MIMO TWO-WAY RELAYING WITH TRANSMIT CSI AT THE RELAY

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ABSTRACT

Conventional half-duplex relaying schemes suffer from the loss in spectral efficiency due to the two channel uses required for the transmission from the source to the destination. Two-way relaying is an efficient means to reduce this loss in spectral efficiency by bidirectional simultaneous transmission of data between the two nodes. In this paper we study the impact of transmit channel state information at the relay in a MIMO two-way decode-and-forward relaying scheme. We investigate and compare two different re-encoding schemes at the relay. The first is based on superposition coding, whereas the second one is based on the bitwise XOR operation.

1. INTRODUCTION

Conventional half-duplex relaying schemes (cf. [1] and the references therein) suffer from the loss in spectral efficiency due to the two channel uses required for the transmission from the source to the destination, which cause a pre-log factor 1/2 in the corresponding rate expressions.

In [2], the authors propose to spatially reuse the relay slot to reduce the loss in spectral efficiency. They consider a base station that transmits K messages to K users and their corresponding relays in K orthogonal time slots. In time slot $K + 1$ all relays retransmit their received signal, causing interference to the other users. The capacity of a single connection (base station to user) has then a pre-log factor $K/(K+1)$ instead of $1/2$. A similar scheme, considering the uplink to a base station, has been investigated in [3].

In [4], a network coding scenario is considered, where two nodes communicate via an access point in three time slots. In the first and second slot, the two nodes transmit their messages via orthogonal channels to the relay. The relay combines both messages on bit-level by means of the XOR operation and retransmits it to both nodes. The nodes use the XOR operation on the decoded message and the own transmitted message to obtain the message from the other node. The resulting pre-log factor with respect to the sum-rate of this scheme is thus given by $2/3$. Note that this scenario is also considered in [5].

In [6, 7], the authors reduce the number of required time slots for the communication between the two nodes even further. The relaying scheme, which is referred to as *two-way relaying*, consists only of two time slots. In the first time slot both nodes transmit their messages simultaneously via a multiple access channel scenario to the relay. In the second time slot the messages are linearly combined by means of superposition coding (and not by means of XOR as in [4]). Since the nodes know their own transmitted signal they subtract the back-propagating *self-interference* prior to decoding. Note that the achievable rate in one direction suffers still from the pre-log factor 1/2. However, two connections are realized in the same physical channel which improves the spectral efficiency. In [8], the integration of two-way relaying in 802.11n WLANs is investigated with respect to coverage issues.

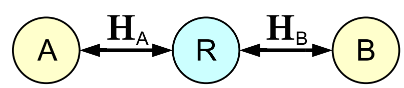

Fig. 1. Bidirectional connection via a relay station R.

In this work, we extend the two-way relaying scheme of [6] to multiple antennas at all nodes and assume further the knowledge of transmit CSI at the decode-and-forward relay. This is a reasonable assumption since the relay has to estimate the MIMO channels for decoding in the first time slot anyway. So, in the second time slot this knowledge can be used for precoding if the bursts are short enough compared to the coherence time of the involved MIMO channels. Furthermore, we compare both approaches of combining the messages in the second time slot at the relay, the XOR precoding of [4] and the superposition coding (SPC) of [6]. We show that two-way relaying achieves a quite substantial improvement in spectral efficiency compared to conventional relaying with and without transmit CSI at the relay. We show that the difference in sum-rate compared to the case where no CSIT is used, increases with increasing ratio between number of relay antennas and number of node antennas. We further show that XOR precoding always achieves higher minimum user rates than SPC if CSIT is used. In our considered simulation scenarios, this leads also to higher sum-rates of the XOR precoding compared to SPC.

Notation: Bold uppercase letters denote matrices and bold lowercase letters vectors. Furthermore $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^*$ denote the transpose, Hermitian transpose, and the element-wise complex conjugate of a matrix or vector, respectively, $|\cdot|$ the determinant of a matrix. The singular value decomposition of the $(n \times m)$ matrix $\mathbf{A}$ with rank r is given by $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$. The dimensions of $\mathbf{U}$, $\mathbf{\Sigma}$, and $\mathbf{V}^H$ are $(n \times r)$, $(r \times r)$, and $(r \times m)$, respectively.

2. MIMO TWO-WAY RELAYING

We consider a low mobility communication scenario as depicted in Fig. 1. Two nodes, node A and B, establish a bidirectional connection via a relay station R. The nodes are equipped with N_A, N_B, N_R antennas, respectively. We consider frequency-flat block fading between all nodes, where $\mathbf{H}_\mathsf{A} \in \mathbb{C}^{N_\mathsf{R} \times N_\mathsf{A}}$ and $\mathbf{H}_\mathsf{B} \in \mathbb{C}^{N_\mathsf{R} \times N_\mathsf{B}}$ denote the corresponding channel matrices. We assume that node A wants to transmit the bit-sequence $\mathbf{x}_\mathsf{AB}$ to node B, whereas node B wants to transmit $\mathbf{x}_\mathsf{BA}$ to node A, respectively.

In the following we present the principle of two-way relaying and derive expressions for the sum-rate as well as the rate-region for the considered scenario. The traffic pattern for two-way relaying is divided into two phases (or time slots). The channels are assumed to be constant over both phases.

