# Robust Time-of-Arrival Estimation with an Energy Detection Receiver

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***Abstract*— In this paper we derive and analyze a novel time-of-arrival estimation rule, which is robust to channel length uncertainty and requires a-priori knowledge of noise variance and signal-to-noise ratio only. This estimator can easily be implemented using the output of a standard energy detection receiver. Moreover, we show by means of computer simulations applying various channel models that the proposed rule outperforms state of the art low complexity time-of-arrival estimation rules in terms of root mean squared error.**

## I. Introduction

The large bandwidth of UWB signals offers the potential to improve the accuracy of positioning techniques through the high temporal resolution of the multipath propagation channel. The most promising localization techniques exploiting the wide bandwidth are based on time-of-arrival (ToA) estimation [1]. The unknown position of a transmitter is calculated by multi-lateration using the estimated ToA (distances) to at least three reference receivers with known positions.

In general, coherent receivers performing channel estimation are used to obtain accurate ToA estimates. However, UWB channel estimation requires high hardware complexity such as high rate analog-to-digital converters and broadband mixers. Since localization is an important feature in wireless sensor networks, it is crucial to obtain accurate ToA estimates even with low complexity non-coherent receivers. We review state-of-the-art low complexity solutions for ToA estimation in Section III with emphasis on required a-priori knowledge and implementation issues.

The contribution of this paper is the systematic derivation of a novel ToA estimation rule with advantages in terms of performance and implementation complexity over existing solutions. The most distinct features of this rule are the robustness to uncertainty in the channel length and the minimization of the number of observation samples required for ToA estimation. In order to facilitate a low complexity solution, we seek to minimize the required a-priori knowledge.

Root mean squared error (RMSE) performance results based on the IEEE 802.15.4a channel model (CM) 3, CM 4 [2], and Gaussian CMs show that this novel ToA estimation rule achieves better accuracy than state of the art solutions.

## II. Problem Formulation

We consider the real-valued received signal $r(t)$ after an ideal lowpass filter of bandwidth $B$ in an observation window $[0, T]$ given by

$$r(t) = h(t-\tau) + w(t).$$

The filtered Gaussian process $w(t)$ has zero mean and power spectral density $N_0/2$ for $-B \leq f \leq B$ and zero otherwise. The convolution of transmit pulse and channel impulse response is denoted by $h(t)$ and has the average energy $E_p = \mathrm{E}\left\{\int_0^T h^2(t)\mathrm{d}t\right\}$, where the expectation is with respect to the random channel. The parameter $\tau$ describes the ToA of the received pulse.

The received signal $r(t)$ is sampled with Nyquist rate $(2B)$ according to $r[n] = r\left(\frac{n}{2B}\right)$, which produces a set of $N = \lfloor T2B \rfloor$ samples

$$r[n] = h[n-m^\star] + w[n] \text{ for } n = 0, \ldots, N-1,$$

where the noise samples $w[n]$ are independently and identically distributed (i.i.d.) Gaussian with zero mean and variance $\sigma^2 = BN_0$. We assume that $h[n]$ is approximately zero for $n < 0$ and for $n \geq N_C$, where $N_C$ denotes the channel length. The discrete time ToA parameter is defined by $m^\star \triangleq \mathrm{round}\,(\tau 2B)$, which is the parameter we want to estimate in the remaining part of the paper.[1] It is assumed that $m^\star$ is uniformly distributed within $\{0, 1, \ldots, M-1\}$, where $M \leq N$ is the length of the uncertainty window. The goal is to estimate $m^\star$ based on the observed samples $r[n]$, which are stacked into the vector $\mathbf{r}$.

This estimation problem can be formulated as $M$-ary composite hypothesis testing problem [3], where all hypotheses have the same a-priori probability. Therefore, the maximum a-posteriori probability (MAP) decision rule is equivalent to the maximum likelihood (ML) decision rule. The probability density function (PDF) of $\mathbf{r}$ conditioned on hypothesis $m$ and $\mathbf{h}$ is given by

$$p\left(\mathbf{r}|m, \mathbf{h}\right) = \prod_{n=0}^{N-1} p\left(r[n]|h[n-m]\right),$$

where the vector $\mathbf{h}$ collects all channel taps. The joint PDF $p\left(\mathbf{r}|m, \mathbf{h}\right)$ can be factorized into the marginal PDFs $p\left(r[n]|h[n-m]\right)$ due to the statistically independent noise

