On the Trade-Off Between Transmit and Leakage Power for Rate Optimal MIMO Precoding

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***Abstract*—In this paper, we study the interdependency between leakage reduction and transmit power control for rate maximization in leakage based precoding (LBP) for a multi user multiple-input multiple-output (MU-MIMO) scenario. We propose a LBP scheme which provides the rate optimal precoding for each MIMO link under a joint leakage and transmit power constraint and derive an iterative closed form solution for it. Depending on the constraints and the strength of the leakage power, the leakage power and/or the transmit power needs to be adjusted. Any leakage level between the egoistic case, where the leakage is not reduced at all, and the altruistic case, where the leakage is minimized, can be achieved. This allows to optimize the network performance in a MU-MIMO setup by optimally choosing the trade-off between leakage power and transmit power constraints. Using numerical simulations, we show the interdependency between the leakage power and the transmit power and demonstrate the gain we can achieve with the optimal trade-off.**

I. Introduction

The ever increasing demand for higher data rates and throughput has set high requirements on future cellular networks. Data rates and spectral efficiency which are larger by orders of magnitude than in nowadays systems have to be provided [1]. One technology to meet these requirements is massive multiple-input multiple-output (MIMO) arrays at the base stations (BSs) [2], possibly combined with coordinated multipoint (CoMP) transmission [1]. In order to efficiently use the resources, the BSs may serve multiple mobile stations (MSs) in parallel on the same physical channel, i.e. multi user MIMO (MU-MIMO). This causes strong co-channel interference (CCI) which degrades the performance of the system. To benefit from the high gains in spectral efficiency which MIMO processing promises, it is necessary to efficiently suppress the CCI.

Various approaches have been proposed to mitigate the CCI [3]. A very promising one is leakage based precoding (LBP) which reduces the leakage power to the unintended users. In [4], a LBP scheme which maximizes the signal-to-leakage-plus-noise-ratio (SLNR) for each individual link is proposed and a closed form solution based on the generalized Rayleigh-Ritz quotient is derived. Due to its low computational complexity and its efficiency in reducing the CCI, it is a very practical method to improve the spectral efficiency. Furthermore, it is well suited for the application in cellular networks combined with CoMP [5]. However, [4] applies equal power allocation among the substreams. Hence, it is not rate optimal. In [6], a power allocation over the SLNR values of the substreams is introduced, which allows to optimize the achievable rate by adaptively selecting the number of substreams for each link and weighting them accordingly. In [7], an iterative transmit filter design is proposed to maximize the weighted sum capacity in MU-MIMO systems, based on a modified definition of SLNR which also includes the receiver structure, and power allocation over the substreams.

The aforementioned schemes always minimize the leakage power produced by the desired signal and transmit at full power. In this paper, we show that this is suboptimal in terms of the sum rate of the system. This is due to the fact that when there is strong leakage for a link, it is better not to transmit at full power, in order to decrease the interference into the network. On the other hand, when the leakage power is low for a link, it is better not to minimize it, but to put the available resources into the desired signal. Hence, the network performance can be improved by optimizing the trade-off between leakage power and transmit power.

In this context, we propose a LBP scheme which provides the rate optimal precoding for each link under a joint leakage and transmit power constraint. A similar approach has been followed in [8] as relaxed zero forcing (RZF), and solved using standard optimization tools. Different to [8], we derive an iterative closed form solution for such a problem. Furthermore, we provide a thorough analysis on the interdependency between the leakage power and the transmit power. Our study covers any leakage level between the egoistic approach, where the leakage power is not reduced at all, and the altruistic approach where the leakage is minimized. Depending on the strength of the leakage on a link and the given constraints, the leakage power and/or the transmit power is adjusted. This approach is very flexible. The constraints can easily be adapted to different network setups or requirements and the system performance can be optimized by finding the optimal trade-off between the leakage and transmit power constraints. Using numerical simulations, we show the relation of the leakage power and the transmit power in LBP and demonstrate the gain which can be achieved with the optimal trade-off.

Notation: In the following $(\cdot)^{\mathsf{H}}$ denotes the hermitian transpose of a matrix, $\mathrm{tr}(\cdot)$ the trace of a matrix and $\mathsf{E}[\cdot]$ the expected value. For the singular value decomposition (SVD) of a matrix $\mathbf{X} = \mathbf{U_X S_X V_X^{\mathsf{H}}}$, $\bar{\mathbf{S}}_\mathbf{X}$ denotes the part of $\mathbf{S_X}$ containing the non-zero singular values, and $\bar{\mathbf{V}}_\mathbf{X}$ the corresponding columns of $\mathbf{V_X}$.