Phase 1: In the first phase, node A and B transmit simultaneously to the relay, whereby both nodes do not have transmit CSI. The transmit signals of node A and B are denoted by $\mathbf{s}_{\mathsf{AB}} \in \mathbb{C}^{N_{\mathsf{A}}}$ and $\mathbf{s}_{\mathsf{BA}} \in \mathbb{C}^{N_{\mathsf{B}}}$, respectively. Both vectors are subject to a power constraint, i.e. $\mathrm{E}\{\mathbf{s}_{\mathsf{AB}}^H \mathbf{s}_{\mathsf{AB}}\} = P_{\mathsf{A}}$ and $\mathrm{E}\{\mathbf{s}_{\mathsf{BA}}^H \mathbf{s}_{\mathsf{BA}}\} = P_{\mathsf{B}}$. The received signal $\mathbf{r}_{\mathsf{R}}$ at the relay can be expressed as

$$\mathbf{r}_{\mathsf{R}} = \mathbf{H}_{\mathsf{A}}\mathbf{s}_{\mathsf{AB}} + \mathbf{H}_{\mathsf{B}}\mathbf{s}_{\mathsf{BA}} + \mathbf{n}_{\mathsf{R}}, \tag{1}$$

where the noise contribution at the relay is given by $\mathbf{n}_{\mathsf{R}} \sim \mathcal{CN}\left(0, \sigma_{\mathsf{R}}^2 \mathbf{I}_{N_{\mathsf{R}}}\right)$. Based on (1) the relay decodes the information from node A and B. Using a Gaussian codebook, the achievable rates of both nodes are theoretically described by the MIMO multiple access channel (MAC) rate region [9, 10], which is defined by the three inequalities

$$R_{\mathsf{A},1} \le I_{\mathsf{A},1} = \log_2 \left| \mathbf{I} + \frac{P_{\mathsf{A}}}{N_{\mathsf{A}}\sigma_{\mathsf{R}}^2} \mathbf{H}_{\mathsf{A}}\mathbf{H}_{\mathsf{A}}^H \right| \tag{2}$$

$$R_{\mathsf{B},1} \le I_{\mathsf{B},1} = \log_2 \left| \mathbf{I} + \frac{P_{\mathsf{B}}}{N_{\mathsf{B}}\sigma_{\mathsf{R}}^2} \mathbf{H}_{\mathsf{B}}\mathbf{H}_{\mathsf{B}}^H \right| \tag{3}$$

$$R_{\mathsf{A},1} + R_{\mathsf{B},1} \le I_1 = \log_2 \left| \mathbf{I} + \frac{\frac{P_{\mathsf{A}}}{N_{\mathsf{A}}}\mathbf{H}_{\mathsf{A}}\mathbf{H}_{\mathsf{A}}^H + \frac{P_{\mathsf{B}}}{N_{\mathsf{B}}}\mathbf{H}_{\mathsf{B}}\mathbf{H}_{\mathsf{B}}^H}{\sigma_{\mathsf{R}}^2} \right|. \tag{4}$$

Phase 2: In the second phase, the relay re-encodes the information and broadcasts it to both nodes on the same channels as in the first phase. We assume that the transmit signal of the relay $\mathbf{s}_{\mathsf{R}}$ is determined by the decoded bit-sequences of the first phase, i.e., $(\mathbf{x}_{\mathsf{AB}}, \mathbf{x}_{\mathsf{BA}}) \longrightarrow \mathbf{s}_{\mathsf{R}}$. The nodes decode the signals they receive in the second phase. Since they know the information $\mathbf{x}_{\mathsf{AB}}, \mathbf{x}_{\mathsf{BA}}$ they have transmitted in the first phase (*self-interference*) they can cancel this contribution and decode the other part of the information.

To derive the rate expressions for the second phase we consider two different coding approaches. The first is based on superposition coding as in [6], whereas the second approach is based on the XOR operation as in [4].

Superposition Coding Scheme (SPC): In this approach the relay uses two Gaussian codebooks to encode the information bit sequences $\mathbf{x}_{\mathsf{AB}} \longrightarrow \tilde{\mathbf{s}}_{\mathsf{AB}}$ and $\mathbf{x}_{\mathsf{BA}} \longrightarrow \tilde{\mathbf{s}}_{\mathsf{BA}}$. Note that the codebook of $\tilde{\mathbf{s}}_{\mathsf{AB}}$ and $\tilde{\mathbf{s}}_{\mathsf{BA}}$ can differ from the codebooks of $\mathbf{s}_{\mathsf{AB}}$ and $\mathbf{s}_{\mathsf{BA}}$ in the first phase. Since the relay has learned the channel matrices $\mathbf{H}_{\mathsf{A}}, \mathbf{H}_{\mathsf{B}}$ from the decoding procedure of the first phase, it can use this knowledge for precoding in the second phase. Thus, the transmit signal in the second phase is given by

$$\mathbf{s}_{\mathsf{R}} = \mathbf{W}_{\mathsf{B}}\tilde{\mathbf{s}}_{\mathsf{AB}} + \mathbf{W}_{\mathsf{A}}\tilde{\mathbf{s}}_{\mathsf{BA}}, \tag{5}$$

where $\mathbf{W}_{\mathsf{A}}$ and $\mathbf{W}_{\mathsf{B}}$ denote precoding matrices. The power constraint at the relay is given by $\mathrm{E}\{\mathbf{s}_{\mathsf{R}}^H\mathbf{s}_{\mathsf{R}}\} = P_{\mathsf{R}}$ The received signals at node A and B are given by