[1] In this formulation, the sampling time poses a fundamental lower bound on the estimation accuracy of $\tau$.

samples. Consequently, the ML decision rule is given by

$$\hat{m} = \underset{m\in\{0,\ldots,M-1\}}{\text{argmax}} \left(\prod_{n=0}^{N-1} p\left(r[n]|h[n-m]\right)\right). \qquad (1)$$

## III. State of the Art ToA Estimation Rules

If the statistics of the channel are known a-priori, the conditional PDF in (1) can be averaged across all possible channel realizations, which leads to the averaged maximum likelihood (AML) decision rule [3]. The AML rule is the Bayesian solution to the composite hypothesis testing problem in (1). Assuming statistically independent channel taps, the AML rule is given by

$$\hat{m} = \underset{m\in\{0,\ldots,M-1\}}{\text{argmax}} \left(\prod_{n=0}^{N-1} p\left(r[n]|m\right)\right), \text{ with} \qquad (2)$$

$$p\left(r[n]|m\right) = \int p\left(r[n]|h[n-m]\right) p\left(h[n-m]\right) \mathrm{d}h[n-m].$$

Note that $p\left(h[n-m]\right)$ is deterministically zero, if $n-m<0$ and $n-m \geq N_C$. The AML rule is used as benchmark result, since it minimizes the total error probability $\mathrm{E}\left\{\mathrm{P}\left(\hat{m} \neq m^\star\right)\right\}$ provided the channel taps are distributed according to the assumed PDF.[2] The complexity of the AML rule can be reduced by dropping the requirement of a-priori known channel statistics.

A popular low complexity ToA estimation rule, which does not require channel statistics, is the threshold crossing (TC) detection rule (cf. [4], [5], [6]). It is formulated as

$$\hat{m} = \min\left\{n \in \{0,1,\ldots,M-1\} | r^2[n] > \delta_{\mathrm{TC}}\right\}, \qquad (3)$$

and selects as estimate for $m^\star$ the smallest index $n$ for which $r^2[n]$ exceeds the threshold $\delta_{\mathrm{TC}}$. The selection of this threshold is the crucial part of this method and a lot of research, e.g. [6], is devoted to this problem. A big advantage of threshold-based estimators is the potential for full analog implementation.

If the channel taps are treated as unknown and deterministic parameters, a joint optimization of (1) with respect to $\mathbf{h}$ and $m$ can be performed, which leads to the generalized maximum likelihood (GML) decision rule [3]. Although there exists no proof of optimality for the GML solution [7], this approach works well for many practical problems. For the ToA estimation problem in (1), the GML decision rule simplifies to the sliding window (SW) integration rule [8] given by

$$\hat{m} = \underset{m\in\{0,\ldots,M-1\}}{\text{argmax}} \left(\sum_{n=m}^{m+N_C-1} r^2[n]\right), \qquad (4)$$

where the ML estimates for the channel taps given hypothesis $m$ are $\hat{h}[n] = r[n+m]$. The derivation of this rule requires the knowledge of the channel length $N_C$. Furthermore, the observation window must be large enough, i.e. $N \geq N_C + M - 1$, such that the full channel impulse response is in the observation window for all hypotheses. Analog implementation of the SW integration rule is rather complex, since $M$ parallel integrate and dump devices are required [8].

[2] Due to their high complexity we do not consider decision rules assuming known channel taps.

## IV. Increasing Window Integration Rule

The TC detection rule (3) is not derived from the ML decision rule (1) and therefore performs strictly suboptimal. Furthermore, its performance depends heavily on the threshold selection, which itself depends on the channel and the signal-to-noise ratio (SNR).

On the other hand the SW integration rule (4) is based on (1) and it can be shown that the AML rule (2) reduces to (4), if the channel taps are i.i.d. and zero mean Gaussian distributed and $N \geq N_C + M - 1$. Thus, we can conclude that the SW integration rule performs optimal for i.i.d. and zero mean Gaussian channel taps.

