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RANDOM BEAMFORMING WITH SYSTEMATIC BEAM SELECTION

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ABSTRACT

In this paper, we propose a new random beamforming scheme for multiuser multiple-input single-output block fading channels, e.g., a wireless downlink in a cellular communication system. Unlike previous methods it achieves spatial multiplexing gain over the complete range of realistic signal-to-noise ratios (SNRs) even for moderate user numbers. The main shortcoming of the random beamforming schemes for multiple datastreams proposed so far is their interference limitation, especially under medium and high SNR. The key to mitigate this problem in our scheme is to allow feedback of each user's channel gain in each random beam. This information suffices to determine the sum-rate maximizing subset of beams and users at the base station. In this sense, our scheme, referred to as orthonormal random beamforming with systematic beam selection, can be considered as optimal among all random beamforming schemes that do not make use of additional preprocessing at the transmitter (apart from power normalization). It achieves a significant performance gain compared to previous schemes particularly for realistic SNR values.

I. INTRODUCTION

In recent years, dirty paper coding (DPC) [1] has been shown to achieve the sum-rate capacity of the Gaussian MIMO broadcast channel [3][4], and moreover, also its capacity region [5]. However, from a practical point of view, its value is limited. One of the main shortcomings of DPC turns out to be the large amount of feedback required to make all complex channel coefficients available at the base station (BS) with high accuracy. DPC is therefore impractical for many applications, especially if the coherence time of the channel is short.

These circumstances encouraged an extension of opportunistic beamforming (OBF) [2], which had originally been proposed to accelerate and increase the variation in the users' channel gains in slow fading or poor scattering environments. The main principle of OBF is to randomly vary the complex weights at an antenna array, to advertise the resulting random beam to the mobile stations (MS) in a separate pilot mode, and to schedule the MS with highest SNR for data transmission in data mode. In [6] the authors extended this idea to multiple random beams. They proposed a scheme that generates $B = N$ orthonormal random beams in pilot mode, and schedules B out of overall K users with best signal-to-interference-plus-noise ratios (SINRs) for transmission in data mode. Indeed, the sum-rate achieved by this scheme called orthonormal random beamforming (ORBF) follows the same scaling law as DPC, namely $B \log \log K$. However, it turns out that even for small B, when realistic user numbers are assumed, the scheme operates far away from the sum-rate achieved by DPC. Furthermore, under medium and high SNR it becomes even inferior to the OBF scheme, since the system is interference dominated when multiple beams are used simultaneously.

In this paper, we will propose a new multi-beam scheme, that performs well even in the medium and high SNR regime. Basically, it has a similar flavor as the scheme proposed in [8], where the generation of multiple pilot beams and the use of only one beam in data mode is suggested as an extension of the OBF scheme. Thereby, the multiuser diversity can be enhanced significantly in the case that only few users are in the cell. This scheme is termed opportunistic beamforming with multiple weighting vectors (OBF/MWV). Accordingly, we generalize the ORBF scheme to generate $1 \leq B \leq N$ orthonormal random beams in pilot mode, while only making use of $B_0 \leq B$ of them in data mode. Thus, we can efficiently balance multiuser diversity with spatial multiplexing gain, and thereby enhance the sum-rate capacity significantly. We allow $B < N$, as, in general, the pilot overhead needs to be taken into account if B is large.

In order to enable such a scheme we require the MSs to feedback all channel gains in all beams. As these are real instead of complex numbers and, moreover, they need not be available at the BS with high accuracy, e.g., as needed to enable DPC or coherent beamforming, this overhead is only moderate, and can also be mitigated through efficient feedback reduction if necessary. The beam and user selection at the BS is done such that the sum-rate is maximized. If we allow for an optimal choice of B_0 for each individual fading block this scheme is termed *dynamic* orthonormal random beamforming with systematic beam selection (ORBF/SBS).

If B_0 is fixed in advance depending on the user number and the average SNR such that the ergodic sum-rate is maximized the scheme is called *static* ORBF/SBS. Indeed, the dynamic scheme is the optimal one among all random beamforming schemes that do not make use of additional preprocessing (apart from power normalization according to the number of used beams). On the other hand, static ORBF/SBS will turn out to perform close to optimal, while being of lower computational complexity. Also the feedback complexity of static ORBF/SBS can be reduced significantly while close to optimal performance is sustained, as we will demonstrate at the end of this paper.

