$$\begin{aligned} \max_{\mathbf{x},t,\boldsymbol{\gamma},\mu} f \qquad \text{subject to} \quad & -\sum_{i=1}^{N_{\mathrm{SD}}}\gamma_i\mathbf{H}_{\mathrm{Z}}^{(i)*} + \mu\mathbf{I}_{\mathrm{SD}} \succeq 0 \\ & \boldsymbol{\gamma} \succeq 0. \end{aligned} \tag{16}$$

With the positive semidefineteness constraint, the dual variable μ is found to be equal to $\lambda_{\max}\Big(\sum_{i=1}^{N_{\mathrm{SD}}}\gamma_i\mathbf{H}_{\mathrm{Z}}^{(i)*}\Big)$. The dual variable μ is dependent on γ_is and also needs to be minimized in order to maximize f. Thus, the solution of (13) turnes out to be a minimization of the maximum eigenvalue problem such that

$$\min_{\boldsymbol{\gamma}} \lambda_{\max}\left(\sum_{i=1}^{N_{\mathrm{SD}}}\gamma_i\mathbf{H}_{\mathrm{Z}}^{(i)*}\right). \tag{17}$$

That is to say, $\tilde{\mathbf{x}}$ is the eigenvector corresponding to the minimum of the possible maximum eigenvalues of $\sum_{i=1}^{N_{\mathrm{SD}}}\gamma_i\mathbf{H}_{\mathrm{Z}}^{(i)*}$ optimized over $\boldsymbol{\gamma}$. Finally, since (12) imposes an average but not an instantaneous power constraint, $\tilde{\mathbf{x}}$ should be scaled as

$$\mathbf{x}_{\mathrm{MMHsd}} = \sqrt{\frac{P_{\mathrm{S}}}{\tilde{\mathbf{x}}^{\mathrm{H}}\mathbf{Q}_{\mathrm{Z}}\tilde{\mathbf{x}}}} \tag{18}$$

to satisfy the general instantaneous power constraint. Equation (17) can be solved by any interior point method or semidefinite programming [11].

Since the *MMHsd* scheme discards the noise effect, it does not achieve the maximum fairness as *MaxMinRate* does. Besides, it does not provide the maximum average sum rate of *MaxSumRate* because of fair distribution of resources. Nevertheless, it is a moderate approach with a reduced-complexity solution with respect to the numerical technique based solutions of *MaxSumRate* and *MaxMinRate*. It will be shown in the simulations section that it also provides a trade-off between sum rate and fairness (diversity).

D. Transmission in the Largest Eigenmode (EigHz)

Instead of maximizing the smallest equivalent as done in the previous section, we now want to maximize the sum of equivalent channel coefficients $\mathbf{H}_{\mathrm{SD}}[i,i]$. The constrained optimization problem can be written as

$$\min_{\mathbf{x}} \quad -\sum_{i=1}^{N_{\mathrm{SD}}} \mathbf{x}^{\mathrm{T}} \mathbf{H}_{\mathrm{Z}}^{(i)\,*} \mathbf{x}^{*} \text{subject to} \qquad \mathbf{x}^{\mathrm{T}}\mathbf{x}^{*} = c \tag{19}$$

where c is a constant. We approach the solution of this non-convex optimization problem by formulating a vector optimization with respect to a bicriterion problem with no constraint (see *Regularized Approximation* in [11]):

$$\min \quad \left(\left(\sum_{i=1}^{N_{\mathrm{SD}}} \mathbf{x}^{\mathrm{T}} \mathbf{H}_{\mathrm{Z}}^{(i)\,*} \mathbf{x}^{*} \right), \ \left(\mathbf{x}^{\mathrm{T}}\mathbf{x}^{*} - c \right) \right) \tag{20}$$

