LINEAR SCALABLE SPACE-TIME CODES: TRADEOFF BETWEEN SPATIAL MULTIPLEXING AND TRANSMIT DIVERSITY

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ABSTRACT

Using a class of recently proposed linear scalable space-time codes we demonstrate the tradeoff between spatial multiplexing and transmit diversity for Rayleigh and Ricean flat fading. The codes are able to use jointly transmit diversity in combination with spatial multiplexing, and they achieve spatial and temporal diversity in Rayleigh and Rice fading environments. Frequency diversity of frequency selective channels can be utilized by combining the considered codes and OFDM. Simulation results for different scenarios show that a system consisting of the considered codes and decoder meets the requirements of future communication systems, e.g. it is possible to exploit high diversity factors and handle high rates with a reasonable and scalable complexity.

1. INTRODUCTION

The use of high rates to support broadband data access will be important for future wireless communication systems. Fading caused by destructive interference of multipaths in the wireless propagation medium affects the average reliability of a communication link. However using multiple transmit and/or receive antennas it is possible to get high data rates on a rich-scattering wireless channel. There are two basic coding methods for this purpose, namely space-time coding to improve *link reliability* and spatial multiplexing to increase *spectral efficiency*.

Space-time codes combat the fading effects by utilizing the *diversity* of the communication channel. Such methods aim to achieve transmit (TX) diversity without channel state information at the transmitter[1]. Space-time block codes based on orthogonal designs [1], [2], space-time trellis codes [3] and unitary space time modulation [4] are important representatives. Usually linear codes are preferred to keep the transmitter and receiver complexity low.

Linear high rate space-time codes increase the data rate over rich-scattering Multiple Input Multiple Output (MIMO) channels without increasing the bandwidth by *spatial multiplexing*, i.e. using the *spatial subchannels* which are available in rich diversity [5], [6], [7]. In uncorrelated Rayleigh fading the channel capacity grows linearly as the number of transmit and receive antennas grow simultaneously [8], [9]. V-BLAST (Vertical Bell Labs Layered Space-Time) is a practical scheme that uses pure spatial multiplexing [6], [7]. It is not designed to achieve additional TX diversity.

[1]To attain receive diversity a coding scheme at the transmitter is not necessary.

In [10] a wide class of space-time codes is presented called *linear dispersion (LD) codes*. Every linear block code can be described as a LD code. The LD codes proposed in [10] are found by a numerical solution of an optimization problem with the mutual information as the objective function. Hence they do not provide any guarantees on the achievable TX diversity gains.

There are important requirements for coding schemes in future wireless networks. These networks will be heterogeneous with regard to node complexity, link level requirements, propagation environments etc. The choice, if it is better to use spatial multiplexing or TX diversity methods, depends on a number of parameters, such as demanded link reliability, demanded data rate, channel conditions (correlated or uncorrelated fading, Rayleigh or Ricean fading), etc. So it is desirable to use a coding scheme that allows a flexible tradeoff between spatial multiplexing and transmit diversity. Such a tradeoff is studied in [11].

Because future wireless communication systems (e.g. next generation wireless LANs) will have to support high data rates, orthogonal frequency division multiplexing (OFDM) seems to be a good choice for these systems. As a result additional frequency diversity will be available.

Probably the used carrier frequencies will increase, this means it will be possible to use a large number of antennas and high diversity factors will be feasible. Therefore a coding scheme is desirable that allows to use a large number of antennas, to exploit high diversity factors and to handle high data rates with reasonable complexity.

Spatial multiplexing shows best performance gains for uncorrelated Rayleigh fading channels. In the presence of a line-of-sight component (or a fixed channel component) these gains decrease, but usually the signal-to-noise ratio increases. Obviously scenarios with line-of-sight components are common (e.g. for indoor wireless LANs). This means that a coding scheme should be able to work in a Rayleigh fading environment as well as in a Ricean fading environment.