$$\mathbf{r}_{\mathsf{A}} = \underbrace{\mathbf{H}_{\mathsf{A}}^T\mathbf{W}_{\mathsf{B}}\tilde{\mathbf{s}}_{\mathsf{AB}}}_{\text{self-interference}} + \mathbf{H}_{\mathsf{A}}^T\mathbf{W}_{\mathsf{A}}\tilde{\mathbf{s}}_{\mathsf{BA}} + \mathbf{n}_{\mathsf{A}} \tag{6}$$

$$\mathbf{r}_{\mathsf{B}} = \mathbf{H}_{\mathsf{B}}^T\mathbf{W}_{\mathsf{B}}\tilde{\mathbf{s}}_{\mathsf{AB}} + \underbrace{\mathbf{H}_{\mathsf{B}}^T\mathbf{W}_{\mathsf{A}}\tilde{\mathbf{s}}_{\mathsf{BA}}}_{\text{self-interference}} + \mathbf{n}_{\mathsf{B}}, \tag{7}$$

where the noise contributions at node A and B are given by $\mathbf{n}_{\mathsf{A}} \sim \mathcal{CN}\left(0, \sigma_{\mathsf{A}}^2\mathbf{I}_{N_{\mathsf{A}}}\right)$ and $\mathbf{n}_{\mathsf{B}} \sim \mathcal{CN}\left(0, \sigma_{\mathsf{B}}^2\mathbf{I}_{N_{\mathsf{B}}}\right)$. Since node A and node B know the information bit sequences ($\mathbf{x}_{\mathsf{AB}}$ and $\mathbf{x}_{\mathsf{BA}}$, respectively) they can cancel (i.e., subtract) this self-interference part. Note that this requires knowledge of the MIMO channels, the precoding matrices and the codebooks used by the relay. Thus, assuming perfect cancelation the mutual information between the relay and the receiving nodes is given by

$$I_{\mathsf{A},2}^{(\text{spc})} = \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{\mathsf{A}}^2}\mathbf{H}_{\mathsf{A}}^T\mathbf{W}_{\mathsf{A}}\mathbf{\Lambda}_{\mathsf{BA}}\mathbf{W}_{\mathsf{A}}^H\mathbf{H}_{\mathsf{A}}^* \right| \tag{8}$$

$$I_{\mathsf{B},2}^{(\text{spc})} = \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{\mathsf{B}}^2}\mathbf{H}_{\mathsf{B}}^T\mathbf{W}_{\mathsf{B}}\mathbf{\Lambda}_{\mathsf{AB}}\mathbf{W}_{\mathsf{B}}^H\mathbf{H}_{\mathsf{B}}^* \right|, \tag{9}$$

with $\mathbf{\Lambda}_{\mathsf{AB}} = \mathrm{E}\{\tilde{\mathbf{s}}_{\mathsf{AB}}\tilde{\mathbf{s}}_{\mathsf{AB}}^H\}$ and $\mathbf{\Lambda}_{\mathsf{BA}} = \mathrm{E}\{\tilde{\mathbf{s}}_{\mathsf{BA}}\tilde{\mathbf{s}}_{\mathsf{BA}}^H\}$. Note, that both rate expressions are only coupled via the transmit power constraint at the relay. The overall rate region of the presented two-way relaying scheme with superposition coding is defined by (2), (3), (4), (8), and (9) as

$$R_{\mathsf{AB}} \le 1/2 \ \min\{I_{\mathsf{A},1}, I_{\mathsf{B},2}^{(\text{spc})}\} \tag{10}$$

$$R_{\mathsf{BA}} \le 1/2 \ \min\{I_{\mathsf{B},1}, I_{\mathsf{A},2}^{(\text{spc})}\} \tag{11}$$

$$R_{\mathsf{AB}} + R_{\mathsf{BA}} \le 1/2 \ I_1, \tag{12}$$

where R_{AB} and R_{BA} denote the achievable rates from node A to B and from node B to A, respectively. The factor $1/2$ refers to the two channel uses which are required for the relaying traffic pattern. The maximum sum-rate of the two-way relaying scheme is thus given by

$$\begin{aligned} R_{\text{sum}} &= R_{\mathsf{AB}} + R_{\mathsf{BA}} \\ &= \frac{1}{2}\min\{\min\{I_{\mathsf{A},1}, I_{\mathsf{B},2}^{(\text{spc})}\} + \min\{I_{\mathsf{B},1}, I_{\mathsf{A},2}^{(\text{spc})}\}, I_1\}. \end{aligned} \tag{13}$$

If the sum rate R_{sum} is limited by I_1, different (R_{AB}, R_{AB}) pairs maximize R_{sum}. In this region fairness or QoS issues can be considered for the choice of R_{AB} and R_{AB} without compromising R_{sum}.