One shortcoming of the SW integration rule is that good estimates for $N_C$ are hard to obtain with low complexity receivers. Moreover, if the channel is time-varying, then also $N_C$ is time-varying. Therefore, it would be beneficial to have a ToA estimation rule, which does not require the knowledge of $N_C$. In the following we derive a novel ToA estimation rule named *increasing window integration* (IWI) rule, which tackles the problem of unknown channel length $N_C$.

We start the derivation from (1) and assume that the channel taps are i.i.d. Gaussian with zero mean and variance $\sigma_h^2$. Intuitively, these channel assumptions resemble the least possible statistical channel knowledge, which can be seen as statistical equivalence to unknown and deterministic channels. Moreover, we assume that we do not know $N_C$ but we require that $N \leq N_C$. This imposes a short observation window, which is desirable due to a reduced number of required observation samples.

With these assumptions the AML rule in (2) follows as

$$\hat{m} = \underset{m\in\{0,\ldots,M-1\}}{\text{argmax}} \left(\prod_{n=0}^{m-1} \sqrt{\frac{1}{2\pi\sigma^2}} \exp\left(-\frac{r^2[n]}{2\sigma^2}\right) \times \prod_{n=m}^{N-1} \sqrt{\frac{1}{2\pi\left(\sigma^2+\sigma_h^2\right)}} \exp\left(-\frac{r^2[n]}{2\left(\sigma^2+\sigma_h^2\right)}\right)\right). \qquad (5)$$

The multipliers for $n \geq M-1$ in the second product of (5) are identical for all $m$, which means that they do not influence the result of the optimization and can be dropped. We observe that this is true for arbitrary channel distributions as long as the taps are i.i.d., which can be seen from the identity

$$\int p\left(r[n]|h[i]\right) p\left(h[i]\right) \mathrm{d}h[i] = \int p\left(r[n]|h[j]\right) p\left(h[j]\right) \mathrm{d}h[j],$$

for $p\left(h[i]\right) = p\left(h[j]\right)$. Therefore, a function of $r[n]$ for $n = 0,1,\ldots,M-2$ constitutes a sufficient statistic, if the channel taps are i.i.d. and $N \leq N_C$.

Taking the natural logarithm of (5) and collecting terms leads to

$$\hat{m} = \underset{m\in\{0,\ldots,M-1\}}{\text{argmax}} \left(\ln\left(\frac{\sigma^{-2m}}{\left(\sigma^2+\sigma_h^2\right)^{M-1-m}}\right) - \frac{1}{\sigma^2+\sigma_h^2}\left(\sum_{n=0}^{M-2} r^2[n] + \sum_{n=0}^{m-1} \frac{\sigma_h^2}{\sigma^2} r^2[n]\right)\right).$$

We note that $\sum_{n=0}^{M-2} r^2[n]$ is independent of the hypothesis $m$ and can be dropped. The IWI decision rule follows as

$$\hat{m} = \underset{m\in\{0,\ldots,M-1\}}{\operatorname{argmax}} \underbrace{\left(m\alpha - \sum_{n=0}^{m-1} \frac{r^2[n]}{\sigma^2}\right)}_{\mathrm{LL_{IWI}}(m)}, \text{ with} \qquad (6)$$

$$\alpha = \frac{\sigma^2+\sigma_h^2}{\sigma_h^2} \ln\left(\frac{\sigma^2+\sigma_h^2}{\sigma^2}\right).$$

This rule searches for the maximum of a linear function in $m$ with slope $\alpha$ minus the cumulative energy of the normalized received signal up to sample $m-1$. On average, the function $\mathrm{LL_{IWI}}(m)$ increases linearly in $m$ with slope $\alpha - 1$ as long as $r[n]$ consists of noise-only samples. Once the first channel tap is observed, the cumulative energy of noise and channel forces $\mathrm{LL_{IWI}}(m)$ to decrease. If the channel is sparse, i.e., between the paths are many noise-only samples, then the expected value of $\mathrm{LL_{IWI}}(m)$ increases during these samples again with slope $\alpha-1$. The fraction of energy of the first channel tap to the noise variance times $\alpha$, i.e. $\alpha h^2[0]/\sigma^2$, determines the maximum number of consecutive succeeding noise-only samples, such that the expected value of $\mathrm{LL_{IWI}}(m^\star)$ is still the maximum.

