Impact of Cooperative Relays on the Capacity of Rank-Deficient MIMO Channels

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***Abstract*— We study the impact of multiple linear amplify-and-forward relays on the capacity of rank-deficient MIMO channels. We derive a general system model for wireless networks with one source/destination pair and several linear amplify-and-forward relay nodes which assist the communication between source and destination. All nodes may be equipped with multiple antennas. For a given allocation of gain factors at the relay nodes we give an analytical expression of the capacity by generalizing the results in [1], [2]. We compare the performance of a relay assisted MIMO link in a line-of-sight environment with a MIMO link without relay nodes. Our results show that the proposed cooperative signaling scheme solves a fundamental problem of MIMO systems: the rich scattering requirement.**

I. Introduction

Antenna arrays at the transmit and/or receive side introduce additional degrees of freedom (spatial dimension) into a wireless communication system. Space-time processing utilizes these degrees of freedom to boost link capacity and/or to enhance diversity. A system with N transmit and M receive antennas constitutes a $M \times N$ multiple-input multiple-output (MIMO) communication system. For i.i.d. flat Rayleigh fading channel coefficients the ergodic capacity of a MIMO link increases proportional by $min\{M, N\}$. In view of the cost of bandwidth this observation has initiated a vast body of research on MIMO systems in recent years. There is however a major obstacle in the practical exploitation of MIMO technology: the capacity increase depends on the propagation environment and diminishes with increasing correlation of the channel coefficients [3].

Node cooperation at the Physical Layer (PHY) is the natural extension of space-time processing to multiple distributed nodes. The distributed nodes essentially form a virtual macro antenna array. Completely new possibilities arise due to the large form factor of this virtual array, the number of cooperating nodes in dense networks, the availability of (limited) signal processing capability at some nodes and the possibility to exchange some control information between adjacent nodes.

Previous Work. The most basic form of PHY node cooperation is a linear *amplify-and-forward relay*, which assists the communication between a source and a destination: The relay amplifies the received signal and forwards it to the designated destination. The impact of cooperative relay nodes on the outage capacity of a wireless point-to-point link has been investigated in [4]. The authors compare the performance of a *decode-and-forward* and an *amplify-and-forward* strategy by means of outage regions (in terms of SNR random variables) and associated outage probabilities.

First capacity results on relay channels have been found in the seventies by Van der Meulen [5], Sato [6], and Cover and Gamal in [7]. More recently upper and lower bounds on the capacity of wireless networks with a relay traffic pattern have been determined in [8] and [9]. In both papers the system model consists of one source/destination pair, while all other nodes operate as relay stations in order to assist this transmission using arbitrary complex network coding.

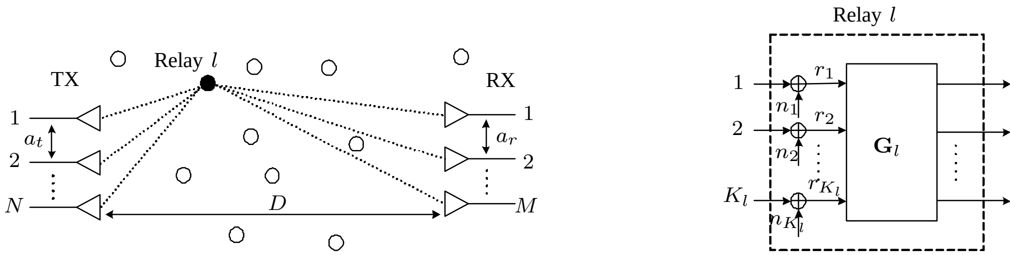
Our Contribution. We derive a general signal model for a wireless network with one source/destination pair and several linear amplify-and-forward relays, where all nodes may employ multiple antennas. Capacity of the relay assisted MIMO link (source/destination pair) for a given gain allocation is then presented. We compare different protocol scenarios and gain allocation strategies by numerical simulations.

