

# MIMO Signaling for Low Rank Channels

A. Wittneben, B. Rankov

Swiss Federal Institute of Technology (ETHZ), Communication Technology Laboratory, Sternwartstr. 7,  
CH-8092 Zurich, Switzerland; [wittneben@nari.ee.ethz.ch](mailto:wittneben@nari.ee.ethz.ch)

**Abstract:** Future Wireless LANs will consist of heterogeneous nodes and will require more than 1Gbps peak throughput between nodes. This makes spatial multiplexing (MIMO) technology indispensable to achieve the required bandwidth efficiency. Due to available spectrum future WLANs will operate beyond 2.4GHz. Increased path loss and poor scattering are major problems for MIMO beyond 2.4GHz. We discuss the basic tradeoffs and outline the paradigm shift from “rich scattering/poor array” to “poor scattering/rich array” as the carrier frequency increases. We show, that (i) a two-hop traffic pattern, which involves an access point with distributed antennas, solves the poor scattering/rich array problem and (ii) linear processing at the access point suffices in many practical cases. Finally some aspects of the source/destination antenna array design are addressed.

## I. INTRODUCTION

Wireless communication systems enable tetherless communication between a variety of nodes ranging from humans to computers. They may roughly be classified by their geographical coverage area. Cellular systems provide global coverage, international roaming and unconstrained mobility. Wireless access systems in contrast support locally constrained tetherless connectivity and mobile access to a backbone network (typically the Internet). The slow take-off of UMTS as opposed to the explosive growth of 802.11 based Wireless Local Area Networks (WLAN) indicates the growing importance of wireless access. Currently most WLAN nodes are portable computers, which makes throughput the major QoS parameter. For this reason the application of Multiple Input/Multiple Output (MIMO) technology [1] to IEEE 802.11 has attracted considerable attention. It is expected, that future WLANs will be used by a more heterogeneous range of nodes. Spatial multiplexing is indispensable to achieve the required peak throughput and it provides scalability with acceptable hardware (IC) reuse across nodes. Thus a future WLAN has to integrate nodes with varying antenna array size. Due to the ubiquitous deployment we expect a much higher node density than in current WLANs. The availability of bandwidth makes operation beyond 2.4GHz mandatory. Spatial multiplexing requires rich scattering. The increased path loss and the poor scattering are major problems for MIMO beyond 2.4GHz. In Section II we discuss the basic tradeoffs and outline the paradigm shift from “rich scattering/poor array” to “poor scattering/rich array” as the carrier frequency increases. In Section III we consider the communication between a MIMO source and destination for (i) a peer-to-peer traffic pattern and (ii) a two-hop traffic pattern with a distributed antenna system (DAS). Our results are based on a propagation environment without scattering and we conclude, that DAS efficiently solves the poor scattering/rich array problem.

## II. THE RICH ARRAY/POOR SCATTERING PARADIGM

To evaluate the potential of MIMO signaling beyond 2.4GHz we consider the radio link from a source to a destination at distance  $d_0$ . The antenna arrays in both nodes occupy a given area  $A$  (fixed physical dimensions). The antenna elements are spaced at a minimum distance of  $\lambda/2$ . To estimate the possible number of antenna elements in  $A$  as a function of the carrier frequency  $f$  we impose a circular dead zone with radius  $\lambda/4$  around each element. Let  $N_{a,0}$  be the number of antenna elements at the reference frequency  $f_0 = 2.4\text{GHz}$ . Then the number  $N_a$  of antenna elements at frequency  $f$  is approximately given by

$$N_a \approx N_{a,0} \cdot \left(f/f_0\right)^2 \equiv N_{a,0} \cdot f_N^2 \quad \text{with} \quad N_{a,0} = \left\lfloor 16 \cdot A / (\pi \cdot \lambda_0^2) \right\rfloor \quad (1)$$

Let  $\gamma$  denote the path loss exponent and  $d_0$  the reference distance. Then the path gain under a path loss channel model is given by

$$x_{\text{PL}} = \left(\lambda / (4\pi \cdot d_0)\right)^2 \cdot G_{\text{TX}} \cdot G_{\text{RX}} \cdot x_{\text{PL},0} \cdot \left(d/d_0\right)^{-\gamma} \quad (2)$$