II. SIGNAL MODEL

We consider a block fading Gaussian MIMO broadcast channel, where the transmitter is equipped with N antennas and the K users are equipped with a single antenna each. Thus, the received signal at the k-th MS is given by

$$y_k = \sum_{i=1}^{B_0} \mathbf{h}_k \boldsymbol{\phi}_i s_i + n_k, \tag{1}$$

where $\mathbf{h}_k$ is the $(1 \times N)$ channel vector for the k-th user containing independent zero mean circularly symmetric complex Gaussian (ZM-CSCG) random variables with unit variance, and $\{\boldsymbol{\phi}_i\}_{i=1}^{B_0}$ are B_0 out of B $(N \times 1)$ orthonormal random vectors containing the beamforming weights chosen according to an isotropic distribution.The s_i's are the symbols transmitted through the B_0 random beams, and n_k is the additive white Gaussian noise at the k-th receiver modelled as ZM-CSCG with unit variance.

In pilot mode, the BS broadcasts B orthonormal training sequences (e.g., Walsh-Hadamard codes) through the generated random beams. These sequences are known at the MSs and used to estimate the channel gains $|\mathbf{h}_k \boldsymbol{\phi}_i|$, $i = 1, \ldots, B$, necessary to compute the users' SINRs at the BS. The power budget $\rho = \text{trace}\left(\mathsf{E}[\mathbf{s}^{\mathrm{H}}\mathbf{s}]\right)$, where $\mathsf{E}[\cdot]$ denotes expectation, is equally distributed over all beams, i.e., the average signal power at each receiver is given by ρ/B. Also in data mode, the power is uniformly distributed, but normalized according to the number of simultaneously scheduled users, B_0. Thus, the signal power received at the MSs is ρ/B_0. Note that we assume a homogeneous channel, i.e., equal $\mathsf{E}[|\mathbf{h}_k|], \forall k$.

III. Orthonormal Random Beamforming with Systematic Beam Selection

In this section, we introduce the class of random beamforming schemes without additional preprocessing (e.g., DPC) at the transmitter.

Definition 1. *The class of random beamforming schemes without additional preprocessing at the transmitter is defined to comprise all schemes that proceed according to the subsequent protocol:*

- *In pilot mode the BS broadcasts pilot sequences through $B \leq N$ random beams to the MSs.*
- *Each MS measures the information required by the scheme and feeds it back to the BS in a vector* $\mathbf{u}$.
- *In data mode, the BS schedules the (based on the available information) optimal subset of $B_0 \leq B$ beams and users for transmission.*
- *Thereby, the fixed power budget is distributed equally over the assigned data streams.*

A scheme within the class is then completely identified by the number of pilot beams B generated by the BS, the number of beams B_0 used for data transmission, and the feedback from the MSs in $\mathbf{u}$.

In such a scheme, the sum-rate is determined by the SINRs of the scheduled users. Accordingly, the information necessary for the BS to determine the optimal – i.e., sum-rate maximizing – subsets of users and beams comprises all channel gains $|\mathbf{h}_k \boldsymbol{\phi}_b|^2$. Once these values are available at the BS, the SINRs

Table 1: Characterization of Schemes

	B	B_0	$\mathbf{u}_k$
dyn. ORBF/SBS	$1 \leq B \leq N$	$1 \leq B_0 \leq B$ (variable)	$(\lvert\mathbf{h}_k \boldsymbol{\phi}_1\rvert, \ldots, \lvert\mathbf{h}_k \boldsymbol{\phi}_B\rvert)$
st. ORBF/SBS	$1 \leq B \leq N$	$1 \leq B_0 \leq B$ (fixed)	$(\lvert\mathbf{h}_k \boldsymbol{\phi}_1\rvert, \ldots, \lvert\mathbf{h}_k \boldsymbol{\phi}_B\rvert)$
ORBF	N	N	$(\max_i \frac{\lvert\mathbf{h}_k \boldsymbol{\phi}_i\rvert^2}{\frac{N}{\rho}+\sum_{j \neq i} \lvert\mathbf{h}_k \boldsymbol{\phi}_j\rvert^2}, i)$
OBF/MWV	$1 \leq B \leq N$	1	$(\max_i \lvert\mathbf{h}_k \boldsymbol{\phi}_i\rvert, i)$
OBF	1	1	$(\lvert\mathbf{h}_k \boldsymbol{\phi}\rvert)$