Organization of the Paper. The remainder of the paper is organized as follows. The next section discusses the approach we consider in this work and gives a typical result. In section III we derive a signal model for a relay assisted MIMO link and give the corresponding capacity expression. Section IV presents and discusses simulation results. Conclusions and an outlook to future work are given in the last section.

II. Approach

Fig. 1(a) illustrates the proposed cooperative MIMO system. The source (TX) has N and the destination (RX) has M antennas. In contrast the relays do not necessarily feature multiple antennas. In this example the transmission of a data packet from the source to the destination occupies two time slots. The first time slot is allocated to the source exclusively. The relays receive during the first time slot and retransmit an amplified version of the received analog signal during the second time slot. The second time slot is also available to the source. Note that all concurrent transmissions utilize the same physical channel and therefore interfere at the destination. The goal of the node cooperation is to increase the rank of the compound (two time slots) channel matrix and to shape the eigenvalue distribution such that the capacity of the MIMO link improves.

Fig. 2 compares the mean capacity of an $N \times N$ MIMO link with and without relay nodes assisting the communi-



(a) A 2-hop relay network with antenna arrays at source and destination

(b) Multi-antenna relay model

Fig. 1. Relay network with linear relay nodes

cation. We assume free space propagation and the capacity is averaged with respect to the random location of the relays. Without relaying the channel matrix has rank one. The capacity increases roughly with $\log N$ because the SNR improves proportionally to N (transmit array gain). With relaying the compound channel matrix has full rank and the capacity increases almost proportionally to N. A typical scenario to which Fig. 2 applies is a future wireless LAN (WLAN) in the 17 GHz band (Hiperlan). The quasi-optical propagation and the high data rates mandate a high density of access points (APs). For cost reasons it would be beneficial to connect the APs by a wireless backbone rather having to install a dense wired backbone infrastructure. For this purpose each access point is equipped with an antenna array to increase the capacity of the wireless backbone. Due to the quasi-optical propagation the channel matrix has low rank. The dashed line in Fig. 2 is indicative for the relation between capacity and array size. According to our proposed scheme the mobile nodes (supporting one or more antennas) act as relays. Note, that the relay nodes in Fig. 1(a) can be viewed as "active" omni-directional scatterers which establish a sort of multipath channel. A major difference to passive scattering is that the relay nodes add noise to the forwarded signal.

III. System Model

Fig. 1(a) depicts the communication link for which we want to determine the capacity. A source with N transmit antennas wishes to send information to a destination with M receive antennas. It is assumed that the source and destination are within range. R relay nodes assist the communication in order to increase the channel rank and the capacity. Fig. 1(b) depicts the signal model of the lth amplify-and-forward relay with K_l antennas. The received signal vector $\mathbf{r}_l = [r_1, \ldots, r_{K_l}]^t$ is perturbed by additive white Gaussian noise, $\mathbf{n}_l = [n_1, \ldots, n_{K_l}]^t \sim \mathcal{CN}(\mathbf{0}, \sigma_\mathrm{r}^2 \mathbf{I}_{K_l})$. The superscript t denotes transpose. The vector $\mathbf{r}_l$ is multiplied by the gain matrix $\mathbf{G}_l$ prior to retransmission.

In time slot k the source sends the baseband equivalent discrete-time vector $\mathbf{x}^{(k)} = \left[x_1^{(k)}, x_2^{(k)}, \ldots, x_N^{(k)}\right]^t$. Relay l receives the $K_l \times 1$ vector

$$\mathbf{r}_l^{(k)} = \mathbf{H}_{1,l}\mathbf{x}^{(k)} + \mathbf{n}_l^{(k)}. \tag{1}$$