$x_{\text{PL},0}$  is the excess path gain at the reference distance  $d_0$  and  $G_{\text{TX}}/G_{\text{RX}}$  is the transmit/receive antenna gain. Note that for a given distance the path gain is always proportional to  $\lambda^2$  even if  $\gamma \neq 2$ . We want to study the impact of the carrier frequency on the capacity of a wireless link for a given source-destination distance  $d_0$ . The path loss between source and destination thus is given by

$$x_{\text{PL}} = \left(\lambda / (4\pi \cdot d_0)\right)^2 \cdot G_{\text{TX}} \cdot G_{\text{RX}} \cdot x_{\text{PL},0} \equiv b^2 / f_N^2 \quad (3)$$

**Ergodic capacity:** we express the  $(N_a \times N_a)$  source-destination channel matrix  $H_{\text{SD}}$  as the product of a i.i.d. random matrix  $H_{\text{SD},N}$  and the path gain from Eq. (3)

$$H_{\text{SD}} = H_{\text{SD},N} \cdot b / f_N \equiv H_{\text{SD},N} / f_N \quad (4)$$

For simplicity we will assume in the sequel  $b=1$  and incorporate the reference path loss in the noise variance. The elements of  $H_{\text{SD},N}$  have unit power. Without scattering  $H_{\text{SD},N}$  is equivalent to the all-one-matrix; for the rich scattering case (Rayleigh fading) the elements are  $\text{CN}(0,1)$ . We assume a source transmit power  $P_s$ . The AWGN at each

destination antenna element has variance  $\sigma_w^2$ . Let  $\{\sigma_{SD}^{(k)}\}$  be the singular values of  $H_{SD}$ . Without channel state information (CSI) at the source the instantaneous capacity per complex dimension thus follows as [4]

$$C_{SD}(H_{SD}) = \sum_{k=1}^{N_a} \log_2 \left( 1 + (P_s / N_a) \cdot (\sigma_{SD}^{2(k)} / \sigma_w^2) \right) \quad (5)$$

The ergodic capacity is the mean of the instantaneous capacity

$$C_{SD} = E_{H_{SD}} [C_{SD}(H_{SD})]$$

**No scattering:** a  $(N_a \times N_a)$  all-one-matrix has one nonzero singular value  $\sigma_{SD,N}^{(1)} = N_a \Rightarrow \sigma_{SD}^{(1)} = N_a / f_N$ . With Eqs. (1) and (5) the ergodic capacity follows as

$$C_{SD} = \log_2 \left( 1 + P_s \cdot N_{a,0} / \sigma_w^2 \right)$$

It is independent of the frequency, because the destination array gain compensates the increasing path loss.

**Rich scattering:** let  $\{\sigma_{SD}^{(k,m)}\}$  be the  $N_a$  singular values of a realization  $H_{SD}^{(m)}$  of  $H_{SD}$ . Without loss of generality we consider each  $\sigma_{SD}^{(k,m)}$  as independent realization of a single random variable  $\sigma$  and obtain the ergodic capacity

$$C_{SD}(N_a) = N_a \cdot E_{\sigma} \left[ \log_2 \left( 1 + (P_s / N_a) \cdot (\sigma^2 / \sigma_w^2) \right) \middle| N_a \right]$$

In terms of the singular values  $\sigma_N$  of a normalized  $(N_a \times N_a)$  i.i.d. random matrix with elements  $CN(0, 1/N_a)$  we obtain with Eq. (1)

$$C_{SD}(N_a) = f_N^2 \cdot N_{a,0} \cdot E_{\sigma_N} \left[ \log_2 \left( 1 + (P_s / f_N^2) \cdot (\sigma_N^2 / \sigma_w^2) \right) \middle| N_a \right] \quad (6)$$

For  $N_a \geq 4$  the probability density function (pdf) of  $\sigma_N$  depends only marginally on  $N_a$ . On this basis Eq. (6) implies, that the spatial degrees of freedom increase with the square of the frequency. On the other side the source has to distribute the transmit power uniformly across the spatial subchannels and the subchannel SNR decreases accordingly. With the area  $A$  of the antenna array (Eq. (1)) we obtain asymptotically for  $f_N \rightarrow \infty$  (compare to [6])

$$C_{SD}^{\infty} = (N_{a,0} / \ln 2) \cdot (P_s / \sigma_w^2) \approx A \cdot (P_s / \sigma_w^2) \cdot 16 / (\pi \cdot \lambda_0^2 \cdot \ln 2)$$