II. System Model

The precoding is obtained for each link separately. That is, for every MS we consider a single user MIMO (SU-MIMO)

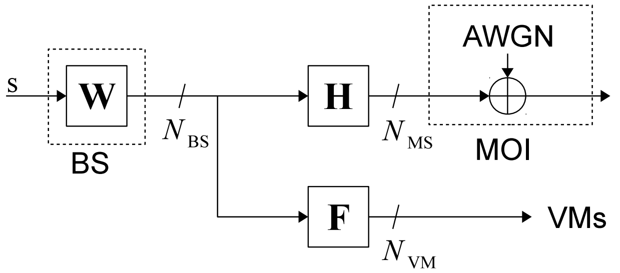

Fig. 1. System model.

setup as in Fig. 1, with one BS with N_{BS} antennas, one mobile of interest (MOI) with N_{MS} antennas, where $N_{\mathrm{BS}} \geq N_{\mathrm{MS}}$, and several victim mobiles (VMs), with in total N_{VM} antennas, suffering from the leakage of the MOI's signal. At the MOI only additive white Gaussian noise (AWGN) is considered. The interference originating from the signals for all other MSs in the network can be included into the noise, under the assumption, that for a large number of MSs, the received interference becomes white and Gaussian as well. The channel from the BS to the MOI is denoted by $\mathbf{H} \in \mathbb{C}^{N_{\mathrm{MS}} \times N_{\mathrm{BS}}}$ and the one from the BS to all VMs is denoted by $\mathbf{F} \in \mathbb{C}^{N_{\mathrm{VM}} \times N_{\mathrm{BS}}}$. The elements of $\mathbf{H}$ and $\mathbf{F}$ are assumed to be i.i.d. circularly symmetric complex Gaussian random variables with zero mean and unit variance $\mathcal{CN}(0,1)$. Both $\mathbf{H}$ and $\mathbf{F}$ are assumed to be full rank matrices. The signal transmitted from the BS to the MOI is the signal vector $\mathbf{s} \in \mathbb{C}^{N_{\mathrm{MS}}}$ with the covariance matrix $\mathbf{\Lambda_s} = \mathsf{E}\left[\mathbf{s}\mathbf{s}^{\mathsf{H}}\right]$, multiplied with the precoding matrix $\mathbf{W} \in \mathbb{C}^{N_{\mathrm{BS}} \times N_{\mathrm{MS}}}$. Hence, the received signal at the MOI can be written as

$$\mathbf{r}_{\mathrm{MOI}} = \mathbf{H}\mathbf{W}\mathbf{s} + \mathbf{n}_{\mathrm{MOI}}, \tag{1}$$

with the noise vector $\mathbf{n}_{\mathrm{MOI}} \in \mathbb{C}^{N_{\mathrm{MS}}}$ whose entries are assumed to be i.i.d. $\mathcal{CN}(0,1)$. The received signal at the VMs is given by

$$\mathbf{r}_{\mathrm{VM}} = \mathbf{F}\mathbf{W}\mathbf{s} + \mathbf{n}_{\mathrm{VM}}, \tag{2}$$

with the noise vector $\mathbf{n}_{\mathrm{VM}} \in \mathbb{C}^{N_{\mathrm{VM}}}$. This leads to the leakage power $P_{\mathrm{L}} = \mathrm{tr}\left(\mathbf{F}\mathbf{W}\mathbf{\Lambda_s}\mathbf{W}^{\mathsf{H}}\mathbf{F}^{\mathsf{H}}\right)$ and the BS transmit power $P_{\mathrm{BS}} = \mathrm{tr}\left(\mathbf{W}\mathbf{\Lambda_s}\mathbf{W}^{\mathsf{H}}\right)$. The channels are considered to be block fading. We assume that the BS has full knowledge of the channel $\mathbf{H}$ and the noise level at the MOI through feedback. From the channel $\mathbf{F}$ only the knowledge of the matrix $\mathbf{F}^{\mathsf{H}}\mathbf{F}$ is required.

For a precoding matrix $\mathbf{W}$ and a signal covariance matrix $\mathbf{\Lambda}_{\mathrm{s}}$ the achievable rate is given by

$$R = \log\det\left(\mathbf{I} + \frac{1}{N_0}\mathbf{H}\mathbf{Q}\mathbf{H}^{\mathsf{H}}\right), \tag{3}$$

with the transmit covariance matrix $\mathbf{Q} = \mathbf{W}\mathbf{\Lambda}_{\mathrm{s}}\mathbf{W}^{\mathsf{H}}$, $\mathbf{I}$ the identity matrix and N_0 the noise power spectral density at the MOI.

III. LBP Under Joint Transmit and Leakage Power Constraint

LBP has two goals, namely to reduce the leakage power to all other users and to serve the MOI with as high rate as possible. Both goals are important for a good network performance. However, they may contradict each other. With respect to the maximization of the achievable rate, we want to allocate a large portion of the transmit power to the desired signal. This may result in a high leakage power however. Leakage minimization in turn implies the use of a fraction of the total transmit power for the leakage reduction. This directly decreases the achievable rate. Hence, it is important to find a good trade-off between these two goals.