XOR Precoding: The XOR precoding scheme combines the two information bit sequences on bit-level prior to encoding. Specifically the relay applies bitwise XOR operation on both decoded bit-sequences $\mathbf{x}_{\mathsf{AB}}$ and $\mathbf{x}_{\mathsf{BA}}$ and codes the resulting bit-sequence $\mathbf{x}_{\mathsf{R}}$, i.e.,

$$\mathbf{x}_{\mathsf{AB}} \oplus \mathbf{x}_{\mathsf{BA}} = \mathbf{x}_{\mathsf{R}} \quad \longrightarrow \quad \mathbf{s}_{\mathsf{R}}. \tag{14}$$

Note that by the XOR precoding scheme the transmit bitrate of the relay is reduced in comparison to SPC. Thus the transmit energy per bit is increased, which yields a SNR gain at nodes A and B. The received signals at both nodes are given by

$$\mathbf{r}_{\mathsf{A}} = \mathbf{H}_{\mathsf{A}}^T\mathbf{s}_{\mathsf{R}} + \mathbf{n}_{\mathsf{A}} \tag{15}$$

$$\mathbf{r}_{\mathsf{B}} = \mathbf{H}_{\mathsf{B}}^T\mathbf{s}_{\mathsf{R}} + \mathbf{n}_{\mathsf{B}}, \tag{16}$$

where both nodes first decode $\mathbf{x}_{\mathsf{R}}$. The self-interference cancelation is done after the decoding by applying a simple XOR operation, i.e.,

$$\mathbf{x}_{\mathsf{R}} \oplus \mathbf{x}_{\mathsf{BA}} = \mathbf{x}_{\mathsf{AB}} \tag{17}$$

$$\mathbf{x}_{\mathsf{R}} \oplus \mathbf{x}_{\mathsf{AB}} = \mathbf{x}_{\mathsf{BA}}. \tag{18}$$

Since both nodes have to be able to decode $\mathbf{x}_{\mathsf{R}}$ the maximum data rate in the second phase with XOR precoding is given by

$$I_2^{(\text{xor})} = \min\left\{I_{\mathsf{A},2}^{(\text{xor})}, I_{\mathsf{B},2}^{(\text{xor})}\right\}, \tag{19}$$

with

$$I_{\mathsf{A},2}^{(\mathrm{xor})} = \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{\mathsf{A}}^2} \mathbf{H}_{\mathsf{A}}^T \mathbf{\Lambda}_{\mathsf{R}} \mathbf{H}_{\mathsf{A}}^* \right| \tag{20}$$

$$I_{\mathsf{B},2}^{(\mathrm{xor})} = \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{\mathsf{B}}^2} \mathbf{H}_{\mathsf{B}}^T \mathbf{\Lambda}_{\mathsf{R}} \mathbf{H}_{\mathsf{B}}^* \right|, \tag{21}$$

where $\mathbf{\Lambda}_{\mathsf{R}} = \mathrm{E}\{\mathbf{s}_{\mathsf{R}}\mathbf{s}_{\mathsf{R}}^H\}$ and $\mathrm{tr}(\mathbf{\Lambda}_{\mathsf{R}}) = P_{\mathsf{R}}$. By replacing $I_{\mathsf{A},2}^{(\mathrm{spc})}$ and $I_{\mathsf{B},2}^{(\mathrm{spc})}$ by $I_2^{(\mathrm{xor})}$ in (10), (11), (12), and (13) we obtain the rate region and sum-rate expressions for the XOR precoding scheme. Note that although the information rate in the second phase (i.e., $I_2^{(\mathrm{xor})}$) is the same for both directions, R_{AB} and R_{BA} do not have to be equal. If we choose $R_{\mathsf{AB}} \neq R_{\mathsf{BA}}$, the relay pads zeros to one bit-sequence (e.g., to $\mathbf{x}_{\mathsf{BA}}$ if $R_{\mathsf{BA}} < R_{\mathsf{AB}}$). Thus in the I_1 limited region XOR precoding can as well support different rate pairs R_{AB}, R_{AB} (e.g. for fairness issues).

3. TRANSMIT CSI AT THE RELAY

In the last section we have derived the rate expressions for two-way relaying without CSI at the nodes A and B. In this section we present how these rate expressions can be optimized (i.e., maximized) if the relay exploits the knowledge of the channel matrices to the nodes in the second phase.

3.1. Superposition Coding

In general there are two optimization goals, maximization of spectral efficiency (i.e., the sum-rate $R_{\mathsf{AB}} + R_{\mathsf{BA}}$) or providing fairness among users (i.e., maximization of minimum rate $\min\{R_{\mathsf{AB}}, R_{\mathsf{BA}}\}$). Here, we primarily focus on the optimization of sum-rate as defined in (13). It can be seen that for maximizing (13), we only can influence $I_{\mathsf{A},2}^{(\mathrm{spc})}$ and $I_{\mathsf{B},2}^{(\mathrm{spc})}$ as defined in (8) and (9), respectively. Both are only coupled via the constrained transmit power at the relay which is shared as $\mathrm{tr}(\mathbf{W}_{\mathsf{B}}\mathbf{\Lambda}_{\mathsf{AB}}\mathbf{W}_{\mathsf{B}}^H) = \alpha P_{\mathsf{R}}$ and $\mathrm{tr}(\mathbf{W}_{\mathsf{A}}\mathbf{\Lambda}_{\mathsf{BA}}\mathbf{W}_{\mathsf{A}}^H) = (1-\alpha)P_{\mathsf{R}}$ with $0 \le \alpha \le 1$. Assuming perfect transmit CSI at the relay it can be easily seen that the precoding matrices in (5) are given by the righthand eigenvectors $\mathbf{W}_{\mathsf{A}} = \mathbf{V}_{\mathsf{A}}$ and $\mathbf{W}_{\mathsf{B}} = \mathbf{V}_{\mathsf{B}}$ of the singular value decompositions of the channels $\mathbf{H}_{\mathsf{A}}^T = \mathbf{U}_{\mathsf{A}}\mathbf{\Sigma}_{\mathsf{A}}\mathbf{V}_{\mathsf{A}}^H$ and $\mathbf{H}_{\mathsf{B}}^T = \mathbf{U}_{\mathsf{B}}\mathbf{\Sigma}_{\mathsf{B}}\mathbf{V}_{\mathsf{B}}^H$, respectively. For each value of α the covariance matrices $\mathbf{\Lambda}_{\mathsf{AB}}$ and $\mathbf{\Lambda}_{\mathsf{BA}}$ are calculated via the waterfilling algorithm [11]; they are diagonal and describe the power loading across the spatial sub-channels to each node.

