A new concatenated linear high rate space-time block code

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Abstract - **Future wireless communication systems are characterized by heterogeneity in terms of node capabilities, link level requirements, propagation environments, cell designs and subnet structures. High carrier frequencies and large system bandwidth will make extreme diversity factors feasible. This imposes new requirements on space-time codes. In this paper we present a new concatenated linear space-time block coding scheme, which meets these requirements. The code consists of a linear time variant inner code and a linear time invariant outer code. We intentionally decouple the designs of each constituent code in order to achieve the required adaptivity. The inner code is optimized with respect to the variation of the instantaneous channel capacity over the block length of the outer code. Efficient inner coding matrices are given for pure TX diversity and joint spatial subchanneling and TX diversity. The linear outer code is optimized for diversity performance. We describe an efficient approach for the optimization of an unitary coding matrix for any given block length. Simulation results for different system parameters are presented. It is concluded that the proposed concatenated linear high rate space-time block coding scheme is a promising candidate for heterogeneous future mobile communication systems.**

I. INTRODUCTION

In mobile communications diversity can be achieved by multiple antenna arrays. The most obvious approach to diversity are spatially separated antennas. For this reason signal processing methods, that utilize diversity, are commonly referred to as space processing. Recently coding schemes have been devised, which jointly utilize the spatial and temporal dimension [1, 4]. These schemes are known as space-time processing/coding schemes. The incorporation of the diversity dimension helps to cope with some fundamental issues of wireless communications: signal fading, throughput and cochannel interference (CCI). The performance of space-time processing/coding depends to a large extend on the a priori channel knowledge at the transmitter. If the channel matrix is known a priori at the transmitter, we can achieve full TX diversity and (in the case of multiple RX antennas) realize orthogonal spatial subchannels by appropriate TX beamforming. CCI nulling is possible for arbitrary angular spread. Without a priori channel knowledge at the transmitter, we need to consider the spatial and the temporal domain in order to achieve TX diversity [1]. It is possible to utilize spatial subchanneling without TX channel knowledge and without incorporating the temporal domain [2]. This introduces block intersymbol interference at the receiver. Without utilization of the temporal domain it is not possible to jointly implement TX diversity and spatial subchanneling.

Future wireless communication systems will feature increasing carrier frequencies. This makes multi-antenna arrays with an acceptable form factor feasible. The required data rate and system bandwidth will increase. This introduces frequency diversity because the channel exhibits large propagation delay differences. Future networks will have a heterogeneous network structure and propagation environment. A network may include ad-hoc subnets, a multiplicity of cell designs (pico- to macro-cells; overlay cells) and basestation elevations and the network may be designed jointly for indoor and outdoor applications. The communication nodes will have heterogeneous capabilities in terms of signal processing power, number of RX and TX antennas etc.. The networks furthermore will have to cope with heterogeneous requirements to the quality of service (data rate, reliability). This imposes some new requirements on future space-time methods:

- ability to utilize large and potentially huge diversity.
- ability to continuously adapt between diversity mode, spatial subchannel mode and CCI nulling mode subject to the capabilities of the communication nodes, the propagation conditions, the required quality of service, the multiuser scenario and available (partial) a-priori channel knowledge at TX
- ability to utilize TX-diversity in the spatial subchannel mode because the basestation may feature a very large number of antennas

Known space-time codes fulfill these requirements only partially. New space-time coding/decoding schemes are required. This is the goal of the new linear high rate space-time block codes described in this paper.

II. BASIC APPROACH TO CODE CONSTRUCTION

Fig. 1 illustrates our approach to the code construction. The code consists of a concatenation of a <u>time variant</u> inner linear code, <u>time invariant</u> outer linear code and an optional FEC code. We intentionally decouple the designs of each constituent code in order to facilitate the adaptation to the heterogeneous requirements, node capabilities, channel conditions and levels of a priori channel knowledge. We constrain our attention to *linear* inner and outer codes, because this allows us to preserve the mean capacity of the MIMO channel X_v and enables a variety of low complexity decoders. An additional benefit is the extremely simple receiver structure in the limiting case of a diagonal channel matrix.

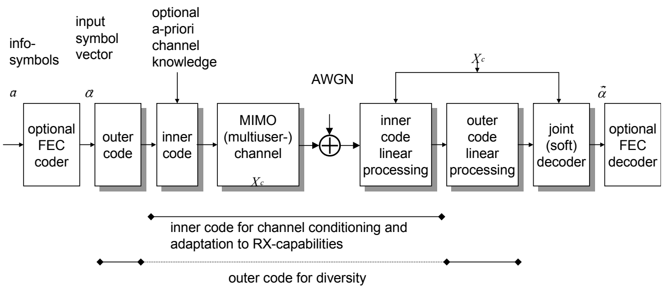

Fig. 1: Approach to code construction

Some objectives of the code design are

- outer code decoupled of channel conditions (e. g. antenna correlation), a priori channel knowledge and TX/RX capabilities
- use the inner code for adaptation to channel conditions, a priori channel knowledge and TX/RX capabilities

- decouple Forward Error Correction (FEC) design from channel conditions. This is facilitated by a linear code and soft decision joint decoder.