In the following, we will formulate a scheme which combines the two goals of LBP and allows to flexibly adapt the trade-off to the system setup and requirements in order to optimize the network performance. At first, we note that LBP can be separated into a signal term $\mathbf{W}_{\mathrm{s}}$, serving the MOI, and a compensation term $\mathbf{W}_{\mathrm{c}}$ to reduce the leakage power to all VMs in an orthogonal subspace to $\mathbf{H}$ without disturbing the desired signal at the MOI,

$$\mathbf{W} = \mathbf{W}_{\mathrm{s}} + \mathbf{W}_{\mathrm{c}}. \tag{4}$$

Assuming that the columns of the matrix $\mathbf{U} \in \mathbb{C}^{N_{\mathrm{BS}} \times N_{\mathrm{MS}}}$ represent an orthonormal basis of $\mathbf{H}$, $\mathbf{W}_{\mathrm{s}}$ can be found as the projection of $\mathbf{W}$ onto $\mathbf{H}$,

$$\mathbf{W}_{\mathrm{s}} = \mathbf{U}\mathbf{U}^{\mathsf{H}}\mathbf{W}. \tag{5}$$

$\mathbf{W}_{\mathrm{c}}$ is then the component of $\mathbf{W}$ orthogonal to $\mathbf{H}$,

$$\mathbf{W}_{\mathrm{c}} = \mathbf{W} - \mathbf{U}\mathbf{U}^{\mathsf{H}}\mathbf{W}. \tag{6}$$

As these two terms are orthogonal to each other, we can also separate the transmit power into the power allocated to the desired signal, $P_{\mathrm{s}} = \mathrm{tr}\left(\mathbf{W}_{\mathrm{s}}\mathbf{\Lambda_s}\mathbf{W}_{\mathrm{s}}^{\mathsf{H}}\right)$, and the power allocated to the leakage reduction, $P_{\mathrm{c}} = \mathrm{tr}\left(\mathbf{W}_{\mathrm{c}}\mathbf{\Lambda_s}\mathbf{W}_{\mathrm{c}}^{\mathsf{H}}\right)$,

$$P_{\mathrm{s}} + P_{\mathrm{c}} = P_{\mathrm{BS}}. \tag{7}$$

That is, to reduce the leakage power, the transmit power has to be invested partially into the compensation term, leading to a lower signal power and hence a lower rate at the MOI. Depending on the amount of transmit power allocated to P_{c}, any leakage power can be achieved between the egoistic approach where no energy at all is invested into the leakage reduction and the altruistic approach, where the leakage power is minimized.

One way to combine the two goals of LBP and to find the optimal trade-off is the optimization problem

$$\begin{aligned} \max_{\mathbf{Q}} \quad & \log\det\left(\mathbf{I} + \tfrac{1}{N_0}\mathbf{H}\mathbf{Q}\mathbf{H}^{\mathsf{H}}\right), \\ \text{s.t.} \quad & \mathrm{tr}\left(\mathbf{F}\mathbf{Q}\mathbf{F}^{\mathsf{H}}\right) \leq \tilde{P}_{\mathrm{L}}, \\ & \mathrm{tr}\left(\mathbf{Q}\right) \leq \tilde{P}_{\mathrm{BS}}, \end{aligned} \tag{8}$$

which was similarly proposed in [8] for RZF, however without providing a closed form solution and in-depth discussion. Instead of simply minimizing the leakage power as in [4], [6] and [7], a leakage power constraint is introduced additionally to the transmit power constraint.

Before we derive a closed form solution for (8), we first revisit the solutions if only one constraint is present.

a) The egoistic case: In this case, we consider only a transmit power constraint $\tilde{P}_{\mathrm{BS}}$. That is, the precoding is designed in an egoistic way without any considerations about the interference produced for the rest of the system. Hence, the optimization problem reduces to

$$\max_{\mathbf{Q}} \log\det\left(\mathbf{I} + \frac{1}{N_0}\mathbf{H}\mathbf{Q}\mathbf{H}^{\mathsf{H}}\right) \text{ s.t. } \mathrm{tr}\left(\mathbf{Q}\right) \leq \tilde{P}_{\mathrm{BS}}. \tag{9}$$

This corresponds to the standard point-to-point MIMO problem with the well known solution of waterfilling over the eigenmodes of the channel $\mathbf{H}$ [9]. By diagonalizing the problem using the SVD of $\mathbf{H}$, $\mathbf{H} = \mathbf{U_H S_H V_H^H}$, and precoding with $\mathbf{W} = \bar{\mathbf{V}}_\mathbf{H}$, the optimization problem can be brought into the equivalent form

$$\max_{\boldsymbol{\Lambda}_\mathrm{s}} \log\det\left(\mathbf{I} + \frac{1}{N_0}\bar{\mathbf{S}}_\mathbf{H}\boldsymbol{\Lambda}_\mathrm{s}\bar{\mathbf{S}}_\mathbf{H}^\mathsf{H}\right) \text{ s.t. } \mathrm{tr}\left(\boldsymbol{\Lambda}_\mathrm{s}\right) \leq \tilde{P}_\mathrm{BS}. \quad (10)$$