The equation for $\alpha$ in (6) follows from the derivation and states the optimal $\alpha$ for i.i.d. Gaussian channel taps. However, the parameter $\alpha$ has to be adapted, if the true channel deviates from this assumption. It will be shown in Section VI that the optimal value for $\alpha$ mainly depends on the SNR.

The implementation of the IWI rule with analog hardware requires only one squaring device, one integration device, and one analog-to-digital converter. A typical non-coherent UWB receiver (e.g. energy detector) is equipped with all these devices.

## V. A Two-Hypotheses Example

In this section, we demonstrate the robustness of the IWI decision rule against channel length uncertainty by means of a simple toy example. In order to model this uncertainty, we assume that the channel length $N_C$ is one with probability $1-p$ and with probability $p$ it is larger than one. We restrict the number of hypotheses to $M = 2$ and the number of observed samples to $N = 2$. This leads to the following two hypotheses:

$$H_0 : \mathbf{r} = \begin{cases} [h[0]+w[0], h[1]+w[1]] & \text{w.p. } p \\ [h[0]+w[0], w[1]] & \text{w.p. } 1-p \end{cases}$$
$$H_1 : \mathbf{r} = [w[0], h[0]+w[1]]\,.$$

The channel taps $h[0]$ and $h[1]$ are i.i.d. Gaussian with zero mean and variance $\sigma_h^2$. With this, the AML decision rule is given by

$$p\sqrt{\frac{\sigma^2}{\sigma^2+\sigma_h^2}}\exp\left(-\frac{r^2[0]}{2\left(\sigma^2+\sigma_h^2\right)} - \frac{r^2[1]}{2\left(\sigma^2+\sigma_h^2\right)}\right)$$
$$+ (1-p)\exp\left(-\frac{r^2[0]}{2\left(\sigma^2+\sigma_h^2\right)} - \frac{r^2[1]}{2\sigma^2}\right) \underset{H_1}{\overset{H_0}{\gtrless}}$$
$$\exp\left(-\frac{r^2[0]}{2\sigma^2} - \frac{r^2[1]}{2\left(\sigma^2+\sigma_h^2\right)}\right). \qquad (7)$$

For $p = 0$ the rule in (7) simplifies to

$$r^2[0] \underset{H_1}{\overset{H_0}{\gtrless}} r^2[1], \qquad (8)$$

and for $p = 1$ it reduces to

$$\frac{r^2[0]}{\sigma^2} \underset{H_1}{\overset{H_0}{\gtrless}} \delta_{\mathrm{AML}}, \qquad (9)$$

where the threshold $\delta_{\mathrm{AML}}$ is given by

$$\delta_{\mathrm{AML}} = \frac{\sigma^2+\sigma_h^2}{\sigma_h^2}\ln\left(\frac{\sigma^2+\sigma_h^2}{\sigma^2}\right).$$

From rule (9) it can be seen that the second observation $r[1]$ is not considered, since in this case a function of $r[0]$ is a sufficient statistic. This is an example for the general observation from Section IV right after (5).

If the channel taps are assumed to be unknown and deterministic, the GML decision rule is given by

$$(1-p)\exp\left(-\frac{r^2[1]}{2\sigma^2}\right) - \exp\left(-\frac{r^2[0]}{2\sigma^2}\right) \underset{H_1}{\overset{H_0}{\gtrless}} -p, \qquad (10)$$

where the ML estimates for the channel taps are

$$H_0 : \begin{cases} \hat{h}[0] = r[0] \\ \hat{h}[1] = r[1] \end{cases} \quad \text{and } H_1 : \hat{h}[0] = r[1].$$

For $p = 0$ the rule in (10) simplifies to (8). However, for $p = 1$ it reduces to (9) with $\delta_{\mathrm{GML}} = 0$, which estimates always $H_0$. The GML rule is therefore not applicable for $p = 1$.