$\mathbf{H}_{1,l} \in \mathbb{C}^{K_l \times N}$ contains the channel coefficients between the source and relay l (first hop). The elements of $\mathbf{H}_{1,l}$ are determined by the relay location (path loss) and the channel model. The destination receives $\mathbf{y}^{(k)} = \left[y_1^{(k)}, y_2^{(k)}, \ldots, y_M^{(k)}\right]^t$. For ease of notation we define the following compound vectors and matrices:

$$\mathbf{r}^{(k)} = \left[\mathbf{r}_1^{(k)t}, \ldots, \mathbf{r}_R^{(k)t}\right]^t, \tag{2}$$

$$\mathbf{n}_\mathrm{r}^{(k)} = \left[\mathbf{n}_1^{(k)t}, \ldots, \mathbf{n}_R^{(k)t}\right]^t, \tag{3}$$

$$\mathbf{G} = \mathrm{diag}\left(\mathbf{G}_1, \ldots, \mathbf{G}_R\right), \tag{4}$$

$$\mathbf{H}_1 = \left[\mathbf{H}_{1,1}^t \ldots \mathbf{H}_{1,R}^t\right]^t. \tag{5}$$

The signals received by the relays and the destination in time slot k are given by

$$\mathbf{r}^{(k)} = \mathbf{H}_1\mathbf{x}^{(k)} + \mathbf{n}_\mathrm{r}^{(k)}, \tag{6}$$

$$\mathbf{y}^{(k)} = \mathbf{H}_0\mathbf{x}^{(k)} + \mathbf{n}^{(k)}. \tag{7}$$

$\mathbf{H}_0 \in \mathbb{C}^{M \times N}$ contains the channel coefficients of the direct link between source and destination. $\mathbf{n}^{(k)} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I}_M)\ \forall\ k$ denotes the AWGN contribution at the destination receiver front end.

In the next time slot $k+1$, the relays send $\mathbf{G}\mathbf{r}^{(k)}$ to the destination. In addition the source may use time slot $k+1$ to send a new data vector $\mathbf{x}^{(k+1)} = \left[x_1^{(k+1)}, x_2^{(k+1)}, \ldots, x_N^{(k+1)}\right]^t$ directly to the destination. Note that the relays are not able to receive in time slot $k+1$, since they are already forwarding the signals from the previous time slot and practical considerations (antenna coupling) prevent the relay nodes from

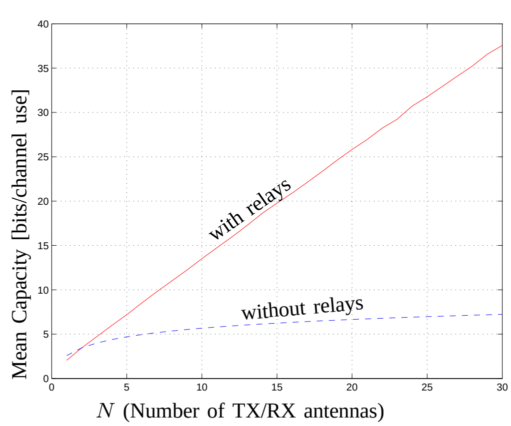
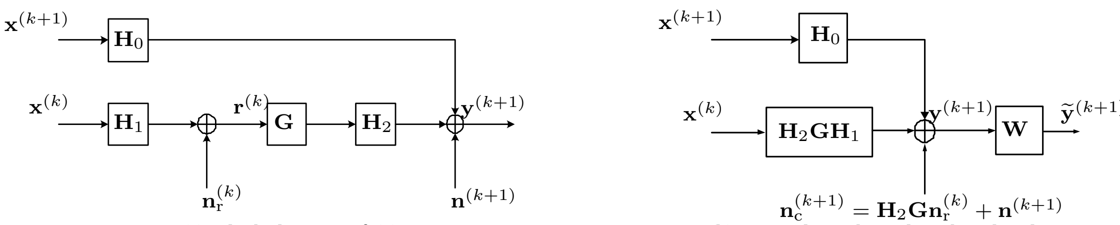


Fig. 2. Mean capacity of a $N \times N$ MIMO link with and without relay nodes for free space propagation and number of relays $R = 112$



(a) Block diagram of (9)

(b) Equivalent channel with colored noise and whitening filter $\mathbf{W}$

Fig. 3. Block diagrams of received signal at the destination in time slot $k+1$

transmitting and receiving concurrently at the same physical channel.