Our goal now is to find the optimal trade-off between the two objectives. To this end we minimize the weighted sum

$$L\left(\mathbf{x}, \boldsymbol{\mu}\right) = -\mu_1 \sum_{i=1}^{N_{\mathrm{SD}}} \mathbf{x}^{\mathrm{T}} \mathbf{H}_{\mathrm{Z}}^{(i)\,*} \mathbf{x}^{*} + \mu_2 \left(\mathbf{x}^{\mathrm{T}}\mathbf{x}^{*} - c \right) \tag{21}$$

consisting of the two objectives in (20). The factors $\mu_1 \geq 0$ and $\mu_2 \geq 0$ determine the weighting of the individual objectives. Taking the derivative of (21) with respect to $\mathbf{x}$ delivers

$$\left(\sum_{i=1}^{N_{\mathrm{SD}}} \mathbf{H}_{\mathrm{Z}}^{(i)} \right) \mathbf{x} = \frac{\mu_2}{\mu_1} \mathbf{x} \tag{22}$$

This is a simple eigenvalue problem. We choose $\tilde{\mathbf{x}}$ to be the eigenvector of $\left(\sum_{i=1}^{N_{\mathrm{SD}}} \mathbf{H}_{\mathrm{Z}}^{(i)} \right)$ belonging to the largest eigenvalue, because a large eigenvalue corresponds to a large ratio $\frac{\lambda_2}{\lambda_1}$. In this case, the solution $\tilde{\mathbf{x}}$ comes as close as possible to the allowed length $\sqrt{c}$. This solution will not satisfy the instantenous relay sum power constraint, $\tilde{\mathbf{x}}$ has still to be scaled as in (18) to deliver the final result $\mathbf{x}_{\mathrm{EigHZ}}$. Note that this is the same result one would get when maximizing the Rayleigh quotient $Q = \mathbf{x}^{\mathrm{T}} \left(\sum_{i=1}^{N_{\mathrm{SD}}} \mathbf{H}_{\mathrm{Z}}^{(i)*} \right) \mathbf{x}^{*} / (\mathbf{x}^{\mathrm{T}}\mathbf{x}^{*})$.

V. Simulation Results

In this section, results of Monte-Carlo simulations are presented for all proposed optimization schemes. We assume that the relays have perfect CSI, the destination have perfect compound CSI (knowledge of $\mathbf{H}_{\mathrm{SD}}$) and the sources have no channel knowledge at all. Furthermore, a global phase reference is assumed to be available at all relays. Unless otherwise stated, the elements of the channel matrices $\mathbf{H}_{\mathrm{SR}}$ and $\mathbf{H}_{\mathrm{RD}}$ are i.i.d. zero mean complex normal random variables with variance $\sigma_{\mathrm{h}}^2 = 1$. In order to calculate the signal-to-noise ratio, we consider a reference scenario with a single S/D pair and only one relay in between. The relay transmit power equals σ_s^2 in this case. We determine the source transmit power which is needed to achieve a defined signal-to-noise ratio $\mathrm{SNR}_{\mathrm{def}}$ on average by simulation. Then we apply this transmit power to the configuration that is to be evaluated at $\mathrm{SNR}_{\mathrm{def}}$.

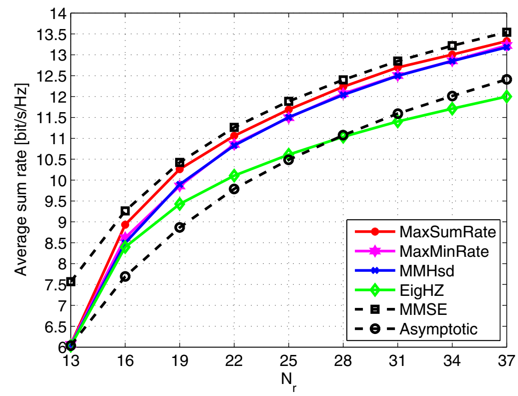

Fig. 4. Average sum rate vs. N_{R} for $N_{\mathrm{SD}} = 4$ and $\mathrm{SNR}_{\mathrm{def}}$ = 10 dB.