for arbitrary combinations of beams and users can be computed according to

$$\text{SINR}_{k_i, b_i} = \frac{\left|\mathbf{h}_{k_i} \boldsymbol{\phi}_{b_i}\right|^2}{\frac{B_0}{\rho} + \sum_{j=1, j \neq i}^{B_0} \left|\mathbf{h}_{k_i} \boldsymbol{\phi}_{b_j}\right|^2}, \tag{2}$$

where k_i denotes the i-th selected user and b_i its assigned beam. The user's rate in bits per channel use is then given by

$$R_{k_i, b_i} = \log_2(1 + \text{SINR}_{k_i, b_i}). \tag{3}$$

If we allow the BS to optimize over user combinations of different sizes, we obtain the optimal scheme within the class under consideration. This scheme is termed dynamic ORBF/SBS, and chooses the beams and users according to the following criterion:

$$\{(b_1^{(\text{dyn})}, k_1^{(\text{dyn})}), \ldots, (b_{B_0}^{(\text{dyn})}, k_{B_0}^{(\text{dyn})})\} = \underset{\{(b_1,k_1),\ldots,(b_{B_0},k_{B_0})\} \in \cup_{B_0=1}^{B} \mathcal{B}_{B_0} \times \mathcal{K}_{B_0}}{\operatorname{argmax}} \sum_{i=1}^{B_0} R_{k_i, b_i}, \tag{4}$$

where $\mathcal{B}_{B_0}$ and $\mathcal{K}_{B_0}$ are the sets of all possible combinations of B_0 beams and users respectively. The term "dynamic" refers to the dynamically chosen number of data streams.

Fixing the number of data streams to

$$B_0 = \underset{1 \leq B_\text{d} \leq B}{\operatorname{argmax}} \; \mathsf{E}\left[\max_{\{(b_1,k_1),\ldots,(b_{B_\text{d}},k_{B_\text{d}})\} \in \mathcal{B}_{B_\text{d}} \times \mathcal{K}_{B_\text{d}}} \sum_{i=1}^{B_\text{d}} R_{k_i, b_i}\right] \tag{5}$$

in advance yields the selection criterion for the static ORBF/SBS scheme:

$$\{(b_1^{(\text{stat})}, k_1^{(\text{stat})}), \ldots, (b_{B_0}^{(\text{stat})}, k_{B_0}^{(\text{stat})})\} = \underset{\{(b_1,k_1),\ldots,(b_{B_0},k_{B_0})\} \in \mathcal{B}_{B_0} \times \mathcal{K}_{B_0}}{\operatorname{argmax}} \sum_{i=1}^{B_0} R_{k_i, b_i}, \tag{6}$$

Here the optimization space is reduced significantly.

Finally, we compare the related parameters used in the different schemes discussed so far in Table 1. It points out that both ORBF/SBS schemes basically can be seen as generalizations of the previous schemes.

IV. IMPACT OF USER NUMBER AND SNR

In this section we consider the dependence of the data stream number B_0 in static ORBF/SBS on the number of users and the SNR. We provide analytical results for the asymptotical cases $K \to \infty$, $\rho \to 0$ and $\rho \to \infty$. We will denote the achieved sum-rate when using x out of B pilot beams in data mode by R_x in the following.

We start with the discussion of the impact of the user number. The more users are requesting data from the BS, the more of them we can expect to reside in (close to) orthogonal beams. Accordingly, it is intuitive that B_0 increases with K. Indeed, the following lemma holds:

Lemma 1. *For fixed ρ, $\lim_{K\to\infty} B_0 = B$, i.e., all pilot beams are used in data mode if the user number is sufficiently large.*

Proof. We upper-bound the sum-rate of a scheme that makes use of B_d (optimally chosen) out of B beams, R_{B_d}, by B_d times the rate of the strongest out of K users in beamforming configuration in absence of any interference with full power budget ρ assigned, $R^{(\mathrm{BF})}$. The channel gain $\|\mathbf{h}_k\|^2$ of each user is χ^2 distributed with $2N$ degrees of freedom. In [6] (Example 1) it has been shown that for $X \triangleq \max_{1\le i\le K} \|\mathbf{h}_i\|^2$

$$\begin{aligned}\mathsf{Pr}\left[X \le \log K + N\log\log K + O(\log\log\log K)\right] \\ = 1 - O\left((\log K)^{-1}\right). \qquad (7)\end{aligned}$$