Thus, the maximization of (13) can be reduced to the problem of finding the optimum value of α. Fig. 2 shows an example of the rates changing over α for a specific realization of $\mathbf{H}_{\mathsf{A}}$ and $\mathbf{H}_{\mathsf{B}}$. Here the sum-rate (13) is not limited by I_1 in (12), and there exists a unique $\alpha^\star$ which maximizes (13), $\alpha^\star = 0.07$ in Fig. 2. For this $\alpha^\star$ only one pair of possible R_{AB} and R_{BA} exists. However, if the sum-rate is limited by I_1, R_{sum} is constant over a certain range of α. As α also determines R_{AB} and R_{BA}, multiple choices of the user rates are possible.

3.2. XOR Precoding

In the XOR precoding scheme the relay broadcasts common information (i.e. $\mathbf{x}_{\mathsf{R}}$ in (14)) to both nodes. The goal is to maximize (19) which is the minimum of the mutual information between the relay and both nodes subject to the transmit power constraint of the relay.

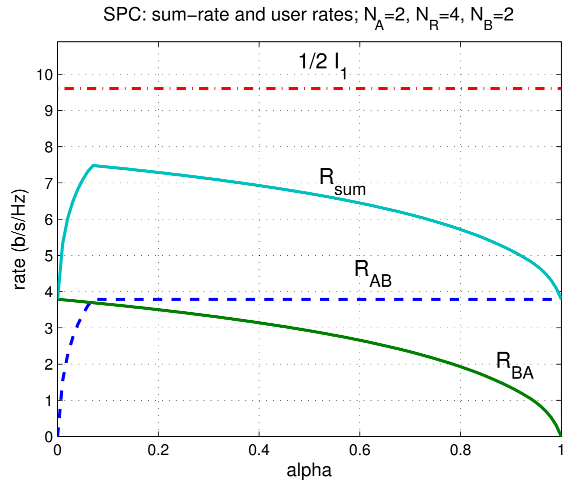

Fig. 2. Superposition Coding (SPC): Sum-rate and user rates for a specific realization of $\mathbf{H}_{\mathsf{A}}$, $\mathbf{H}_{\mathsf{B}}$ vs. α.

The optimization can be stated as

$$\begin{aligned} I_2^{(\mathrm{xor})\star} \quad &= \quad \max_{\mathbf{\Lambda}_{\mathsf{R}}} \ \min\left\{I_{\mathsf{A},2}^{(\mathrm{xor})}, I_{\mathsf{B},2}^{(\mathrm{xor})}\right\} \\ &\quad\ \text{subject to } \ \mathrm{tr}(\mathbf{\Lambda}_{\mathsf{R}}) = P_{\mathsf{R}} \\ &\quad\ \mathbf{\Lambda}_{\mathsf{R}} \succeq 0, \end{aligned} \tag{22}$$

where $\succeq$ denotes the positive semidefinite generalized inequality. This optimization problem is equivalent to the problem to determine the capacity of a Gaussian broadcast channel with common information [12], which is also denoted as *multicast* scenario. It can be seen that different to the superposition coding scheme we have to find one covariance matrix for two links, instead of one matrix for each link. Since $\mathbf{\Lambda}_{\mathsf{R}}$ has to be positive semidefinite, the optimization can be solved by *semidefinite programming* techniques [13, 14]. Note that in this case we do not have to consider the first hop rates during the optimization. This is not the case for the superposition coding scheme where (13) has to be evaluated for each α.

3.2.1. Suboptimal Beamforming for XOR Precoding

Although the optimal $\mathbf{\Lambda}_{\mathsf{R}}$ can be found by semidefinite programming, this optimization is still complex. In this section we present a suboptimal beamforming approach for $\mathbf{\Lambda}_{\mathsf{R}}$ which is inspired by superposition coding. However, instead of combining independent information we linearly combine common information. The derivation of this scheme gives also insights why the XOR precoding scheme performs better than the superposition coding scheme.