Thus, in time slot $k+1$ the destination receives

$$\mathbf{y}^{(k+1)} = \mathbf{H}_2\mathbf{G}\mathbf{r}^{(k)} + \mathbf{H}_0\mathbf{x}^{(k+1)} + \mathbf{n}^{(k+1)}. \tag{8}$$

The compound channel matrix $\mathbf{H}_2 = [\mathbf{H}_{2,1} \dots \mathbf{H}_{2,R}]$ contains the channel coefficients between the relays and the destination (second hop). By inserting (6) into (8) and separating signal and noise terms we obtain

$$\begin{aligned}\mathbf{y}^{(k+1)} = [\mathbf{H}_2\mathbf{G}\mathbf{H}_1 \quad \mathbf{H}_0] \begin{bmatrix}\mathbf{x}^{(k)} \\ \mathbf{x}^{(k+1)}\end{bmatrix} \\ + [\mathbf{H}_2\mathbf{G} \quad \mathbf{I}_M] \begin{bmatrix}\mathbf{n}_\mathrm{r}^{(k)} \\ \mathbf{n}^{(k+1)}\end{bmatrix}.\end{aligned} \tag{9}$$

Fig. 3(a) illustrates (9). The signal component of $\mathbf{y}^{(k+1)}$ is a linear combination of the source signals in time slots k and $k+1$. The noise component is a linear combination of the destination noise $\mathbf{n}^{(k+1)}$ and the relay noise $\mathbf{n}_\mathrm{r}^{(k)}$. Due to the gain matrix $\mathbf{G}$ and the channel matrix $\mathbf{H}_2$ the resulting noise at the destination in time slot $k+1$ is in general not white. Let $\mathbf{n}_\mathrm{c}^{(k+1)}$ denote this noise component (Fig. 3(b)). Its autocorrelation matrix $\mathbf{\Lambda}$ follows as

$$\begin{aligned}\mathbf{\Lambda} &= \mathrm{E}\left[\mathbf{n}_\mathrm{c}^{(k+1)} {\mathbf{n}_\mathrm{c}}^{(k+1)\dagger}\right] \\ &= \sigma^2 \left(\mathbf{H}_2\mathbf{G}\mathbf{G}^\dagger\mathbf{H}_2^\dagger \frac{\sigma_r^2}{\sigma^2} + \mathbf{I}_M\right) = \sigma^2\mathbf{\Lambda}^{'}.\end{aligned} \tag{10}$$

where $\dagger$ denotes conjugate transpose. The matrix $\mathbf{\Lambda}^{'}$ can be diagonalized by a unitary similarity transformation: $\mathbf{\Lambda}^{'} = \mathbf{V}\mathbf{D}\mathbf{V}^\dagger$, where $\mathbf{V}$ contains the eigenvectors of $\mathbf{\Lambda}^{'}$ and the diagonal matrix $\mathbf{D}$ the corresponding eigenvalues. $\mathbf{V}$ is a unitary matrix, i.e., $\mathbf{V}\mathbf{V}^\dagger = \mathbf{V}^\dagger\mathbf{V} = \mathbf{I}_M$. The whitening matrix

$$\mathbf{W} = \mathbf{D}^{-\frac{1}{2}}\mathbf{V}^\dagger \tag{11}$$

decorrelates the noise vector $\mathbf{n}_\mathrm{c}^{(k+1)}$, because

$$E\left[\left(\mathbf{W}\mathbf{n}_\mathrm{c}^{(k+1)}\right)\left(\mathbf{W}\mathbf{n}_\mathrm{c}^{(k+1)}\right)^\dagger\right] = \mathbf{W}\mathbf{\Lambda}\mathbf{W}^\dagger = \sigma^2\mathbf{I}_M. \tag{12}$$