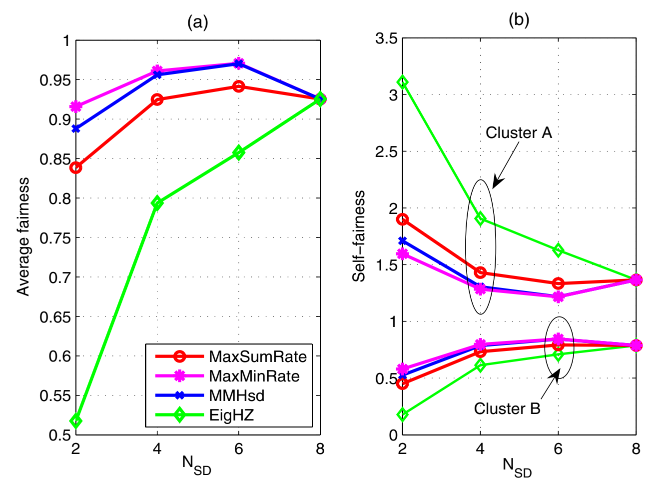

Fig. 5. Average fairness and self-fairness vs. N_{SD} for $N_{\mathrm{R}} = 57$ and $\mathrm{SNR}_{\mathrm{def}}$ = 10 dB. Clusters A and B denote S/D pairs with good ($\sigma_{\mathrm{h}}^2 = 10$) and bad ($\sigma_{\mathrm{h}}^2 = 1$) channel conditions, respectively.

In **Fig. 4**, the average sum rates of the proposed schemes are plotted versus N_{R} ranging from 13 (minimum relay configuration for $N_{\mathrm{SD}} = 4$) to 37. As reference we also added curves for multiuser minimum mean square error (denoted by *MMSE*) based relaying [1] and asymptotic multiuser ZF relaying [2] (denoted by *Asymptotic*). The *MaxMinRate* and the *MMHsd* schemes perform surprisingly almost the same, which shows that the effect of correlated noise is negligible to determine the gain factors when max-min fairness is considered. Besides, their performace is close to the maximum achievable sum rate, i.e. the curve for *MaxSumRate*. Compared to MMSE relaying, this maximal rate for ZF based relaying schemes is about 0.17 bit/s/Hz worse for the given configuration. As N_{R} increases the performance gap between the simple *EigHZ* and the other schemes grows. However, it outperforms the asymptotic ZF relaying in terms of sum rate up to the point where $N_{\mathrm{R}} = 28$. All ZF based schemes perform equal at minimum relay configuration. This is reasonable as the gain vector is uniquely determined in this case, and no optimization can be performed. **Fig. 5** depicts the fairness measures introduced in Section III versus N_{SD}. We create two clusters of S/D pairs: In Cluster A the S/D are close to the relays ($\sigma_{\mathrm{h}}^2 = 10$) and in Cluster B, they are further away ($\sigma_{\mathrm{h}}^2 = 1$). In **Fig. 5-a** we can see that the *MaxMinRate* scheme is closest to the fair-share resource allocation in which all links have the same rate, i.e. $\overline{F} = \overline{U} = 1$. Moreover, the *MMHsd* scheme reaches the fairness of *MaxMinRate* as N_{SD} increases. The decrease of average fairness at $N_{\mathrm{SD}} = 8$ is because the gain allocation vector is uniquely defined for the minimum relay configuration. As depicted in **Fig. 5-b** from all investigated schemes, *MaxMinRate* results the self-fairness closest to 1 for both clusters. The Cluster A pairs sacrifice resource to support pairs in Cluster B. Since they are allocated less resource their self-fairness is larger than 1, which tends to reach 1 as N_{SD} increases. On the other hand, Cluster B pairs have self-fairness smaller than 1, because of the more resource allocation to them than the fair-share resource allocation. Lastly, as N_{SD} increases, all schemes are inclined to provide fair-share resource allocation.