$\mathsf{E}[R^{(\mathrm{BF})}]$ now can be upper-bounded by

$$\begin{aligned}&\log\left(1+\rho\left[\log K + N\log\log K + O(\log\log\log K)\right]\right)\\ \cdot\;& \mathsf{Pr}\left[X \le \log K + N\log\log K + O(\log\log\log K)\right]\\ +\;& O(N\log K)\\ \cdot\;& \mathsf{Pr}\left[X > \log K + N\log\log K + O(\log\log\log K)\right]\\ =\;& \log\left(1+\rho\log K\right) + O(\log\log\log K), \qquad (8)\end{aligned}$$

where we used the fact that $R^{(\mathrm{BF})} \le O(N\log K)$ which is the single user MIMO capacity with full CSI at the transmitter, N transmit and K receive antennas [6]. Thus,

$$\begin{aligned}\lim_{K\to\infty}\frac{\mathsf{E}[R_{B_\mathrm{d}}]}{B_\mathrm{d}\log\log K} &\le \lim_{K\to\infty}\frac{B_\mathrm{d}\mathsf{E}[R^{(\mathrm{BF})}]}{B_\mathrm{d}\log\log K}\\ &= \lim_{K\to\infty}\frac{\mathsf{E}[R^{(\mathrm{BF})}]}{\log\log K}\\ &\le \lim_{K\to\infty}\frac{\log\log K + O(\log\log\log K)])}{\log\log K}\\ &= 1. \qquad (9)\end{aligned}$$

On the other hand, we know from Theorem 1 in [6] that $\lim_{K\to\infty}\mathsf{E}[R_B]/B\log\log K = 1$ if all B beams are used for data transmission. Finally, we conclude that for $B_\mathrm{d} < B$

$$\begin{aligned}\lim_{K\to\infty}\frac{\mathsf{E}\left[R_{B_\mathrm{d}}\right]}{\mathsf{E}\left[R_B\right]} &= \frac{B_\mathrm{d}}{B}\cdot\lim_{K\to\infty}\frac{\frac{\mathsf{E}\left[R_{B_\mathrm{d}}\right]}{B_\mathrm{d}\log\log K}}{\frac{\mathsf{E}\left[R_B\right]}{B\log\log K}}\\ &= \frac{B_\mathrm{d}}{B}\cdot\frac{\lim_{K\to\infty}\frac{\mathsf{E}\left[R_{B_\mathrm{d}}\right]}{B_\mathrm{d}\log\log K}}{\lim_{K\to\infty}\frac{\mathsf{E}\left[R_B\right]}{B\log\log K}}\\ &\le \frac{B_\mathrm{d}}{B} < 1. \qquad (10)\end{aligned}$$

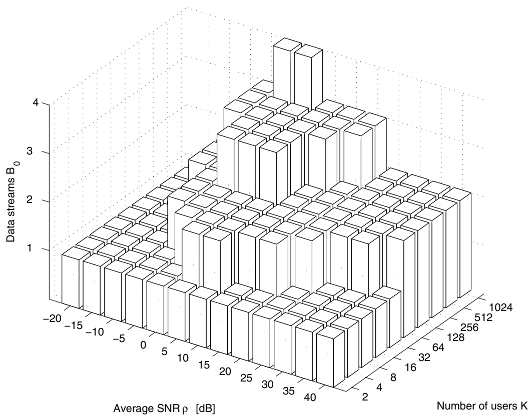

Figure 1: B_0 in static ORBF/SBS as a function of K and ρ for N=B=4.

Consequently, B data beams are optimal, if $K \to \infty$. □

Fig. 1 confirms this result (for small and large ρ the plot needs to be extended to arrive at $B_0 = B$).

Next, we consider the dependence of the number of data streams on the average SNR. We state the following lemma, that might look counterintuitive at first sight:

Lemma 2. *For fixed K, $\lim_{\rho\to 0} B_0 = 1$, i.e., one data stream is used under sufficiently low SNR.*