It can be shown that the TC decision rule (3) is equivalent to (8) and that the IWI decision rule (6) is equivalent to (9) for $\alpha = \delta_{\mathrm{AML}}$. We compare the performance of the mentioned rules in terms of their total error probability $P_e$ defined as

$$P_e = 0.5\left(\mathrm{P}\left(\hat{m}=1|H_0\right) + \mathrm{P}\left(\hat{m}=0|H_1\right)\right).$$

The SNR for this toy example is defined as $\mathrm{SNR_{toy}} = \frac{\sigma_h^2}{\sigma^2}$ and is set to 30 dB. We note that the optimal $\alpha$ for i.i.d. Gaussian channel taps is completely specified by $\mathrm{SNR_{toy}}$ according to

$$\alpha = \left(1 + \frac{1}{\mathrm{SNR_{toy}}}\right)\ln\left(1+\mathrm{SNR_{toy}}\right). \qquad (11)$$

This dependence will proof useful also for more realistic channel models, which are discussed in the next section. Fig. 1 depicts $P_e$ as a function of $p$ for all discussed decision rules. It can be seen that for $p = 0$ GML, SW, and TC coincide with the AML performance and IWI shows a small performance loss. For increasing $p$ the performance of GML, SW, and TC drops drastically. The reason for SW performing poorly is the wrong assumption on the channel length. However, the GML rule accounts for $p$, i.e., it knows that the channel has two taps with probability $p$. Nevertheless, it performs poorly, which indicates that the concept of GML is not applicable to ToA estimation problems, where the $N_C$ is uncertain.

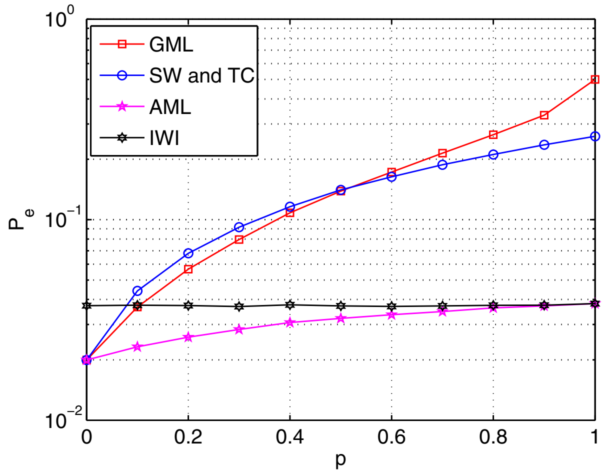


Fig. 1. $P_e$ for GML (10), SW and TC (8), AML (7), IWI (9) as function of $p$ and $\mathrm{SNR}_{\mathrm{toy}} = 30$ dB.

## VI. Simulation Results

We simulate the performance of the discussed ToA estimation rules for IEEE 802.15.4a CM 3, CM 4, and for Gaussian channels with an exponential power delay profile (PDP) with a length of $N_C = 150$ taps. The performance metric is the RMSE and SNR $\triangleq \frac{E_p}{N_0}$, where $E_p$ was the average energy of the received pulse (cf. Section II). We set the bandwidth of the lowpass filter to $B = 2.5$ GHz.[3] The uncertainty window size is set to 20 ns, which implies $M = 100$. The observation window size is set to $T = 60$ ns, which implies $N = 300$. Note that the IWI and TC rules require only the observation of the first 99 and 100 samples, respectively. The performance curves are obtained for 1000 channel realizations and 1000 noise realizations for each channel.

We compare the results of the IWI decision rule (6) with the SW integration rule (4) and the TC decision rule (3).[4] All three rules require a proper choice of one parameter $(\alpha, N_I, \delta_{\mathrm{TC}})$, where $N_I$ is the integration window size in (4). Note that $N_I = N_C$ for i.i.d. Gaussian channels. However, for more realistic channel models $N_I$ can be adapted to improve the performance. In order to show the dependency of (3) on $\delta_{\mathrm{TC}}$, we use the normalized threshold $\delta_{\mathrm{normTC}} \in [0, 1]$ and the relationship

$$\delta_{\mathrm{TC}} = \min_n \left(r^2[n]\right) + \delta_{\mathrm{normTC}} \left(\max_n \left(r^2[n]\right) - \min_n \left(r^2[n]\right)\right),$$

which ensures that the threshold $\delta_{\mathrm{TC}}$ is between the smallest and the largest value of $r^2[n]$.

First we find the optimal values $(\alpha^\star, N_I^\star, \delta_{\mathrm{normTC}}^\star)$, which provide minimal RMSE for each channel model, based on exhaustive search.

Fig. 2 depicts the RMSE of the IWI rule (6) as a function of the parameter $\alpha$ for two SNR values of 15 dB and 30 dB.