We obtain an equivalent signal model with white noise $\mathbf{w}^{(k+1)} = \mathbf{W}\mathbf{n}_\mathrm{c}^{(k+1)} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I}_M)\ \forall\, k$ (Fig. 3(b)). Let $\widetilde{\mathbf{y}}^{(k+1)} = \mathbf{W}\mathbf{y}^{(k+1)}$ be the equivalent received signal. With (9) we obtain

$$\widetilde{\mathbf{y}}^{(k+1)} = [\mathbf{W}\mathbf{H}_2\mathbf{G}\mathbf{H}_1 \quad \mathbf{W}\mathbf{H}_0] \begin{bmatrix}\mathbf{x}^{(k)} \\ \mathbf{x}^{(k+1)}\end{bmatrix} + \mathbf{w}^{(k+1)}. \tag{13}$$

The signal received at the destination in time slot k is given by (7). Note that the whitening matrix $\mathbf{W}$ is only effective in time slot $k+1$. It is convenient to define a channel matrix

$$\mathbf{H}_{12} = \mathbf{W}\mathbf{H}_2\mathbf{G}\mathbf{H}_1 \tag{14}$$

and to consider both time slots jointly

$$\begin{bmatrix}\mathbf{y}^{(k)} \\ \widetilde{\mathbf{y}}^{(k+1)}\end{bmatrix} = \underbrace{\begin{bmatrix}\mathbf{H}_0 & \mathbf{0} \\ \mathbf{H}_{12} & \mathbf{W}\mathbf{H}_0\end{bmatrix}}_{\mathbf{H}} \begin{bmatrix}\mathbf{x}^{(k)} \\ \mathbf{x}^{(k+1)}\end{bmatrix} + \begin{bmatrix}\mathbf{n}^{(k)} \\ \mathbf{w}^{(k+1)}\end{bmatrix}. \tag{15}$$

Equation (15) represents a $2N \times 2M$ MIMO channel with additive white Gaussian noise. The capacity in bits per two channel uses for the case that the channel is unknown to the source but perfectly known to the destination follows readily [1] by

$$C_\mathbf{H} = \log_2 \det\left(\mathbf{I}_{2M} + \frac{P}{N \cdot \sigma^2}\mathbf{H}\mathbf{H}^\dagger\right). \tag{16}$$

where $P = P^{(k)} + P^{(k+1)}$ with $P^{(i)} = \mathrm{E}\left[\mathbf{x}^{(i)\dagger}\mathbf{x}^{(i)}\right]$. If $\mathbf{H}$ is a random matrix, the mean capacity C is given by the expectation $\mathrm{E}_\mathbf{H}[C_\mathbf{H}]$.

IV. Numerical Results

Simulation Setup: Fig. 4 depicts an example of a 2-hop relay network. The source node is located at coordinates $(0,0)$ and the destination node at $(0,1)$. The relay nodes are randomly placed in the square $(-2,2) \times (-2,2)$ according to a uniform distribution. The source and the destination node are equipped with multiple antennas. The relay nodes have only one transmit and receive antenna, i.e., $\mathbf{G}$ in (4) is a diagonal matrix containing the gain factors $g_l \in \mathbb{C}$ of the individual relays. We assume for the whole simulation set a path loss channel model (no multipath) with a power path loss exponent $\alpha = 4$.