We assume that the covariance matrix $\mathbf{\Lambda}_{\mathsf{R}}$ in (20) and (21) has a predetermined structure given by $\mathbf{\Lambda}_{\mathsf{R}} = \mathbf{W}\breve{\mathbf{\Lambda}}\mathbf{W}^H$. The matrix $\mathbf{W}$ contains the righthand eigenvectors of the channel matrices between the relay and the nodes, i.e., $\mathbf{W} = [\mathbf{V}_{\mathsf{B}}\mathbf{V}_{\mathsf{A}}]$. Note that $\mathbf{V}_{\mathsf{A}}$ and $\mathbf{V}_{\mathsf{B}}$ have as many columns as the number of nonzero eigenvalues of the corresponding channels. The covariance matrix $\breve{\mathbf{\Lambda}}$ is assumed to be diagonal. This leads to transmit signals of the relay given by

$$\begin{aligned} \breve{\mathbf{s}}_{\mathsf{R}} &= \mathbf{W}\,\breve{\mathbf{s}} = \begin{bmatrix}\mathbf{V}_{\mathsf{B}} & \mathbf{V}_{\mathsf{A}}\end{bmatrix} \begin{pmatrix}\breve{\mathbf{s}}_{\mathsf{RB}} \\ \breve{\mathbf{s}}_{\mathsf{RA}}\end{pmatrix} \\ &= \mathbf{V}_{\mathsf{B}}\breve{\mathbf{s}}_{\mathsf{RB}} + \mathbf{V}_{\mathsf{A}}\breve{\mathbf{s}}_{\mathsf{RA}}, \end{aligned} \tag{23}$$

where the signal vectors $\breve{\mathbf{s}}_{\mathsf{RB}}$ and $\breve{\mathbf{s}}_{\mathsf{RA}}$ denote the contribution of the Gaussian codebook from the relay to node B and A, respectively. At the receivers, after multiplication by $\mathbf{U}_{\mathsf{A}}^{H}$ and $\mathbf{U}_{\mathsf{B}}^{H}$, respectively, the received signals are represented by

$$\breve{\mathbf{r}}_{\mathsf{A}} = \boldsymbol{\Sigma}_{\mathsf{A}}\breve{\mathbf{s}}_{\mathsf{RA}} + \boldsymbol{\Gamma}_{\mathsf{A}}\breve{\mathbf{s}}_{\mathsf{RB}} + \mathbf{n}_{\mathsf{A}} \quad (24)$$
$$\breve{\mathbf{r}}_{\mathsf{B}} = \boldsymbol{\Sigma}_{\mathsf{B}}\breve{\mathbf{s}}_{\mathsf{RB}} + \boldsymbol{\Gamma}_{\mathsf{B}}\breve{\mathbf{s}}_{\mathsf{RA}} + \mathbf{n}_{\mathsf{B}}, \quad (25)$$

where $\boldsymbol{\Gamma}_{\mathsf{A}} = \boldsymbol{\Sigma}_{\mathsf{A}}\mathbf{V}_{\mathsf{A}}^{H}\mathbf{V}_{\mathsf{B}}$ and $\boldsymbol{\Gamma}_{\mathsf{B}} = \boldsymbol{\Sigma}_{\mathsf{B}}\mathbf{V}_{\mathsf{B}}^{H}\mathbf{V}_{\mathsf{A}}$. Thus, the mutual information between the relay and the nodes is given by

$$\breve{I}_{\mathsf{A},2}^{(\mathrm{xor})} = \log_2\left|\mathbf{I} + \frac{1}{\sigma_{\mathsf{A}}^2}\left(\boldsymbol{\Sigma}_{\mathsf{A}}\breve{\boldsymbol{\Lambda}}_{\mathsf{RA}}\boldsymbol{\Sigma}_{\mathsf{A}}^{H} + \boldsymbol{\Gamma}_{\mathsf{A}}\breve{\boldsymbol{\Lambda}}_{\mathsf{RB}}\boldsymbol{\Gamma}_{\mathsf{A}}^{H}\right)\right| \quad (26)$$
$$\breve{I}_{\mathsf{B},2}^{(\mathrm{xor})} = \log_2\left|\mathbf{I} + \frac{1}{\sigma_{\mathsf{B}}^2}\left(\boldsymbol{\Sigma}_{\mathsf{B}}\breve{\boldsymbol{\Lambda}}_{\mathsf{RB}}\boldsymbol{\Sigma}_{\mathsf{B}}^{H} + \boldsymbol{\Gamma}_{\mathsf{B}}\breve{\boldsymbol{\Lambda}}_{\mathsf{RA}}\boldsymbol{\Gamma}_{\mathsf{B}}^{H}\right)\right|, \quad (27)$$

where $\breve{\boldsymbol{\Lambda}}_{\mathsf{RB}} = \mathrm{E}\{\breve{\mathbf{s}}_{\mathsf{RB}}\breve{\mathbf{s}}_{\mathsf{RB}}^{H}\}$ and $\breve{\boldsymbol{\Lambda}}_{\mathsf{RA}} = \mathrm{E}\{\breve{\mathbf{s}}_{\mathsf{RA}}\breve{\mathbf{s}}_{\mathsf{RA}}^{H}\}$.

When (8), (9) are compared with (26), (27), it is realized that the rate expressions of the proposed suboptimal beamforming scheme for XOR precoding has a similar structure as for SPC. In SPC, the mismatched information acting as self-interference is cancelled, so as the power reduces. On the other hand, the proposed scheme yields an additional power gain arising from the mismatched part because of the inequality on determinants $|\mathbf{A}+\mathbf{B}| \geq |\mathbf{A}|$ for any $(m \times m)$ positive definite $\mathbf{A}$ and $(m \times m)$ positive semidefinite $\mathbf{B}$ [15]. Thus, assuming the same power loading as for SPC, i.e. $\breve{\boldsymbol{\Lambda}}_{\mathsf{RA}} = \boldsymbol{\Lambda}_{\mathsf{BA}}$ and $\breve{\boldsymbol{\Lambda}}_{\mathsf{RB}} = \boldsymbol{\Lambda}_{\mathsf{AB}}$, the mismatched part always contributes in terms of power which assures that the proposed suboptimal XOR precoding scheme performs always better or at least equal to SPC if (19) has to be maximized. Hence, we conclude that this suboptimal XOR beamforming scheme is an upper bound on the performance of SPC with respect to the maximal minimum user rate. However, in the performance section we show that suboptimal XOR beamforming also achieves higher sum-rates than SPC.