In this paper we consider a recently proposed class of linear space-time block codes (and associated decoders) [5], which meet these requirements. These codes are highly flexible and adaptive; we refer to them as linear scalable dispersion (LSD) codes. They allow a joint usage of transmit diversity and spatial multiplexing and they enable a flexible tradeoff between transmission rate (the used numbers of spatial subchannels) and diversity gain. Due to the linearity of the LSD codes they are essentially transparent to the symbol alphabet and are readily combined with forward error correction and turbo decoding. Frequency diversity can be utilized by combining the codes with OFDM. In combination with a suitable decoder [12] these codes form a practicable proposal for an

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efficient communication system.

2. SYSTEM MODEL

2.1. Coding scheme

In the following we will give a description of the in [5] proposed linear scalable space-time codes, which are used in this paper.

Such a code consists of two concatenated but decoupled linear block codes, the time variant inner code and the time invariant outer code, given by matrices $\mathbf{R}$ and $\mathbf{C}_\nu$ respectively. No a priori channel knowledge is required at the transmitter; but if it is available it can be applied (e.g. the inner code can be adapted to TX beamforming). In this paper we assume that there is no a priori channel knowledge at the transmitter.

A time series representation of the coding scheme is depicted in Fig. 1. The input symbol vector $\boldsymbol{\alpha}$, consisting of N_I informa-

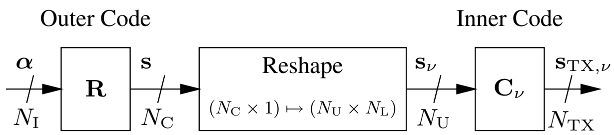

Fig. 1. Symbol discrete model of the encoding scheme

tion symbols, is multiplied with the $(N_C \times N_I)$ outer code matrix $\mathbf{R}$ to form the transmit symbol vector $\mathbf{s}$. The dimensions of the code matrix determine the code rate to $\frac{N_I}{N_C}$. Thereafter the transmit symbol vector $\mathbf{s}$ of dimension $(N_C \times 1)$ is reshaped into a $(N_U \times N_L)$ matrix. The columns of this matrix are the consecutive $N_L = \frac{N_C}{N_U}$ input vectors $\mathbf{s}_\nu$ of the linear time-variant inner code $\mathbf{C}_\nu$, whereas ν is the time index.

The system transmits $N_U \leq \operatorname{rank}(\mathbf{H})$ symbols in one timestep, where $\mathbf{H}$ is the MIMO channel matrix. We refer to N_U as the number of *spatial subchannels* to be used for spatial multiplexing. The remaining spatial dimensions can be used by the code achieving an additional diversity gain.

The inner code $\mathbf{C}_\nu$ is adapted to the configurations of the MIMO system (N_{TX}, N_{RX}, N_U) and the channel statistics (Rayleigh or Ricean fading). Pure TX diversity $(N_U = 1)$, spatial multiplexing $(N_U = \operatorname{rank}(\mathbf{H}))$ or a combination of both $(N_U \leq \operatorname{rank}(\mathbf{H}))$ are possible configurations of the inner code. The inner code is optimized with respect to the outage capacity of the channel conceived by the outer code. In [5] efficient code matrices are presented for TX diversity and joint TX diversity and spatial multiplexing. Using spatial multiplexing an efficient choice of $\mathbf{C}_\nu$ is given by

$$\mathbf{C}_\nu = \operatorname{diag}(\mathbf{c}_\nu) \cdot \mathbf{M} \qquad (1)$$

The N_L vectors $\mathbf{c}_\nu$ are the rows of a $(N_L \times N_{TX})$ Fourier matrix F. The time invariant matrix $\mathbf{M}$ is a $(N_{TX} \times N_U)$ unitary matrix. The determination of the elements of $\mathbf{M}$ is described in [5]. The adaption to a different number N_U of spatial subchannels can simply be done by adding/deleting columns of $\mathbf{M}$.