We consider three different **Protocol Scenarios** (*P1*)-(*P3*):

(*P1*). A signal sent by the source reaches the destination only via the relay nodes, i.e., there is no line-of-sight between source and destination. Two time slots are necessary to transmit the information: in the first time slot the source sends the information signals to the relay nodes. In the second time slot the relay nodes forward an amplified version to the destination, whereas the source is quiet. In this case (15) is written as follows:

$$\begin{pmatrix}\mathbf{0} \\ \widetilde{\mathbf{y}}^{(k+1)}\end{pmatrix} = \underbrace{\begin{bmatrix}\mathbf{0} & \mathbf{0} \\ \mathbf{H}_{12} & \mathbf{0}\end{bmatrix}}_{\mathbf{H}_{(P1)}} \begin{pmatrix}\mathbf{x}^{(k)} \\ \mathbf{0}\end{pmatrix} + \begin{pmatrix}\mathbf{0} \\ \mathbf{w}^{(k+1)}\end{pmatrix}. \tag{17}$$

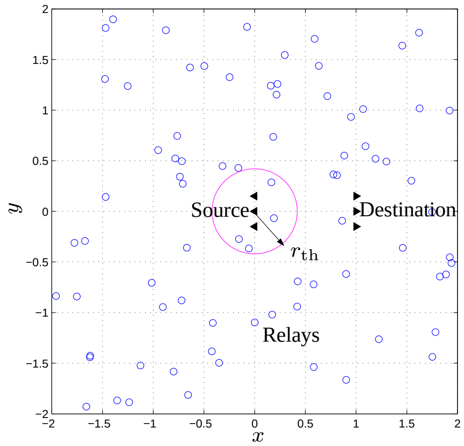


Fig. 4. MIMO communication link assisted by a relay network, $\{N, M, R\} = \{3, 3, 80\}$

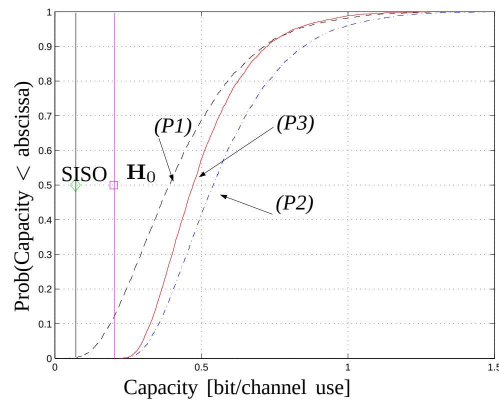


Fig. 5. Cumulative Distribution Functions of the capacity for protocol scenarios (*P1*)-(*P3*), $\{N, M, R, \sigma_r^2, \sigma^2, g_{\rm th}\} = \{3, 3, 80, 10^{-5}, 10^{-5}, 45\text{dB}\}$, mean values are plotted at the 50% percentile of the CDFs

(*P2*). Here we have additionally a line-of-sight channel between source and destination, i.e, the destination receives in the first time slot signals from the source and in the second time slot signals from the relay nodes. Again, the source is quiet in the second time slot. Equation (15) reduces to

$$\begin{pmatrix} \mathbf{y}^{(k)} \\ \tilde{\mathbf{y}}^{(k+1)} \end{pmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_0 & \mathbf{0} \\ \mathbf{H}_{12} & \mathbf{0} \end{bmatrix}}_{\mathbf{H}_{(P2)}} \begin{pmatrix} \mathbf{x}^{(k)} \\ \mathbf{0} \end{pmatrix} + \begin{pmatrix} \mathbf{n}^{(k)} \\ \mathbf{w}^{(k+1)} \end{pmatrix}. \tag{18}$$

(*P3*). In this scenario we allow the source to transmit in both time slots. The received signal vector is given by (15).

Let $\mathbf{x} = \left[\mathbf{x}^{(k)t}, \mathbf{x}^{(k+1)t}\right]^t$ be the compound source transmit symbol vector. The protocol scenarios are compared on the basis of the same source transmit energy $\mathrm{E}\left[\mathbf{x}^\dagger \mathbf{x}\right] = 1$. As the source does not know the channel matrix $\mathbf{H}$, we can not employ adaptive power allocation to maximize the capacity for a given $\mathbf{H}$. Our experience shows that a static power allocation (i.p. between time slots k and $k+1$) may be beneficial nevertheless. The numerical results in this section specifically are based on the following heuristic power allocation for *P3*: 75% of the available power is allocated to time slot k and radiated from all three antennas. The remaining power (25%) is allocated to time slot $k+1$ but radiated from a single antenna only.