Proof. We consider the ratio of the sum-rates achieved for a channel realization when using $B_\mathrm{d} > 1$ beams and only one beam, respectively, i.e., R_{B_d}/R_1. In the first case, we denote the gains of the data beams for the scheduled users by $\alpha_1 = |\mathbf{h}_{k_1}\boldsymbol{\phi}_{b_1}|^2 > \ldots > \alpha_{B_\mathrm{d}} = |\mathbf{h}_{k_{B_\mathrm{d}}}\boldsymbol{\phi}_{b_{B_\mathrm{d}}}|^2$, and the largest individual channel gain by $\beta = \max_{x,y}\{|\mathbf{h}_{k_x}\boldsymbol{\phi}_{b_y}|^2\}$. Clearly, $\beta \ge \alpha_1 > \alpha_2 > \ldots > \alpha_{B_\mathrm{d}} > 0$. Finally, $i_i = \sum_{j\ne i}|\mathbf{h}_{k_i}\boldsymbol{\phi}_{b_j}|^2$ denotes the gain of the interfering data beam of the i-th chosen user. Now, by denoting the mean of the α_i by $\overline{\alpha}$, we can write according to (2) and (3)

$$\begin{aligned}\lim_{\rho\to 0}\frac{R_{B_\mathrm{d}}}{R_1} &= \lim_{\rho\to 0}\frac{\sum_{i=1}^{B_\mathrm{d}}\log\left(1+\frac{\alpha_i}{\frac{B_\mathrm{d}}{\rho}+i_i}\right)}{\log\left(1+\beta\rho\right)}\\ &\le \lim_{\rho\to 0}\frac{\sum_{i=1}^{B_\mathrm{d}}\log\left(1+\frac{\alpha_i\rho}{B_\mathrm{d}}\right)}{\log\left(1+\beta\rho\right)}\\ &\le \lim_{\rho\to 0}\frac{B_\mathrm{d}\log\left(1+\frac{\overline{\alpha}\rho}{B_\mathrm{d}}\right)}{\log\left(1+\beta\rho\right)}\\ &= \lim_{\rho\to 0} B_\mathrm{d}\frac{\frac{\overline{\alpha}}{B_\mathrm{d}}\rho - o(\rho)}{\beta\rho - o(\rho)}\\ &= \frac{\overline{\alpha}}{\beta} < \frac{\alpha_1}{\beta} < 1, \qquad (11)\end{aligned}$$

where we used Jensen's inequality. $o(\cdot)$ is the Landau symbol[1]. As $R_{B_\mathrm{d}} < R_1$ holds for any channel realization, it finally follows that $\mathsf{E}[R_{B_\mathrm{d}}] < \mathsf{E}[R_1]$. □

[1] $f(\phi) = o(\phi)$ means $\lim_{\phi\to\phi_0}\frac{f(\phi)}{\phi} = 0$ for the considered limit.

The result is counterintuitive, as one might think that the impact of the interference will become less severe, if the system is noise dominated, and therefore as many streams as possible are used. The reason for the optimality of one beam, is the linear scaling of capacity under low SNR. We have $R_1 \approx \text{const} \cdot \rho$ in the case of one beam, which is larger than $R_2 \approx \text{const} \cdot (\frac{\rho}{2}/(1+i_1\frac{\rho}{2}) + \frac{\rho}{2}/(1+i_2\frac{\rho}{2}))$.

In the high SNR region, the optimal B_0 can be well understood intuitively, again. In such a scenario, a multi-beam system is interference limited, i.e., the sum-rate capacity remains bounded for arbitrarily high signal power. Accordingly, we have the following lemma:

Lemma 3. *For fixed K, $\lim_{\rho\to\infty} B_0 = 1$, i.e., one data stream is used under sufficiently high SNR.*

Proof. Again, we consider the ratio R_{B_d}/R_1. As all $i_i > 0$ it follows that

$$\lim_{\rho\to\infty} \frac{R_{B_\text{d}}}{R_1} = \lim_{\rho\to\infty} \frac{\sum_{i=1}^{B_\text{d}} \log\left(1 + \frac{\alpha_i}{\frac{B_\text{d}}{\rho} + i_i}\right)}{\log(1+\beta\rho)} = 0. \quad (12)$$

With the same argument as in the proof of Lemma 2 we conclude $\mathsf{E}[R_{B_\text{d}}] < \mathsf{E}[R_1]$ and have shown that the use of one data stream is optimal. □

Again, we find our results confirmed in Fig. 1 (for large K the plot needs to be extended to arrive at $B_0 = 1$).

It is also seen in Fig. 1 that for all reasonable pairs (ρ, K), more than one, but less than B pilot beams are used in data mode. This fact strongly motivates ORBF/SBS.