The outer code $\mathbf{R}$ is optimized for diversity performance and achieves a high diversity gain and an excellent performance in a fading environment even at code rate 1. The considered cost function is the maximal fading averaged pairwise error probability. As described in [5] the cyclic matrix $\mathbf{R}$ of dimension $(N_I \times N_C)$ is unitary. The columns of $\mathbf{R}$ are samples of the cyclic impulse response of a chirp filter. Note that a simple modification of the code rate is possible by adding/deleting columns of the outer code matrix. Because of the interdependence of code rate and required decoder complexity this feature is highly useful in networks with heterogeneous node capabilities [13]. Throughout this paper code rate $r_C = 1$ is used.

Due to the code concatenation the diversity performance optimization and channel conditioning (adaptation to the number of TX and RX antennas, pure TX diversity, pure spatial multiplexing, joint spatial multiplexing and TX diversity, number of spatial subchannels) are decoupled.

2.2. Combination with OFDM

Fig. 2 shows the combination of the LSD codes with OFDM and multiple transmit antennas. The N_{TX} elements of the vector $\mathbf{s}_{\mathrm{TX},\nu}$ are transmitted by the N_{TX} OFDM modulators. If only one TX antenna is used (i.e. $N_U = N_{TX} = 1$), there is no need for the inner code; the system describes a linear precoded OFDM. The coding across the OFDM subcarriers is done by the outer code $\mathbf{R}$. So frequency diversity can be exploited. The used chirp matrix $\mathbf{R}$ shows a better performance (a lower maximal fading averaged pairwise error probability) than the often used Hadamard code matrix [5]. For more than one TX antenna the inner code $\mathbf{C}_\nu$ is used as described above. So spatial multiplexing and TX diversity is accessible. Altogether the system is able to use frequency, spatial and temporal diversity. Performance results for the combination of the LSD codes and OFDM are subject of a further contribution.

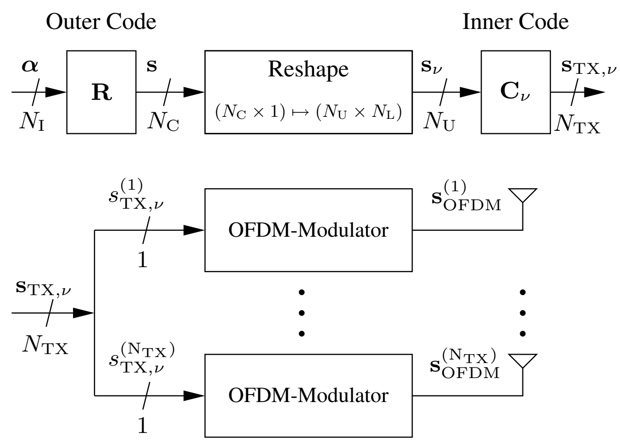

Fig. 2. Symbol discrete model of the encoding scheme including OFDM modulator

2.3. Decoder

One of the tasks of the decoder for the LSD codes is the compensation of intersymbol interference (ISI). The ISI results from interfering spatial subchannels and from the optimized diversity performance of the outer code in fading. In [12] a suboptimal reduced complexity ISI decoder (MAP-DFE) is presented. This decoder is feasible for large block length N_C. The decoder uses a scalable interference cancellation method, that by applying a posteriori information achieves very high performance with low complexity.

The key idea is to use estimated a posteriori probabilities to determine the order of the cancellation process. So a combination of a parallel and serial method is possible: several iterations to process a block of received symbols, in each iteration determining a segment with a posteriori error probability below a certain threshold, parallel decoding of a segment of symbols. For the LSD codes this decoder shows a better performance and a lower complexity than the BLAST decoder (MMSE-DFE) according to [7] (but due to the linearity of the LSD codes a variety of known ISI cancellation methods is applicable).