This problem is maximized by a diagonal $\boldsymbol{\Lambda}_\mathrm{s}$ with entries found by waterfilling over the diagonal elements of $\bar{\mathbf{S}}_\mathbf{H}^\mathsf{H}\bar{\mathbf{S}}_\mathbf{H}$. This optimization leads to the maximal achievable rate at the MOI under the transmit power constraint, while disregarding the leakage power.

b) The altruistic case: In the second case, we only have a leakage power constraint and the optimization problem reduces to

$$\max_{\mathbf{Q}} \log\det\left(\mathbf{I}+\frac{1}{N_0}\mathbf{HQH}^\mathsf{H}\right) \text{ s.t. } \mathrm{tr}\left(\mathbf{FQF}^\mathsf{H}\right) \leq \tilde{P}_\mathrm{L}. \quad (11)$$

It is important to note that we need to have $N_\mathrm{VM} \geq N_\mathrm{BS}$ for this optimization problem. Otherwise the leakage can be nulled and an infinite rate can be achieved. In order to solve (11), we can not directly resort to the SVD, as the $\mathbf{W}^\mathsf{H}\mathbf{F}^\mathsf{H}\mathbf{FW}$ term in the trace would not disappear. Following [4], we resort to the generalized eigenvalue decomposition (GEVD) to diagonalize the system. With the GEVD of the matrix pair $\left(\mathbf{H}^\mathsf{H}\mathbf{H}, \mathbf{F}^\mathsf{H}\mathbf{F}\right)$ we can find a matrix $\mathbf{Z}$ and a diagonal matrix $\mathbf{D}_\mathrm{g}$ such that

$$\begin{aligned} \mathbf{Z}^\mathsf{H}\mathbf{H}^\mathsf{H}\mathbf{HZ} &= \mathbf{D}_\mathrm{g} \\ \mathbf{Z}^\mathsf{H}\mathbf{F}^\mathsf{H}\mathbf{FZ} &= \mathbf{I}, \end{aligned} \quad (12)$$

where $\mathbf{D}_\mathrm{g}$ contains the GEVs of the matrix pair $\left(\mathbf{H}^\mathsf{H}\mathbf{H}, \mathbf{F}^\mathsf{H}\mathbf{F}\right)$ and $\mathbf{Z}$ the corresponding generalized eigenvectors. Setting $\mathbf{W} = \bar{\mathbf{Z}}$, the optimization problem can be brought into the equivalent form

$$\max_{\boldsymbol{\Lambda}_\mathrm{s}} \log\det\left(\mathbf{I} + \frac{1}{N_0}\bar{\mathbf{D}}_\mathrm{g}^{1/2}\boldsymbol{\Lambda}_\mathrm{s}\bar{\mathbf{D}}_\mathrm{g}^{1/2}\right) \text{s.t.} \,\mathrm{tr}\left(\boldsymbol{\Lambda}_\mathrm{s}\right) \leq \tilde{P}_\mathrm{L}, \quad (13)$$

where $\bar{\mathbf{D}}_\mathrm{g}$ denotes the part of $\mathbf{D}_\mathrm{g}$ containing the non-zero GEVs, and $\bar{\mathbf{Z}}$ the corresponding columns of $\mathbf{Z}$. Analogously to (10), this problem is optimized by a diagonal $\boldsymbol{\Lambda}_\mathrm{s}$ with entries found by waterfilling over the values of $\bar{\mathbf{D}}_\mathrm{g}$. This optimization provides the maximal achievable rate at the MOI under a leakage power constraint, disregarding the transmit power at the BS. The resulting precoding can also be interpreted as the precoding which minimizes the leakage power for a given achievable rate. That is, the precoding is designed in an altruistic way in favour of the network performance.