In Fig. 5 we plot the Cumulative Distribution Functions (CDF) of the capacity of the system depicted in Fig. 4 under protocol scenarios (*P1*)-(*P3*). The curves are obtained by uniform random placement of the relays. The mean capacities are also shown in the figure. For comparison the capacity of the corresponding Single-Input Single-Output (SISO) system and the capacity of the direct one-hop channel ($\mathbf{H}_0$) are given.

The gain allocation of the relays modifies the compound channel matrix $\mathbf{H}$ and thus has impact on the capacity. For the results in Fig. 5 the real gain factors g_l of the SISO relays are chosen as

$$g_l = \begin{cases} \frac{1}{a_l}, & d_l < r_{\rm th}; \\ \frac{1}{a_{\rm th}} = g_{\rm th}, & d_l \geq r_{\rm th}. \end{cases} \tag{19}$$

where $a_l = \sqrt{\frac{1}{d_l^\alpha}}$ denotes the channel attenuation from the source to relay l (d_l is the distance). We assume that the attenuations (but not the phase shifts) from a relay to each antenna element at the source/destination are equal (antenna spacing $\ll d_l$). $g_{\rm th} = \sqrt{r_{\rm th}^\alpha}$ denotes a threshold gain which is determined by the radius $r_{\rm th}$ of the disk depicted in Fig. 4. Relays within the disk simply compensate the channel attenuation (but not the phase shifts) and forward the amplified signal to the destination (we assume that the relays know the magnitude of the channel coefficients perfectly). Relays which are located outside the disk do not compensate the channel attenuation because the noise amplification would be too high. Instead they amplify the received signals with a threshold gain as shown in (19). The choice of the threshold gain has considerable impact on the mean capacity. Fig. 6 shows that for the given set of parameters the optimum threshold gain is identical for all protocol scenarios. The results in Fig. 5 are based on the optimum threshold gain. Note that the threshold gain is static: it is not adapted to the actual relay placement.

An intuitive explanation of Fig. 6 is provided by the following observation. If the threshold gain is too small, essentially only nodes cooperate, which are adjacent to the source. For this reason the angular spread of the virtual sources as seen from the destination is small and the compound channel matrix $\mathbf{H}$ has small rank. The larger the threshold gain, the more relays effectively participate. The angular spread and the MIMO capacity increases. At a certain threshold gain the angular spread saturates and the capacity starts to drop because more and more distant relays with small receive SNR get involved.

From Fig. 5 we can see that the capacity is increased if the relays assist the communication between source and destination. The reason is that $\mathbf{H}_0$ has rank one. We see further that protocol (*P2*) performs best for the chosen parameter set $\{N, M, R, \sigma_r^2, \sigma^2, g_{\rm th}\} = \{3, 3, 80, 10^{-5}, 10^{-5}, 45\text{dB}\}$, i.e., the source should use all its available energy to transmit in the first time slot and be quiet in the second time slot. In the case of low cost relays, the local noise variance $\sigma_{\rm r}^2$ at the relay

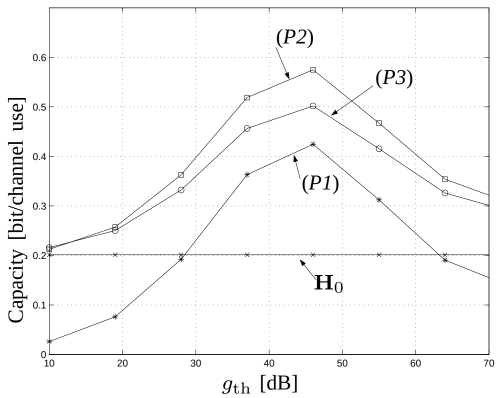


Fig. 6. Mean capacity versus threshold gain $g_{\rm th}$ in dB, $\{N, M, R, \sigma_r^2, \sigma^2\} = \{3, 3, 80, 10^{-5}, 10^{-5}\}$