2.4. Channel model

In the following we will consider a multi-antenna system with N_{TX} transmit antennas and N_{RX} receive antennas. The channel is assumed as constant over one code interval N_L (this is not necessary for the use of the LSD codes) and flat fading. Given a complex valued $(N_{TX} \times 1)$ transmit code vector $\mathbf{x}_t$ with mean energy $E\left\{\mathbf{x}_t^H \mathbf{x}_t\right\} = 1$, the discrete time baseband representation of the $(N_{RX} \times 1)$ received signal vector $\mathbf{r}_t$ at time index t can be expressed as

$$\mathbf{y}_t = \frac{1}{\sqrt{N_{TX}}} \mathbf{H}\mathbf{x}_t + \mathbf{w}_t, \qquad (2)$$

where the $(N_{TX} \times 1)$ vector $\mathbf{w}_t$ is the equivalent baseband noise. Since the channel is constant over one code interval N_L we can compound the transmitted code, the received and the noise vectors into the code matrices $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{W}$.

$$\mathbf{Y} = \frac{1}{\sqrt{N_{TX}}} \mathbf{H}\mathbf{X} + \mathbf{W} \qquad (3)$$

The N_L codewords $\mathbf{s}_{TX,\nu}$ of Fig. 1 build the $(N_{TX} \times N_L)$ block code matrix $\mathbf{X}$ of (3).

3. PERFORMANCE RESULTS

In the following we will analyze the performance of the presented linear scalable dispersion (LSD) coding scheme based on simulations for different scenarios. In all simulations we use the MAP-DFE [12] as decoder.

3.1. Heterogeneous nodes, ad-hoc mode

In a heterogenous network nodes with different capabilities (e.g. number of antennas) transmit data with different link level requirements (e.g. required data rate, demanded link reliability). Using the LSD codes it is possible to adapt to different conditions.

Fig. 3 shows the symbol error rate (SER) over $\frac{E_b}{N_0}$ at the receiver. Two receive (RX) antennas are used, the block length of the outer code is $N_C = 16$, and the number of applied spatial subchannels is $N_U = 2$. The SER decreases as the number of TX antennas increases ($N_{TX} = 2, 3, 4, 5$): the system is able to jointly utilize TX diversity and spatial multiplexing.

Now we consider a MIMO system with eight transmit and receive antennas ($N_{TX} = N_{RX} = 8$) and an outer code with length $N_C = 32$. Fig. 4 depicts the impact of the number of used spatial subchannels N_U on the error performance. The performance results of this system assuming a BLAST system (no outer code, MAP-DFE as decoder) and an AWGN channel are plotted as references here. The figure shows the flexibility of the LSD codes. For $N_U = 8$ spatial subchannels we achieve full rate (pure spatial multiplexing); for $N_U = 1$ we get full diversity. The more

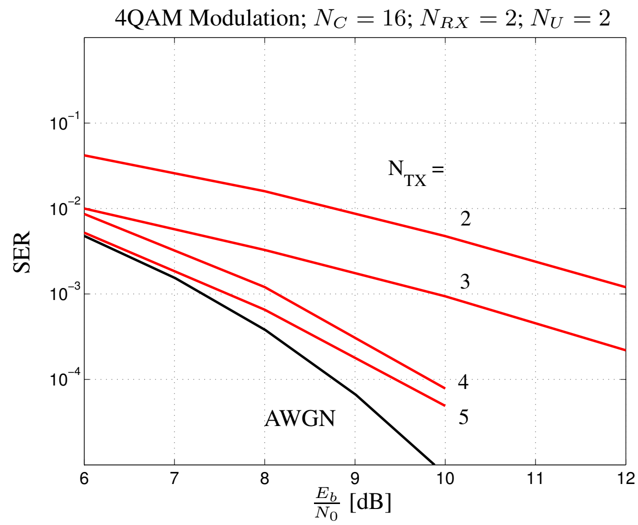

Fig. 3. Symbol Error Rate over $\frac{E_b}{N_0}$; joint TX diversity and spatial multiplexing