In Fig. 1 we give an example of the ergodic capacity versus the normalized frequency for  $N_{a,0} = 4$  and  $P_s / \sigma_w^2 = 10$ . There is a huge difference between the capacity in a rich scattering and a poor scattering environment. Note that for  $f_N = 8$  the channel has  $f_N^2 \cdot N_{a,0} = 8^2 \cdot 4 = 256$  degrees of freedom, i.e. we would need 256 relevant scatterers to achieve the rich scattering capacity. This illustrates a major problem of MIMO at higher frequencies: indoor measurements around 2GHz and 5GHz typically reveal less than 10 relevant scatterers (maximum rank of channel matrix). This number is not expected to increase with the frequency, as the channel becomes increasingly Line-of-Sight. The curve “reality” in Fig. 1 gives a conjecture of the achievable capacity in a realistic propagation environment. With increasing carrier frequency we observe a paradigm shift from “*poor array/rich scattering*” to “*rich array/poor scattering*”. This is a fundamental problem for future WLANs. In the next section we show, that distributed antenna systems may solve this problem.

### III. DISTRIBUTED ANTENNA SYSTEMS AND MIMO

A distributed antenna system (DAS) [2,3] consists of spatially distributed antenna elements (including radio front end), which are connected to a central processor. An efficient implementation may be based on a fibre backbone with radio over fibre. Inside buildings powerline communication is an interesting alternative to connect the distributed radio front ends. In this section we consider a WLAN installation in a  $(12.5m \times 12.5m)$  square room. The MIMO source and destination have  $N_a$  antenna elements as described in the previous section. The antenna elements of the DAS are distributed uniformly along the perimeter of the room [5]. A transmission cycle from the source to the destination requires two channel uses. In the first slot the source transmits to the DAS (uplink). In a decode DAS (DDAS) the central processor decodes the signal prior to retransmission to the destination (downlink). In a linear DAS (LDAS) the central processor performs only linear operations (combining, amplification and beamforming) on the received signal. In contrast to DDAS the LDAS approach is transparent to the source modulation alphabet and thus is readily combined with link adaptation. In both cases we assume (i) no CSI at the source, (ii) perfect uplink CSI at the DAS and (iii) perfect CSI at the destination. For DDAS we assume no downlink CSI at the DAS; the LDAS however has perfect downlink CSI. It performs downlink eigen-beamforming with equal transmit power on each beam. Our LDAS results are a lower bound on the capacity, as they are based on a particular gain allocation and beamforming scheme. Details are out of the scope of this paper.

In Figs. 2, 3 and 4 we present results on the 10% outage capacity per channel use for different system parameters. For each parameter set we consider 1000 uniformly distributed random locations of source and destination in the square room. As we assume low node mobility, the outage capacity is a more appropriate performance measure than the ergodic capacity. For reference we show the capacity of a peer-to-peer (p2p) traffic pattern (direct transmission from source to destination). In contrast to the DAS case this traffic pattern requires only one channel use per transmission cycle. All cases are based on the same average transmit power per channel use; i.e. the sum of source and DAS transmit power (two channel uses) is two times the source transmit power in the p2p case (one channel use). To emphasize the impact of the propagation environment, we have not considered antenna coupling in this exposition. We consider a path

loss channel model with exponent  $\gamma = 2$  (Eq. (2)) and no scattering. Thus the p2p traffic pattern yields no spatial multiplexing gain and we consider an extreme rich array/poor scattering scenario. In Fig. 2 the DAS has 16 elements and we consider the outage capacity as a function of the number elements of source/destination uniform linear arrays (ULA). Due to the path loss channel model (no scattering), the p2p scenario does not achieve a spatial multiplexing gain. The moderate capacity increase is due to the array gain of the destination antenna. In contrast both LDAS and DDAS achieve a substantial spatial multiplexing gain. LDAS outperforms DDAS for  $N_a < 8$ , because it exploits downlink CSI. Fig. 2 characterizes the benefits of LDAS/DDAS in a network with heterogeneous nodes. In Fig. 3 we study the impact of the element spacing of the source/destination ULA. Source, destination and the DAS have 16 elements each. As we do not consider antenna coupling, the array gain of the destination antenna is independent of the spacing and we would expect the p2p capacity to be independent of the spacing. The moderate increase is due to a residual spatial multiplexing gain, which increases with the antenna spacing. Similar to a rich scattering scenario, with LDAS/DDAS the capacity saturates at  $d_a/\lambda = 0.5$ . Note the graceful degradation below this value. Even at a very close element spacing the system achieves a substantial capacity improvement. We conclude, that very compact source/destination antenna arrays are feasible in the DAS scenarios. In Fig. 4 we state the outage capacity versus the carrier frequency for given physical dimensions of the source/destination antenna arrays. At the reference frequency, source, destination and DAS have 4 antenna elements each. As outlined in Section II the number of elements increases proportionally to the square of the normalized frequency. A comparison of Fig. 1 and 4 shows, that DAS is able to solve the rich array/poor scattering problem. As the channel model has no scattering, the p2p traffic pattern compares to the “no scattering” curve in Fig. 1.