V. Performance Comparisons

In this section we compare the sum-rate capacities of both ORBF/SBS schemes to the previous schemes. The corresponding plots are found in Fig. 2. In our simulations we assume $N = 4$. The number of pilot beams is $B = 4$ for ORBF/SBS and OBF/MWV.

We start with a comparison of the sum-rate capacities of the two ORBF/SBS schemes. The dynamic scheme – due to its optimality – is clearly superior to the static scheme. However, the performance gap between both schemes is surprisingly small over the complete range of SNR values and user numbers. The reason for the only moderate performance loss can be seen by examining Fig. 3, which shows the corresponding probability mass functions (PMF) for the number of data streams *dynamically* chosen by the dynamic ORBF/SBS scheme. One can see that the scheme does not make use of the whole spectrum of data stream numbers effectively. Rather, it focuses on one or two different stream numbers mainly. Furthermore, one of these stream numbers is usually dominant. Accordingly, the loss in sum-rate when fixing the beam number is small.

The largest gaps between the static and the dynamic scheme will arise, whenever K and ρ are such that the B_0 in static ORBF/SBS changes from one integer to another. In this case, the data stream number PMF of the dynamic scheme under the same setup will not have a dominant peak, but two almost

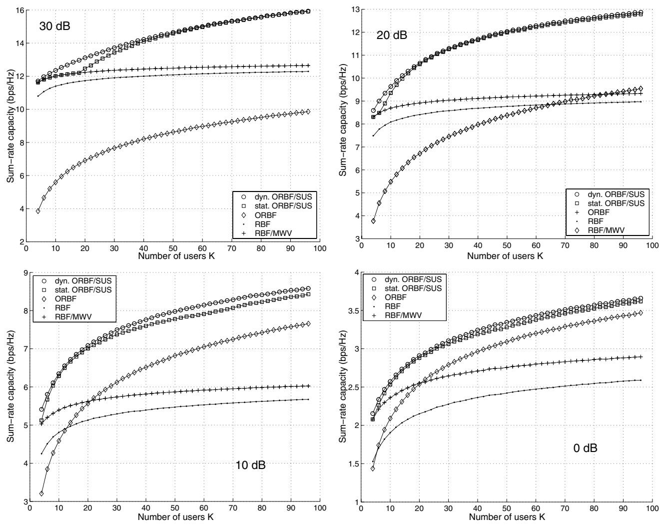

Figure 2: Comparison of random beamforming schemes for $\rho = 0, 10, 20, 30$ dB. $N = B = 4$ for ORBF/SBS, OBF/MWV and ORBF, and $N = 4, B = 1$ for OBF.

equally likely beam numbers. This effect can be identified most obviously through the curve simulated for $\rho = 30$dB in the region around $K = 18$ users, where the optimal static data stream number changes from one to two (results in a sharp bend in the curve). Also one should refer to the respective PMF in Fig. 3. In the simulated scenarios it turns out that the static ORBF/SBS scheme always guarantees at least 93% of the sum-rate achieved by the dynamic scheme, while the computational complexity is reduced.

Finally, we also consider the performance of the existing schemes mentioned earlier, i.e., OBF, OBF/MWV, and ORBF. Three key observations are listed as follows:

- In contrast to static ORBF/SBS, none of the schemes approaches the (optimal) dynamic ORBF/SBS scheme over the complete range of K and ρ.
- For very low/high SNR (0/30 dB) and moderate user numbers, OBF/MWV demonstrates same performance as static ORBF/SBS. This is consistent with Lemmas 2 & 3.
- For (unrealistically) high user number, ORBF converges to static ORBF/SBS. This is consistent with Lemma 1.

Based on the above analysis, we conclude that existing schemes approach the optimal performance in a few unrealistic scenarios. However, in real world applications, i.e., for reasonable user numbers and SNRs, ORBF, OBF/MWV, and OBF operate far away from the optimal sum-rate.