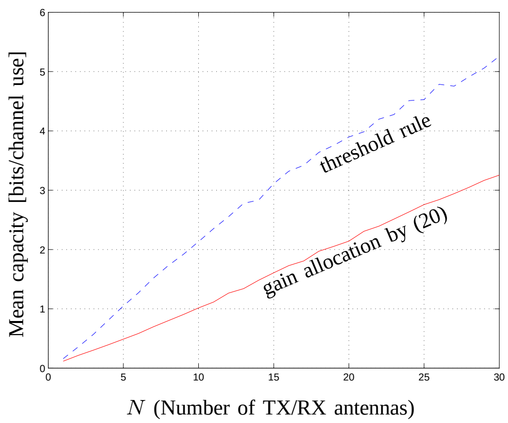


Fig. 7. Comparison of mean capacity for gain allocation with threshold rule and gain allocation by (20) with $Q_l = 1\ \forall l$

may exceed the destination noise variance. Our experience shows that typically in that case (*P3*) outperforms (*P2*).

The proposed gain allocation strategy does not require communication between the relays. Rather is is based on RSSI (Radio Signal Strength Indication) measurements at each relay. It is interesting to compare this simple strategy with another gain allocation scheme . In Figure 7 we compare the threshold rule described in (19) with a gain allocation according to [4], [8]

$$g_l = \sqrt{\frac{Q_l}{a_l^2 + \sigma_r^2}} \tag{20}$$

where Q_l denotes the maximum transmit power of relay l. As seen from Fig. 7 it is not advantageous to allow the relays to send with maximum power. By applying the simple threshold rule we can avoid a high noise amplification by distant relays which only receive a weak signal from the source.

In (19) and (20) the relays only compensate the attenuations of the first hop channel but not the phase shifts. Fig. 8 compares the case where the relays and the destination fully invert the channel (magnitude and phase) and the case where only channel attenuations are compensated but not the phase shifts. We can see that with a phase compensation the capacity grows approximately with $\log R$ as it was derived in [8] for the SISO case. However, in many cases assuming

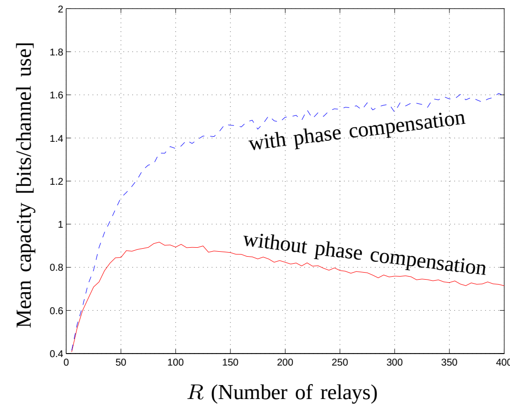


Fig. 8. Mean Capacity with and without phase compensation at the relays and destination with $N = M = 5$, gain allocation by (20)

phase coherence among the relay nodes is not very practical because would require a common frequency reference. Furthermore, small variations in the location of the mobile relay nodes lead immediately to large phase changes which are difficult to track.

V. Conclusion

We derived a general system model for wireless networks with one source/destination pair (MIMO link) and several linear amplify & forward relays with one ore more antennas. Assuming a given gain allocation for the relays we gave the exact capacity expression for such a network by generalizing the results from [1] and [2]. Two gain allocation strategies were compared to study their impact on the capacity. It was shown that with a simple threshold rule better capacity results could be obtained than by allowing all relays to transmit with maximum power. Further investigations will concentrate on the relation between specific gain allocations and the resulting capacity. Numerical and analytical capacity results for optimal gain allocation schemes are under investigation.

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