IV. A Closed Form Solution

To find the rate optimal precoding under the joint constraints, a trade-off has to be found between the egoistic and the altruistic approach. In order to get a better understanding of the relation between leakage power and transmit power, we have to look at the $P_\mathrm{BS} - P_\mathrm{L}$ plane in Fig. 2, which shows the interdependency of the generated leakage power and used transmit power. The data in this figure has been acquired from one channel realization in a SU-MIMO setup as described in Section II. The dashed red line and the dashed green line show the maximal respectively the minimal achievable leakage power for a given P_BS. The points on these lines can be reached by precoding with the eigenvector corresponding to the maximal respectively minimal eigenvalue of $\mathbf{F}^\mathsf{H}\mathbf{F}$. Hence, any achievable point $(P_\mathrm{BS}, P_\mathrm{L})$ lies on or in between these two lines. The solid red curve shows the generated leakage power if the rate is maximized under a transmit power constraint (egoistic case). For any point above this curve (take the red cross as an example) there is a corresponding point (the red dot) on the curve with the same transmit power, a lower leakage power and equal or better rate. Thus, the region between the solid and dashed red curve - even though achievable - is of no practical interest. Contrariwise, the solid green curve shows the resulting transmit power if the rate is maximized under a leakage power constraint (altruistic case). For any point to the right of this curve (take the green cross as an example), there is a corresponding point (the green dot) on the green curve with the same leakage power but a lower transmit power and equal or better rate. Thus, the region between the solid and dashed green curve is as well of no practical interest.

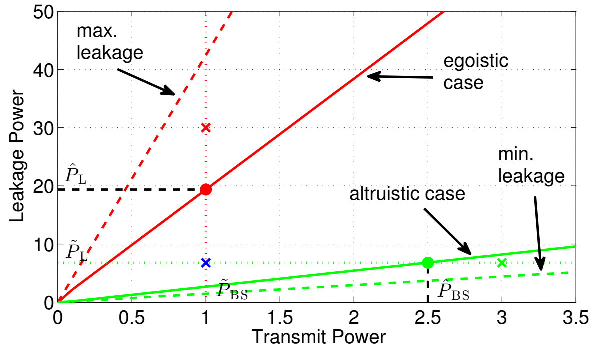

Fig. 2. The $P_\mathrm{BS} - P_\mathrm{L}$ plane.

An arbitrary joint constraint on the leakage and transmit power defines a set of points in the $P_\mathrm{BS} - P_\mathrm{L}$ plane. For the constraints in (8) e.g., the set of points define a rectangular region. Thus, we first consider the problem to find the rate optimal precoder for a given point $(\tilde{P}_\mathrm{BS}, \tilde{P}_\mathrm{L})$. For what was said above, we only need to consider the region constrained by the solid red and green curves in Fig. 2. For any point outside of this region, the rate optimal precoding can either be found by (10) or (13). Note that this region changes with each channel realization. To capture the impact of both power constraints in a single constraint, we consider the weighted sum $\tilde{P}$ of the transmit power and the leakage power,

$$\begin{aligned} \tilde{P} &= cP_\mathrm{L} + (1-c)\,P_\mathrm{BS} \quad (14) \\ &= c\,\mathrm{tr}\left(\mathbf{W}^\mathsf{H}\mathbf{F}^\mathsf{H}\mathbf{FW}\boldsymbol{\Lambda}_\mathrm{s}\right) + (1-c)\,\mathrm{tr}\left(\mathbf{W}^\mathsf{H}\mathbf{W}\boldsymbol{\Lambda}_\mathrm{s}\right) \\ &= \mathrm{tr}\left(\mathbf{W}^\mathsf{H}\left(c\mathbf{F}^\mathsf{H}\mathbf{F} + (1-c)\,\mathbf{I}\right)\mathbf{W}\boldsymbol{\Lambda}_\mathrm{s}\right) \\ &= \mathrm{tr}\left(\mathbf{W}^\mathsf{H}\tilde{\mathbf{F}}^\mathsf{H}\tilde{\mathbf{F}}\mathbf{W}\boldsymbol{\Lambda}_\mathrm{s}\right) \\ &= \mathrm{tr}\left(\tilde{\mathbf{F}}\mathbf{Q}\tilde{\mathbf{F}}^\mathsf{H}\right). \end{aligned}$$

By optimizing the rate subject to the single power constraint $\tilde{P}$ instead of the two individual constraints, we get the same structure in the optimization problem as in the altruistic case. Hence, we can optimize the achievable rate subject to the combined constraint $\tilde{P}$ in a closed form in the same way over the

GEVD of $\left(\mathbf{H}^{\mathsf{H}}\mathbf{H}, \tilde{\mathbf{F}}^{\mathsf{H}}\tilde{\mathbf{F}}\right)$, with $\tilde{\mathbf{F}}^{\mathsf{H}}\tilde{\mathbf{F}} = \left(c\mathbf{F}^{\mathsf{H}}\mathbf{F} + (1-c)\,\mathbf{I}\right)$. Note that for given c and $\tilde{P}$, the set of points that solve (14) defines a line in the P_{BS}–P_{L} plane. The optimization will then yield the point on this line which corresponds to the maximal achievable rate. This point is not necessarily identical with the point of interest $(\tilde{P}_{\mathrm{BS}}, \tilde{P}_{\mathrm{L}})$. For this reason, we consider all lines that pass through the point of interest and search for the one leading to the desired leakage and transmit power pair $(\tilde{P}_{\mathrm{BS}}, \tilde{P}_{\mathrm{L}})$. This set of lines is parameterized by the parameter c and we choose the combined constraint as

$$\tilde{P} = c\tilde{P}_{\mathrm{L}} + (1-c)\,\tilde{P}_{\mathrm{BS}}. \tag{15}$$

Hence, the optimization problem (8) reduces to a one dimensional search for c.