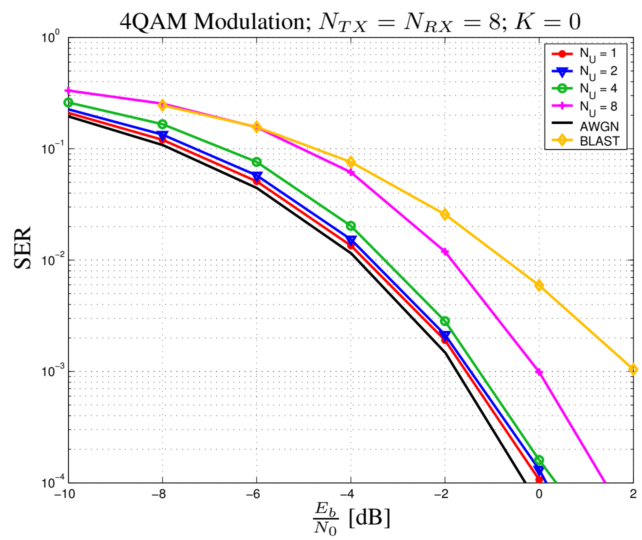

Fig. 4. Symbol Error Rate over $\frac{E_b}{N_0}$; Dimension of outer code: $N_C = 32$; Various number of used subchannels; compared with uncoded (BLAST) and AWGN case (8 RX antennas)

spatial subchannels are used the higher the rate, but the SER increases too: it is possible to trade diversity gain (link reliability) for data rate in a very flexible way. Furthermore it can be seen that the considered scheme nearly achieves AWGN performance if the number of used spatial subchannels N_U is small.

In addition to a Rayleigh fading environment ($K = 0$) the channel statistics is extended to Ricean fading with Ricean factor $K \neq 0$:

$$\mathbf{H} = \sqrt{\frac{K}{1+K}} \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix} + \sqrt{\frac{1}{1+K}} \mathbf{H}_{random}. \qquad (4)$$

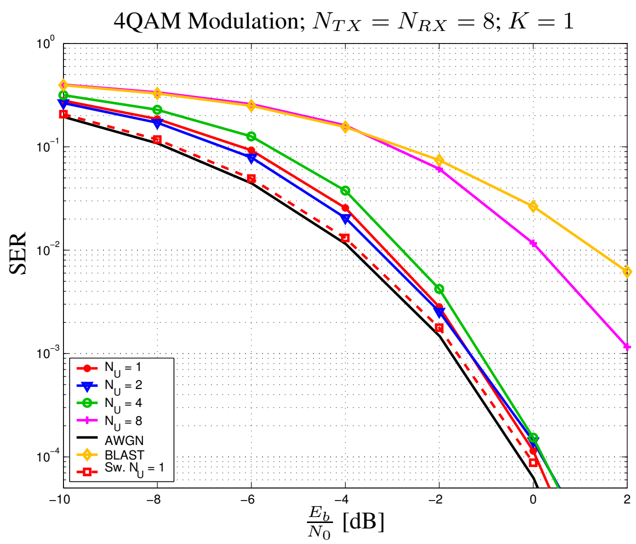

Fig. 5. $K = 1$: Symbol Error Rate over $\frac{E_b}{N_0}$; Dimension of outer code: $N_C = 32$; Various number of used subchannels