## REFERENCES

- [1] D. Gesbert, M. Shafi, D. S. Shiu, P. Smith, and A. Naguib, “From theory to practice: An overview of MIMO space-time coded wireless systems,” *IEEE Journal on Selected Areas in Communications*, vol. 21, April 2003.
- [2] M. V. Clark et al, “Distributed Versus Centralized Antenna Arrays in Broadband Wireless Networks,” *IEEE Vehicular Technology Conference VTC Spring 2001*.
- [3] W. Roh, A. Paulraj, “Outage performance of the distributed antenna systems in a composite fading channel,” *IEEE Vehicular Technology Conference, VTC Fall 2002*.
- [4] G. Foschini, M. J. Gans, “On limits of wireless communication in a fading environment when using multiple antenna arrays,” *Wireless Personal Communication*, vol. 6, pp. 311-335, March 1998.
- [5] A. Wittneben, B. Rankov, “Distributed Antenna Arrays versus Cooperative Linear Relaying for Broadband Indoor MIMO Wireless,” *International Conference on Electromagnetics in Advanced Applications*, Conference Proceedings, Torino, Italia, September 2003.
- [6] Gesbert, D.; Ekman, T.; Christophersen, N., "Capacity limits of dense palm-sized MIMO arrays," *Global Telecommunications Conference, 2002. GLOBECOM '02. IEEE*, Volume: 2, Nov 17 - 21, 2002

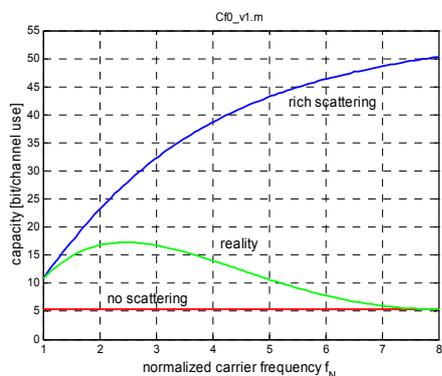


Fig. 1: Ergodic capacity versus the normalized carrier frequency for different propagation models.

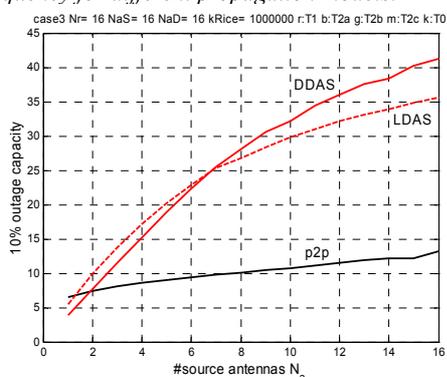


Fig. 2: 10% outage capacity versus the number  $N_a$  of source/destination antenna elements. DAS: 16 elements.

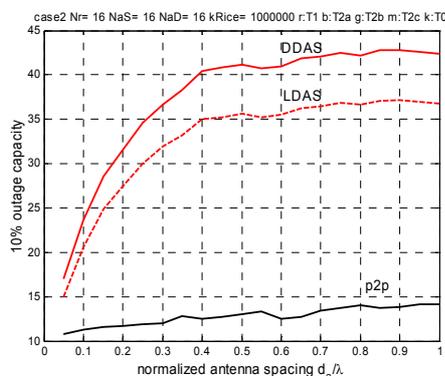


Fig. 3: 10% outage capacity versus the normalized spacing of the source and destination antenna (ULA)

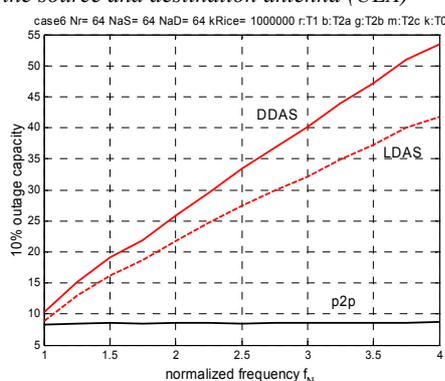


Fig. 4: 10% outage capacity versus the normalized carrier frequency (element spacing  $d_a/\lambda = 0.5$ , s.f. Section II).