We note here, that from any rate optimal precoding matrix $\mathbf{W}_1 = \mathbf{W}_{\mathrm{s}} + \mathbf{W}_{\mathrm{c}}$, leading to a point $(\tilde{P}_{\mathrm{BS}}, P_{\mathrm{L},1})$ inside the region of practical interest, we can construct a new precoding matrix $\mathbf{W}_2 = \alpha\mathbf{W}_{\mathrm{s}} + \beta\mathbf{W}_{\mathrm{c}}$, with $\alpha > 1$ and $0 \le \beta < 1$, such that $P_{\mathrm{BS},2} = \tilde{P}_{\mathrm{BS}}$. As $\mathbf{W}_1$ is rate optimal, i.e. the leakage power is minimal for the corresponding achievable rate and transmit power, both $\alpha > 1$ and $\beta < 1$ lead to an increased leakage power $P_{\mathrm{L},2} > P_{\mathrm{L},1}$. Furthermore, $\alpha > 1$ leads to an increased achievable rate, as $\mathbf{W}_{\mathrm{c}}$ is orthogonal to $\mathbf{H}$ and thus does not affect the desired signal. That is, for a point inside the region of practical interest with transmit power $\tilde{P}_{\mathrm{BS}}$, we can always increase the achievable rate by allowing a higher leakage power up to the leakage power $\hat{P}_{\mathrm{L}}$, resulting from the rate maximization under the transmit power constraint only. Hence, on a vertical line within the region of practical interest the achievable rate is monotonously increasing. In a similar way, it can be shown that the achievable rate is also monotonously increasing on a horizontal line within this region. Thus, all points with $P_{\mathrm{L}} \ge \tilde{P}_{\mathrm{L}}$ and $P_{\mathrm{BS}} \ge \tilde{P}_{\mathrm{BS}}$ allow for a higher achievable rate than the point $(\tilde{P}_{\mathrm{BS}}, \tilde{P}_{\mathrm{L}})$. Therefore, we only have to consider lines with a negative or a zero slope in (15) (i.e. $c \in [0, 1]$). All other lines contain points with $P_{\mathrm{L}} > \tilde{P}_{\mathrm{L}}$ and $P_{\mathrm{BS}} > \tilde{P}_{\mathrm{BS}}$, allowing for a higher achievable rate. For such lines, the optimization would always lead to a solution violating the individual constraints.

To find the optimal c, we use the bisection method and iteratively search for the c leading to $(\tilde{P}_{\mathrm{BS}}, \tilde{P}_{\mathrm{L}})$ up to a tolerance η. The detailed procedure can be found in Algorithm 1. With this algorithm, the optimal solution is found within a few iterations, depending on η.

For any closed convex constraint set, the rate optimal point lies on the boundary of the set, as otherwise we could always increase the leakage and/or the transmit power and achieve a higher rate until we reach the boundary. For the special case of a rectangular constraint set $P_{\mathrm{L}} \le \tilde{P}_{\mathrm{L}}$, $P_{\mathrm{BS}} \le \tilde{P}_{\mathrm{BS}}$, we can distinguish three situations. If $\tilde{P}_{\mathrm{L}}$ is higher than the resulting leakage power $\hat{P}_{\mathrm{L}}$ of the rate maximization under a $\tilde{P}_{\mathrm{BS}}$ constraint, the transmit power is the dominating constraint. Hence, the rate optimal point lies on the solid red curve in Fig. 2 and can be found by (10). If $\tilde{P}_{\mathrm{BS}}$ is higher than the resulting transmit power $\hat{P}_{\mathrm{BS}}$ of the rate maximization under a $\tilde{P}_{\mathrm{L}}$ constraint, the leakage power is the dominating constraint. In this case, the rate optimal point lies on the solid green curve and can be found by (13). For all other cases, both constraints are restraining and the rate optimal point can be found at $(\tilde{P}_{\mathrm{BS}}, \tilde{P}_{\mathrm{L}})$ with Algorithm 1.