Note that the use of the covariance matrices of SPC is not necessarily optimal. As in (22), the maximal rate in the second phase with the suboptimal XOR beamforming approach can be optimized (i.e. maximized) with the following opimization problem

$$\max_{\breve{\boldsymbol{\Lambda}}_{\mathsf{RA}},\breve{\boldsymbol{\Lambda}}_{\mathsf{RB}}} \min\left\{\breve{I}_{\mathsf{A},2}^{(\mathrm{xor})}, \breve{I}_{\mathsf{B},2}^{(\mathrm{xor})}\right\}$$
$$\text{subject to } \operatorname{tr}(\breve{\boldsymbol{\Lambda}}_{\mathsf{RB}}) + \operatorname{tr}(\breve{\boldsymbol{\Lambda}}_{\mathsf{RA}}) = P_{\mathsf{R}}$$
$$\breve{\boldsymbol{\Lambda}}_{\mathsf{RA}}, \breve{\boldsymbol{\Lambda}}_{\mathsf{RB}} \succeq 0, \quad (28)$$

which is convex and can be modeled as a semidefinite programme.

4. PERFORMANCE RESULTS

In the simulations we consider frequency-flat block fading between all nodes; the elements of the channels $\mathbf{H}_{\mathsf{A}}$ and $\mathbf{H}_{\mathsf{B}}$ are i.i.d. Rayleigh fading coefficients with zero mean and variance $\sigma_{H_{\mathsf{A}}}^2$ and $\sigma_{H_{\mathsf{B}}}^2$, respectively. We assume the channels to be constant over the two phases of the two-way relaying scheme. Nodes A and B as well as the relay, use the same transmit power, i.e. $P_{\mathsf{A}} = P_{\mathsf{B}} = P_{\mathsf{R}} = P$. At the nodes and the relay we assume circular symmetric complex Gaussian noise contributions with zero mean; in the simulations we consider the case where all nodes have the same noise variance, i.e. $\sigma_{\mathsf{A}}^2 = \sigma_{\mathsf{B}}^2 = \sigma_{\mathsf{R}}^2 = \sigma^2$. Based on these assumptions we define $\mathrm{SNR}_1 = (P/\sigma^2)\sigma_{H_{\mathsf{A}}}^2$ and $\mathrm{SNR}_2 = (P/\sigma^2)\sigma_{H_{\mathsf{B}}}^2$.

In Fig. 3 the average sum-rate vs. number of relay antennas N_{R} is shown for SPC, XOR and the suboptimal XOR beamforming approach (cf. section 3.2.1) with and without CSIT; the number of antennas at node A and B is fixed to $N_{\mathsf{A}} = N_{\mathsf{B}} = 2$ and

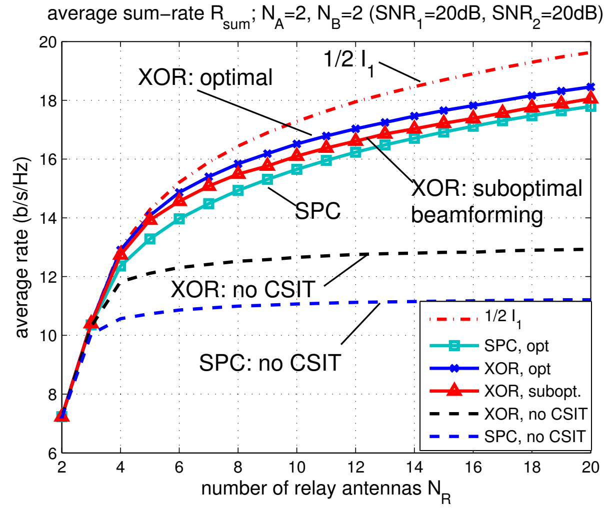

Fig. 3. XOR and SPC with and without waterfilling: average sum-rate R_{sum} vs. number of relay antennas N_{R}.

$\mathrm{SNR}_1 = \mathrm{SNR}_2 = 20\mathrm{dB}$. In the case that all three nodes have 2 antennas there is no significant differences between the SPC and XOR schemes - neither with nor without CSIT. As the $1/2 I_1$ curve shows, this is mainly due to the fact that the rates are limited by I_1 (cf. (4) and (12)) in the first phase of the two-way relaying scheme. For $N_{\mathsf{R}} = 4$ antennas at the relay the SPC scheme without CSIT shows a significantly lower sum rate than the other schemes. This is due to the fact that in this case $\alpha = 1/2$ is chosen, i.e. power $P/2$ is used for $\mathbf{s}_{\mathsf{AB}}$ and $P/2$ for $\mathbf{s}_{\mathsf{BA}}$. The scheme "XOR without CSIT" transmits the same information to both nodes A and B using power P, cf. (14); the difference in sum-rate between these two schemes is due to this 3dB power gain. The sum-rate differences between the schemes without CSIT and the schemes with CSIT increase with an increasing number of relay antennas N_{R}. The XOR scheme with CSIT shows the highest sum rate, followed by the suboptimal XOR beamforming approach, which converges for a large number of relay antennas to the sum-rate of "SPC with CSIT": the contribution of the third summand in (26) and (27) decreases compared to the contribution of the second summand in these equations, because the cross-talk contributes less than the part of the transmission that is coherently combined by the beamforming. This follows from the observation that the relevant eigenvectors in $\mathbf{V}_{\mathsf{A}}$ and $\mathbf{V}_{\mathsf{B}}$ with high probability are quasi-orthogonal for large N_{R}.