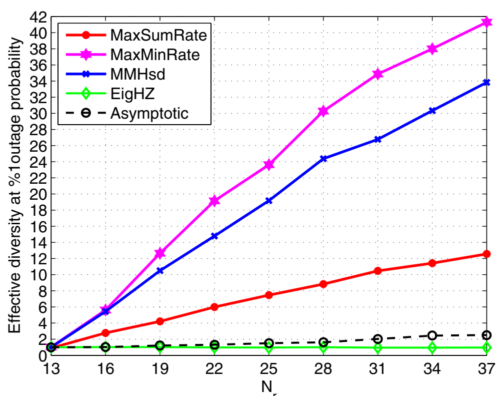

Fig. 6. Effective diversity at %1 outage probability vs. N_{R} for $N_{\mathrm{SD}} = 4$ and $\mathrm{SNR}_{\mathrm{def}}$ = 10 dB.

We plot the effective diversity at %1 outage probability versus the number of relays N_{R} in **Fig. 6**. Diversity increases linearly with N_{R}. The reason is that when adding additional relays, the number of independent channel realizations seen by each S/D pair increases. As expected, the *MaxMinRate* scheme supplies the largest diversity. This is because of the relation of diversity and fairness explained in Section IV. Note that with the common definition of diversity as the number of independent channel realizations over which the same data is transmitted, the diversity should not exceed the number of relays. However, as our definition is not strictly bound to this, we observe a values larger than N_{R} for the *MaxMinRate* scheme.

Figs. 4-6 validate our claim that *MMHsd* is the best to provide a trade-off between sum rate and fairness (diversity). As anticipated in Section IV-C, its sum rate performance approaches the maximum achievable sum rate, and moreover its fairness (diversity) performance is the closest to *MaxMinRate*.

In **Fig. 7** the average sum rate is plotted versus the number of S/D pairs N_{SD} for a fixed number of relays ($N_{\mathrm{R}} = 57$). It can be seen that for small number of S/D pairs, the average sum rate increases nearly linearly with N_{SD}. However, when approaching the maximum number of users that can be orthogonalized ($N_{\mathrm{SD,\,max}} = 8$ for $N_{\mathrm{R}} = 57$) the average sum rate drops again. This is due to the fact that the nullspace of

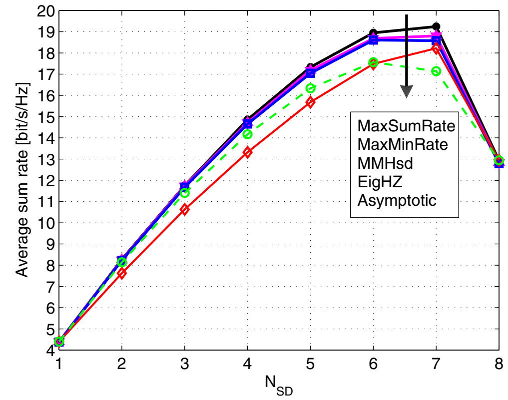

Fig. 7. Average sum rate vs. N_{SD} for $N_{\mathrm{R}} = 57$ and $\mathrm{SNR}_{\mathrm{def}}$ = 10 dB.

$\mathbf{H}_{\mathrm{I}}$ in which we choose the gain vector (see (6)) has ever less dimensions, i.e., the optimization space becomes smaller and smaller. Finally, for $N_{\mathrm{SD}} = N_{\mathrm{SD,\,max}} = 8$, the average sum rate is identical for all schemes. This is because the nullspace now has dimension one and the optimizations reduce to a scaling in order to fulfill the relay transmit power constraint, which is the same for all schemes. For $N_{\mathrm{SD}} = 1$, the average sum rate is also the same for all schemes because the relays do not need to orthogonalize the links.

VI. Conclusions

We considered a two hop wireless coherent multiuser relaying system. The relays apply a ZF scheme based on nullspace projection to orthogonalize all source/destination pairs. The enlarged nullspace with the excess number of relays enables the further optimization of the relay gain factors. The proposed optimization criteria are compared in terms of average sum rate, fairness and diversity. Moreover, the trade-off between the different figures of merit is investigated.

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