Algorithm 1 Finding the rate optimal precoding

Initialization: $c_{\max} = 1$, $c_{\min} = 0$, $\hat{P}_{\mathrm{L}} = 0$, $\hat{P}_{\mathrm{BS}} = 0$, η
while $|\hat{P}_{\mathrm{L}} - \tilde{P}_{\mathrm{L}}| > \tilde{P}_{\mathrm{L}} \cdot \eta$ or $|\hat{P}_{\mathrm{BS}} - \tilde{P}_{\mathrm{BS}}| > \tilde{P}_{\mathrm{BS}} \cdot \eta$ **do**
  $c = \frac{c_{\max} - c_{\min}}{2}$
  $\tilde{P} = c\tilde{P}_{\mathrm{L}} + (1-c)\,\tilde{P}_{\mathrm{BS}}$
  $\tilde{\mathbf{F}}^{\mathsf{H}}\tilde{\mathbf{F}} = c\mathbf{F}^{\mathsf{H}}\mathbf{F} + (1-c)\mathbf{I}$
  $(\mathbf{Z}, \mathbf{D}_{\mathrm{g}}) = \mathrm{GEVD}\left(\mathbf{H}^{\mathsf{H}}\mathbf{H}, \tilde{\mathbf{F}}^{\mathsf{H}}\tilde{\mathbf{F}}\right)$
  $\max_{\mathbf{\Lambda}_{\mathrm{s}}} \log\det\left(\mathbf{I} + \frac{1}{N_0}\bar{\mathbf{D}}_{\mathrm{g}}^{1/2}\mathbf{\Lambda}_{\mathrm{s}}\bar{\mathbf{D}}_{\mathrm{g}}^{1/2}\right)$ s.t. $\mathrm{tr}\left(\mathbf{\Lambda}_{\mathrm{s}}\right) \le \tilde{P}$
  $\mathbf{W} = \bar{\mathbf{Z}}$
  $\hat{P}_{\mathrm{L}} = \mathrm{tr}\left(\mathbf{F}\mathbf{W}\mathbf{\Lambda}_{\mathbf{s}}\mathbf{W}^{\mathsf{H}}\mathbf{F}^{\mathsf{H}}\right)$
  $\hat{P}_{\mathrm{BS}} = \mathrm{tr}\left(\mathbf{W}\mathbf{\Lambda}_{\mathbf{s}}\mathbf{W}^{\mathsf{H}}\right)$
  if $\tilde{P}_{\mathrm{L}} < \hat{P}_{\mathrm{L}}$ **then**
    $c_{\max} = c$
  else
    $c_{\min} = c$
  end if
end while

V. Numerical Simulations

In the following, the impact of the optimal trade-off between transmit power and leakage power will be shown by numerical simulations and compared to SLNR precoding [4] as a baseline of LBP. The simulation setup is a single cell MU-MIMO system with one BS with $N_{\mathrm{BS}} = 8$ antennas and $M_{\mathrm{MS}} = 6$ MSs with $N_{\mathrm{MS}} = 2$ antennas each. The channels to the MSs and the AWGN are chosen as described in Section II. Note, that the MSs are not subject to pathloss, i.e. all channel matrices have i.i.d. elements with unit variance. For the precoding, the system is split up into a SU-MIMO systems with one MOI and 5 VMs for each MS in the system. The MOI is then served with the rate optimal precoding under joint constraints as described in Section IV, with $\tilde{P}_{\mathrm{BS}} = 1$, $\tilde{P}_{\mathrm{L}} \in \{0.5, 1, \ldots, 10\}$ and $\eta = 0.001$. The noise at the MOI consists of AWGN and the interference from the data streams of the other MSs in the cell. For the calculation of the precoding we assume that the interference is spatially white and has a variance of $\sigma_i^2 = \tilde{P}_{\mathrm{L}}/N_{\mathrm{MS}}$ at each MOI antenna. This value would arise in practice, if all interference contributions were distributed uniformly across the VM antennas. For the computation of the achievable rates however, we consider the true interference covariance matrices. For SLNR precoding, the leakage power constraint is irrelevant as it is not considered in this precoding scheme.

Fig. 3 shows the average achievable rate per MS and Fig. 4 the average transmit power and leakage power involved. Fig. 5 and 6 show the corresponding empirical cumulative distribution functions (ECDFs) for a selection of $\tilde{P}_{\mathrm{L}}$ constraints. It can be observed in Fig. 3 and 5, that for a low $\tilde{P}_{\mathrm{L}}$ the performance is inferior compared to higher $\tilde{P}_{\mathrm{L}}$ (up to $\tilde{P}_{\mathrm{L}} = 3$). For most channel realizations $\tilde{P}_{\mathrm{L}}$ is the dominating constraint, allowing only for a low transmit power (c.f. Fig. 6). This leads to a low signal power and hence to a low achievable rate. Increasing $\tilde{P}_{\mathrm{L}}$ allows for higher transmit powers and thus higher signal powers. This can compensate the loss of the additional interference in the network, leading to increased achievable rates. The higher the leakage constraint is, the more channel realizations lead to the case where both constraints are

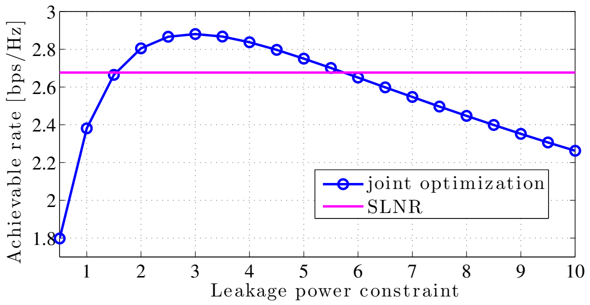

Fig. 3. Average achievable rate per MS.