[3] Lower bound on RMSE for estimation of $\tau$ is $\frac{1}{2B\sqrt{12}} \approx 0.06$ ns.

[4] For the exponential PDP, we provide the minimal RMSE as benchmark, which is obtained by the AML rule (2).

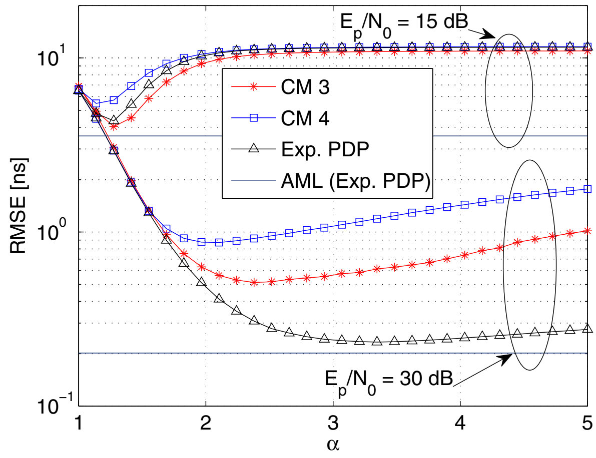


Fig. 2. RMSE for IWI (6) as function of $\alpha$ for all channel models.

The three curves with different markers show the RMSE for CM 3, CM 4, and the exponential PDP. The solid lines without markers show the RMSE of the AML rule for the exponential PDP. It can be seen that there exists a single $\alpha^\star$, which depends on the SNR and the channel model. Fig. 3 and Fig. 4 depict the corresponding RMSE for (4) and (3).

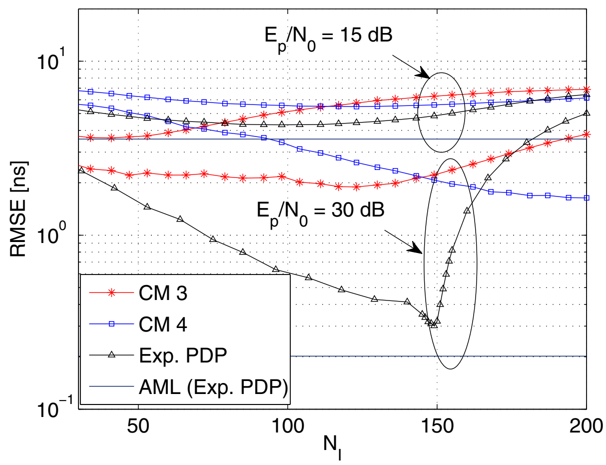


Fig. 3. RMSE for SW (4) as function of $N_I$ for all channel models.

By comparing Figs. 2, 3, and 4, it can be seen that the IWI rule achieves the smallest RMSE for all CMs for $\alpha^\star$ compared to the other rules for $N_I^\star$ and $\delta_{\mathrm{normTC}}^\star$. The IWI rule achieves almost optimal (AML) performance for the exponential PDP. The SW rule performs poor for CM 3 and CM 4 for all $N_I$ due to the fact that the channel realizations do not have a fixed length. From the SW performance curve for the exponential PDP at high SNR in Fig. 3, it can be seen that a small deviations of $N_I$ from the actual length of the channel ($N_C = 150$ samples) cause severe performance degradations.

From (11) we can deduce that $\alpha^\star$ increases approximately linearly with $\mathrm{SNR}_{\mathrm{toy}}$ in dB for i.i.d. Gaussian channels. Simulations examining further SNR operating points show

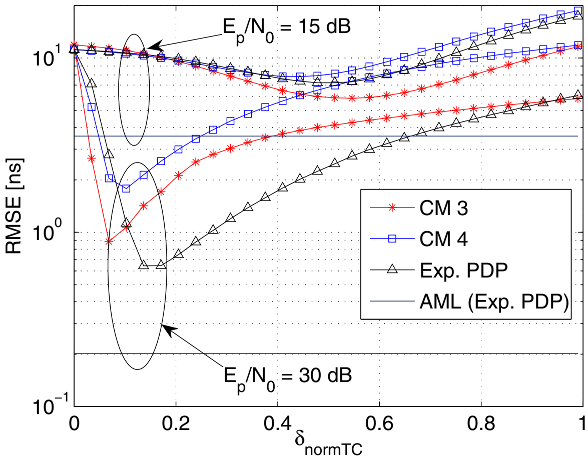


Fig. 4. RMSE for TC (3) as function of $\delta_{\text{normTC}}$ for all channel models.