In Fig. 4 we study the impact of unbalanced link quality, i.e. $\mathrm{SNR}_1 = 20\mathrm{dB}$ and $0\mathrm{dB} \leq \mathrm{SNR}_2 \leq 40\mathrm{dB}$. The average sum-rates are given for a scenario with $N_{\mathsf{A}} = N_{\mathsf{B}} = 2$ antennas at the nodes A and B and $N_{\mathsf{R}} = 4$ antennas at the relay. We compare SPC, the two XOR schemes and the conventional (one-way) relaying scheme; the sum-rates are calculated with the use of CSIT at the relay and without. The sum-rate of the conventional relaying scheme is calculated by averaging over four hops: from node A over the relay to node B and vice versa. The differences between the two-way relaying schemes are small for the case where the relay uses CSIT. Without CSIT the XOR scheme shows an advantage for $13\mathrm{dB} \leq \mathrm{SNR}_2 \leq 27\mathrm{dB}$. Generally, all two-way relaying schemes with CSIT almost double the sum-rate of the conventional relaying scheme with CSIT in this scenario; they are able to avoid the pre-log factor $1/2$. Even without CSIT, XOR and SPC outperform the

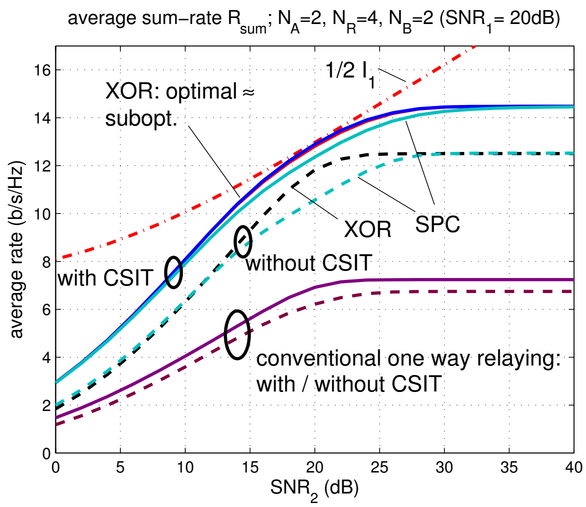

Fig. 4. Average sum-rates with and without CSIT: two-way relaying using XOR and SPC compared to conventional (one-way) decode and forward relaying.

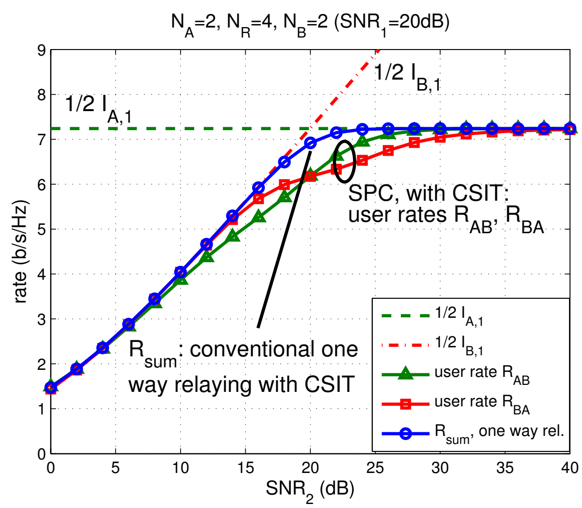

Fig. 5. Average user rates for SPC compared to the sum rate for conventional (one-way) relaying.

conventional scheme with CSIT, too.

Fig. 5 shows for the same scenario as Fig. 4 a comparison of the SPC user rates and the sum-rate of conventional relaying; in addition $1/2I_{\mathrm{A},1}$ and $1/2I_{\mathrm{B},1}$ from (10) and (11) are given, too. In the considered scenario the user rates differ only slightly; the second interesting observation is that both SPC user rates are not far from the sum-rate of the conventional scheme.

5. CONCLUSIONS

In this work we extended the two-way relaying scheme of [6] to the case of multiple antennas at all nodes. We studied the impact of transmit CSI at the relay, which is used for precoding. We showed that two-way relaying achieves a quite substantial improvement in spectral efficiency compared to conventional relaying with and without transmit CSI at the relay. We showed that the difference in sum-rate compared to the case where no CSIT is used, increases with increasing ratio between number of relay antennas and number of node antennas. We conclude that XOR precoding always achieves higher minimum user rates than SPC if CSIT is used, since already the proposed suboptimal beamforming is an upper bound on the performance of SPC. Furthermore, in our considered simulation scenarios, XOR precoding achieved always higher sum-rates than SPC, too.

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