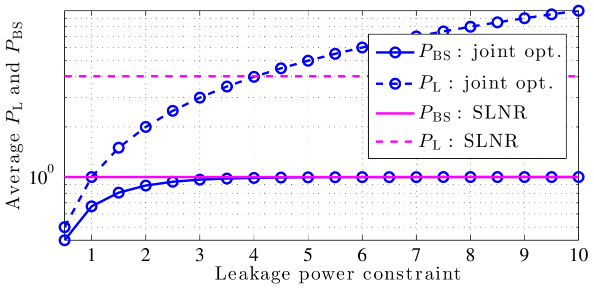

Fig. 4. Average transmit and leakage power per MS.

restraining. However, this does not lead to a consistently increasing performance. For a certain $\tilde{P}_{\mathrm{L}}$ the peak performance is reached (in this setup for $\tilde{P}_{\mathrm{L}} = 3$). For any higher $\tilde{P}_{\mathrm{L}}$ the disadvantage of more interference in the network can not be compensated anymore by the advantage of an increased signal power, and the achievable rates are decreasing again. Compared to SLNR precoding, the optimization under joint constraints leads to a significant average performance gain at the peak performance point. However, it can be seen in Fig. 5, that for the weak users, SLNR precoding outperforms the optimization under joint constraints. This is due to the fact, that for the optimization under joint constraints, the transmit power is reduced if there is strong leakage for a link. Due to the lower available power, the achievable rate suffers. As SLNR precoding does not adapt the transmit power, higher rates can be achieved at the MOI in these cases. However, also more leakage is generated, lowering the performance of all other MSs in the system. Comparing the average transmit power (Fig. 4) of SLNR precoding and the optimization under joint constraints at the point where both schemes achieve about the same average achievable rate (i.e. for $\tilde{P}_{\mathrm{L}} = 1.5$), it can be observed that significantly less transmit power is used for the optimization under joint constraints. That is, with a flexible $\tilde{P}_{\mathrm{L}}$ constraint, the network performance can be optimized and transmit power saved.

Note on complexity: Throughout our simulations in Matlab, the computation time of the proposed algorithm was about 1 to 2 orders of magnitude smaller than with a standard optimization toolbox for semidefinite programming [10]. The average number of iterations has been around 7.

VI. Conclusions and Outlook

In this paper, a LBP scheme has been presented which provides the rate optimal precoding under any transmit and leakage power constraints. An iterative closed form solution to this problem has been derived and a thorough analysis of LBP and the interdependency between the leakage power and transmit power has been provided. This approach allows to optimize the network performance by finding the optimal trade-off between the leakage power and transmit power. We note that this scheme is well suited to be applied in cellular networks with CoMP. The precoding can be flexibly distributed over several BSs and also allows overlapping clusters in CoMP. Due to its leakage power constraint the interference between the clusters can be balanced and the network performance can be optimized. The investigation of this topic is the scope of future work.

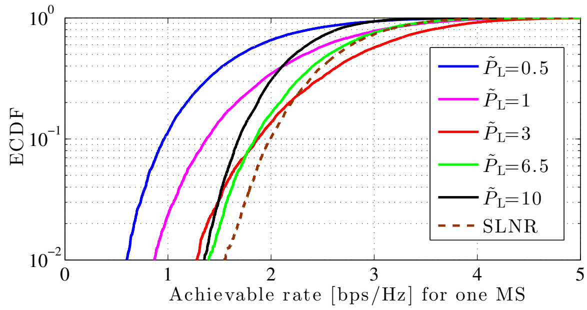

Fig. 5. ECDFs of achievable rates per MS.

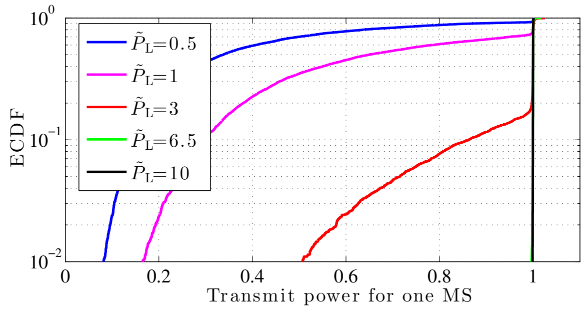

Fig. 6. ECDFs of transmit power per MS.

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