VI. Reducing Feedback & Complexity

In this section we focus on feasibility issues, when ORBF/SBS is applied in practical applications. Particularly, we consider feedback requirements and computational complexity of the search algorithm at the BS, which finds the scheduled beam and user subsets according to (4) or (6). Compared to previous

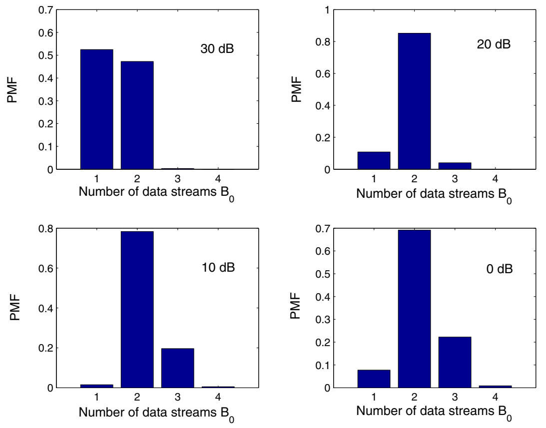

Figure 3: PMF of B_0 as chosen by dyn. ORBF/SBS scheme when K=18 and N=B=4 for $\rho = 0, 10, 20, 30$ dB.

schemes, we increase both if we implement the ORBF/SBS schemes literally. However, it turns out that both can be reduced efficiently while sustaining close to optimal performance.

Requiring feedback of all the channel gains of all B pilot beams sounds very demanding at first sight. Before considering techniques to reduce the feedback efficiently, we point out that even the feedback of B channel gains is far less than the amount required for DPC or coherent beamforming. In the latter cases, we feed back all channel coefficients, which are complex instead of real numbers. Furthermore, these must be made available at BS with high accuracy. In ORBF/SBS the channel gains can be discretized more roughly, as they are not used for precise beamforming weight derivation or interference presubtraction, but only for SINR computation. The robustness of random beamforming to discretization of feedback is shown in [7], where the authors proof that 1-bit feedback suffices to sustain the $\log \log K$ scaling law in a single data beam scheme.

In static ORBF/SBS the amount of feedback can be further reduced without significant performance loss. The idea is to ignore all beams with moderate channel gain, as these – with high probability – are too weak to be used as data beam, and too strong to be scheduled as interfering beam. Thus, the users need to feed back the index and channel gain of the strongest and few indices and channel gains of the weakest beams only. The number of weak beams that should be considered in general depends on K. The a-priori-probability that a user will be scheduled in a beam where $|\mathbf{h}_k \boldsymbol{\phi}_i| < \mathrm{th_u}$ or interfered in a beam where $|\mathbf{h}_k \boldsymbol{\phi}_i| > \mathrm{th_l}$ for some threshold values $\mathrm{th_u}$ and $\mathrm{th_d}$, decreases, when K increases. Thus, we can reduce the feedback most efficiently for large user numbers, i.e., where the amount of feedback is most crucial.

If B is not too large and $B_0 = 2$, it is even possible to feedback a single SINR (each MS computes its SINR when scheduled in its strongest and interfered in its weakest beam) and two beam indices for each user while sustaining close to optimal performance. Computer simulations show, that even for realistic user numbers the performance of static ORBF/SBS can be approached. Under the same setup as in Section V., for $K = 30$ and $\rho = 20$ dB the loss is less than three percent.

A naive approach to perform the optimization in (4) and (6) is to search over all possible user and beam combinations. Assuming $K \gg B$, this algorithm is of complexity $O(K + \ldots + K^B) = O(K^B)$, when applied with the dynamic and of complexity $O(K^{B_0})$ when applied with the static scheme. However, in a practical application the search can be performed as follows: 1. Compute only one SINR for each user assuming that it is scheduled in its strongest beam for each particular beam combination and add it to a list of candidates for its strongest beam. 2. For each beam choose the user with highest SINR from the respective candidate list. If the candidate lists for all beams are non-empty, this algorithm yields the optimal user and beam combinations. As the probability of an empty candidate list goes to zero rapidly for increasing K, the algorithm almost surely yields the optimal result for moderate user numbers already. Both dynamic and static scheme are of complexity $O(K)$ under the above algorithm. More precisely, the static scheme scales like $K\binom{B}{B_0}$, the dynamic like $K\sum_{B_0=1}^{B}\binom{B}{B_0}$.

VII. Conclusions

We have proposed a generalization of several existing random beamforming schemes. Thereby, we obtained the ultimate limit on the sum-rate achievable by the class of random beamforming methods that do not make use of additional preprocessing at the transmitter. With dynamic ORBF/SBS we stated the optimal scheme within this class. Furthermore, with static ORBF/SBS we proposed a close to optimal performing scheme of lower computational complexity. The proposed schemes enable a significant performance improvement in terms of sum-rate capacity as compared to the previously introduced schemes in most practical scenarios, i.e., for realistic user numbers and in reasonable SNR regimes. This is achieved by efficiently balancing spatial multiplexing gain and multiuser diversity.

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