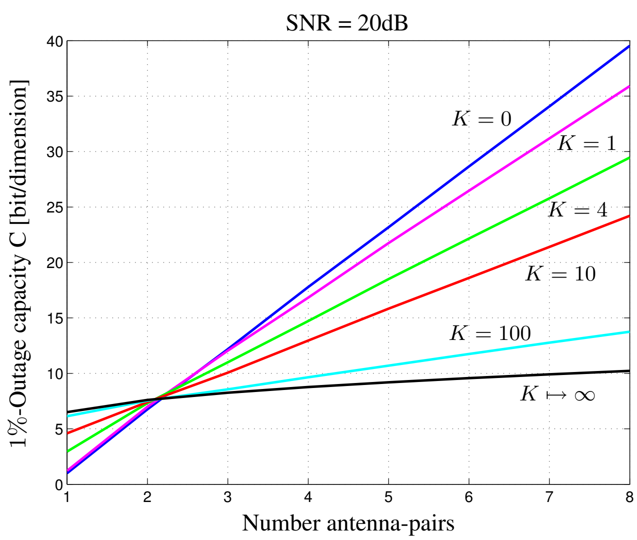

Fig. 6. 1% Outage capacity of Rayleigh and Ricean fading MIMO channels

Fig. 5 shows the performance of the LSD codes in Ricean fading with $K = 1$ and apart from that similar boundary conditions as in Fig. 4. The performance decreases (SER grows) compared to the results for Rayleigh fading. The 1%-outage capacity[2] of MIMO channels decreases as the Ricean K-factor increases (Fig. 6). Therefore the use of spatial subchannels loses efficiency.

3.2. Constant data rate services

In Fig. 7 a MIMO system with four transmit and receive antennas ($N_{TX} = N_{RX} = 4$) and an outer code with length $N_C = 8$ is

[2]The capacity that is at least reached for 99% of the channel realizations.

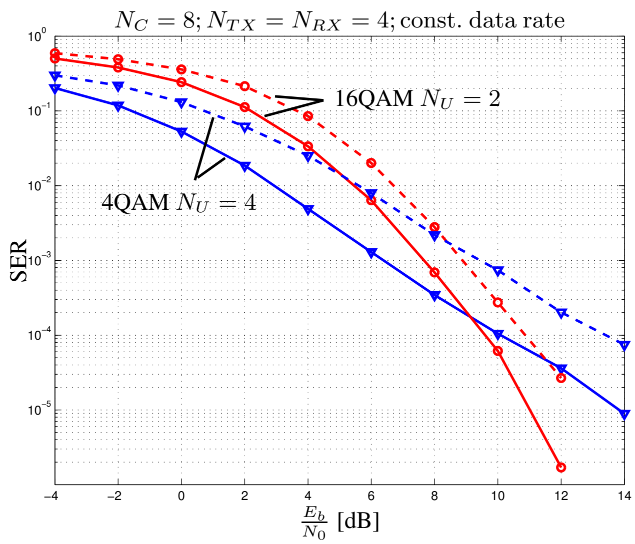

Fig. 7. Symbol Error Rate over $\frac{E_b}{N_0}$; Tradeoff SM vs. joint SM/TXD for same throughput of 8 bit per channel use; solid line: $K = 0$, dashed line: $K = 2$

considered. The figure shows the tradeoff between spatial multiplexing (SM) and joint transmit diversity and spatial multiplexing (SM/TXD) for $K = 0$ (solid) and $K = 2$ (dashed). The effective throughput of 8bit per channel use is held constant for both schemes. The SM/TXD-scheme achieves a higher degree of diversity (steeper slope) compared to the pure SM-scheme and therefore shows a better error performance for higher signal-to-noise ratios. For lower signal-to-noise ratios spatial multiplexing is the better choice. For $K = 2$ the performance of both schemes degrades; the intersection point drifts to a lower signal-to-noise ratio (SNR). With increasing K the performance of the diversity scheme increases in contrast to the performance of spatial multiplexing.