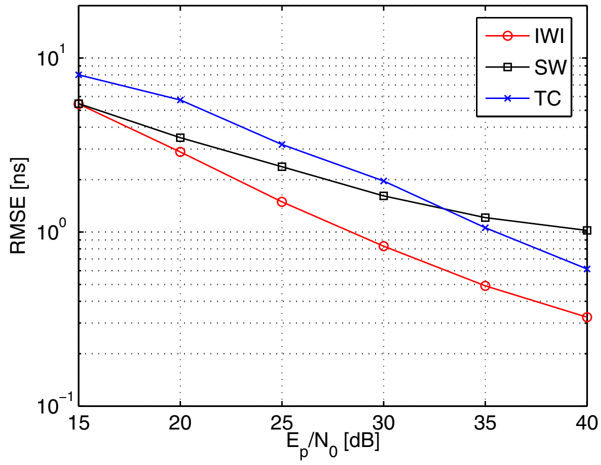


Fig. 6. RMSE for CM 4 as function of $\frac{E_p}{N_0}$.

that this is also approximately true for the here considered CMs. Therefore, we propose to use linear interpolation to approximate $\alpha^\star$ for arbitrary SNR operating points based on two available values $\alpha_l^\star$ for $\mathrm{SNR}_l$ and $\alpha_h^\star$ for $\mathrm{SNR}_h$ according to

$$\tilde{\alpha}^\star\left(\mathrm{SNR_{dB}}\right) = \alpha_h^\star + \frac{\alpha_h^\star - \alpha_l^\star}{\mathrm{SNR}_h - \mathrm{SNR}_l}\left(\mathrm{SNR_{dB}} - \mathrm{SNR}_h\right) \approx \alpha^\star.$$

The corresponding RMSE for CM 3 and CM 4 as functions of the SNR are shown in Fig. 5 and Fig. 6. It can be seen that the IWI rule outperforms the others, although we used $\tilde{\alpha}^\star$ as estimate for $\alpha^\star$. The corresponding $N_I^\star$ and $\delta_{\text{normTC}}^\star$ are computed by exhaustive search for each SNR value and CM. The curves for $\alpha^\star$ (computed by exhaustive search) show only a minor improvement compared to the curves in Fig. 5 and Fig. 6 and are not depicted.

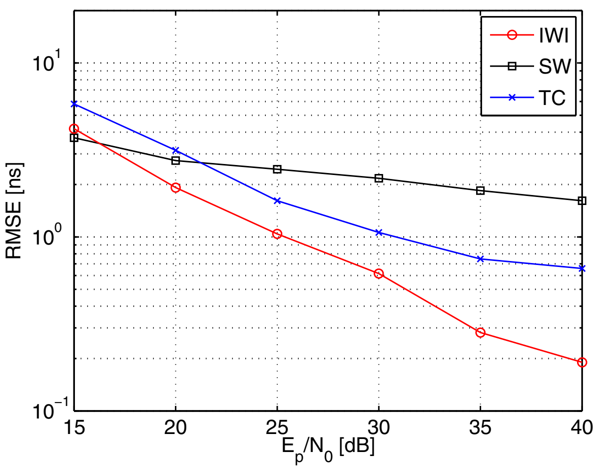


Fig. 5. RMSE for CM 3 as function of $\frac{E_p}{N_0}$.

## VII. Conclusions

We have derived a novel low complexity ToA estimation rule, which provides better performance for various CMs than low complexity ToA estimation rules proposed in literature. The IWI rule requires only minimal a priori knowledge in the form of noise variance and the parameter $\alpha$. The channel length $N_C$ is not required, which makes this rule robust to time-varying $N_C$. We find that the optimal values for $\alpha$ depend approximately linearly on the SNR in dB for various CMs. Finally, the IWI rule is easy to implement with analog hardware, and it requires only the minimal number of observation samples for a given uncertainty window.

## VIII. Acknowledgements


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