3.3. TX diversity: low complexity method

An efficient adaption of the inner code for the pure TX diversity mode is to use TX antenna switching instead of the construction of the inner code described in section 2.1. This leads to a lower transmitter complexity. Fig. 8 shows the 1%-outage capacity conceived by the outer code as a function of the Ricean K-factor (SNR = 10 dB; $N_C = 32$; $N_{TX} = 8$; pure TX diversity: $N_U = 1$). For Rayleigh fading the capacity achieved by an inner code built by the rows of a $(N_L \times N_{TX})$ Fourier matrix is higher. But for $K \geq 0.4$ it is better to use TX antenna switching. In Fig. 5 the dashed line (legend: Sw. $N_U = 1$) shows the influence of this on the SER for $N_C = 32$, $N_{TX} = N_{RX} = 8$ and $K = 1$. Fig. 9 illustrates this again for pure TX diversity, $N_C = 32$, $N_{TX} = 16$, $K = 0$ (solid lines) and $K = 1$ (dashed lines). The SER for switching and $K = 1$ is lower than for switching and $K = 0$ as well as for Fourier vectors as inner coding vectors and $K = 0$.

4. CONCLUSIONS

The considered LSD codes meet the requirements of future wireless communication systems. They allow to use a large number of antennas with reasonable complexity. A rich tradeoff between spatial multiplexing to increase the data rate and TX diversity to

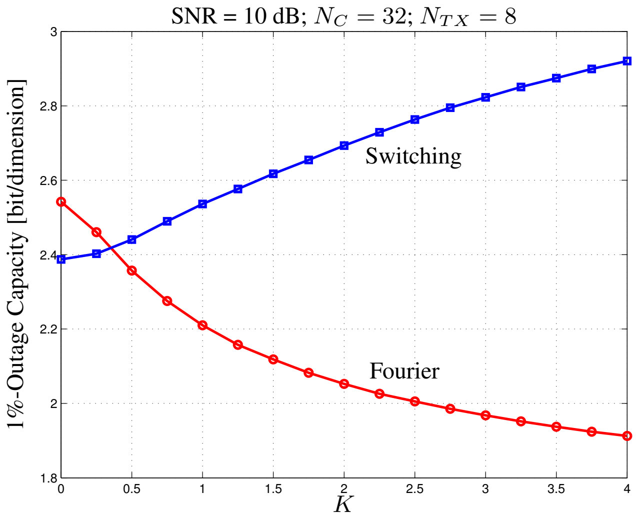

Fig. 8. 1% Outage capacity conceived by the outer code over Ricean K-factor; pure TX diversity, inner code: TX antenna switching vs. rows of a $(N_L \times N_{TX})$ Fourier matrix

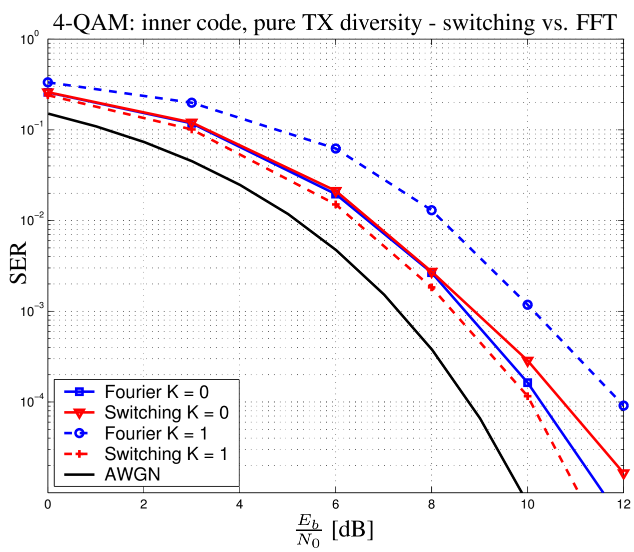

Fig. 9. Performance comparison for pure TX diversity; inner code: TX antenna switching vs. rows of a $(N_L \times N_{TX})$ Fourier matrix; $N_C = 32, N_{RX} = 1, N_{TX} = 16$ K=0 (solid) K=1 (dashed)

improve the link reliability is possible in a very flexible way. The codes are scalable and adaptive to different node complexities, link level requirements and different propagation environments.

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