The complexity of the joint decoder is essential to enable large block length. This further motivates the use of linear codes as a variety of intersymbol interference decoders are known. They offer a wide tradeoff between complexity and performance. The additional FEC is optional and out of the scope of this paper.

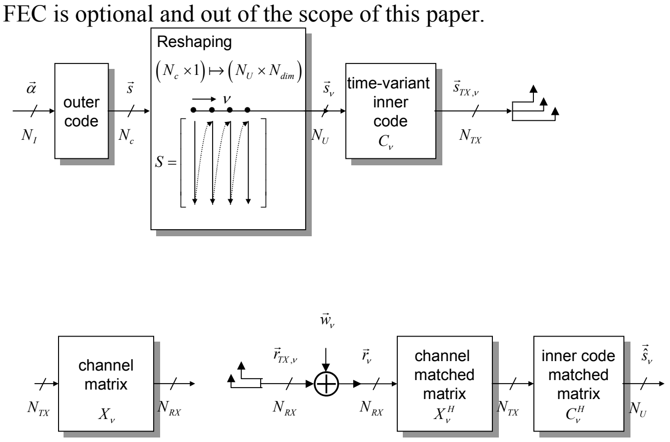

Fig. 2: Symbol discrete time series representation of the concatenated coding scheme

Fig. 2 gives a time series representation of the coding scheme and defines important signal and parameters. The input symbol vector $\vec{\alpha}$ is multiplied with the outer code matrix R to form the transmit symbol vector $\vec{s}$. The dimensions of both symbol vectors determine the code rate. Note that a simple modification of the code rate is possible by adding/deleting columns of the outer code matrix. Due to the interdependence of code rate and required decoder complexity this feature is highly useful in networks with heterogeneous node capabilities. The transmit symbol vector is reshaped into a $\left(N_U \times N_{dim}\right)$ matrix. The columns of this matrix yield the consecutive N_c / N_U input vectors $\vec{s}_\nu$ of the linear time variant inner code C_ν . ν is the time index. The codewords $\vec{s}_{TX,\nu}$ of the inner code are transmitted through the Multiple Input Multiple Output (MIMO) channel X_ν. Typically X_ν is defined by the channel coefficient between each pair of TX and RX antennas (flat fading). Throughout the paper we will assume independent and identically distributed (i.i.d.) complex normal coefficients of X_ν (Rayleigh fading). Note that the channel matrix X_ν may be „time-variant" (time-variant channel; different OFDM subcarriers etc.). The receive vector $\vec{r}_{TX,\nu}$ is perturbed by additive white Gaussian noise $\vec{w}_\nu$. Multiplication with the channel matched matrix X_ν^H and the inner code matched matrix C_ν^H yields a sufficient statistics $\vec{\hat{s}}_\nu$ for the estimation of the inner code input vector $\vec{s}_\nu$. The system transmits N_U symbols in one temporal dimension. We refer to N_U as the number of „spatial subchannels" to be used. If the channel matrix is known at the transmitter, the available orthogonal spatial subchannels would be given by the singular values of the channel matrix. We use the term spatial subchannels, because the orthogonal transmit channels are obtained by TX beamforming. In this paper however we assume no channel knowledge at the TX (unknown channel case).

III. CONSTRUCTION OF INNER CODE

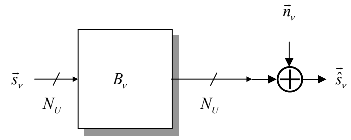

Fig. 3: Equivalent time-variant MIMO system

Inner code, channel and matched matrices form a time-variant $\left(N_U \times N_U\right)$-MIMO-system B_ν :

$$B_\nu = C_\nu^H X_\nu^H X_\nu C_\nu$$

The noise is correlated and has the autocorrelation matrix

$$\Lambda_{nn,\nu} = B_\nu \cdot \sigma_w^2$$

Note that noise vectors with different time indices ν are statistically independent. We will assume in the sequel that B_ν is perfectly known at the RX .

A. *Pure TX Diversity*

In pure TX-diversity the receiver utilizes one antenna and the channel matrix deflates to a channel vector

$$N_{RX} = 1 \quad \Rightarrow \quad X_\nu \mapsto \vec{x}_\nu^H$$

Consequently spatial subchanneling is not possible. With Fig. 2 we obtain

$$N_U = 1 \quad \Rightarrow \quad C_\nu \mapsto \vec{c}_\nu \,; \vec{s}_\nu \mapsto s_\nu \,; \vec{\hat{s}}_\nu \mapsto \hat{s}_\nu$$

and B_ν deflates to a scalar. In time-invariant fading $\vec{x}_\nu^H \mapsto \vec{x}^H$ the equivalent matrix follows as

$$B_\nu = \vec{c}_\nu^H \cdot \vec{x} \cdot \vec{x}^H \cdot \vec{c}_\nu = \left\| \vec{x}^H \cdot \vec{c}_\nu \right\|^2$$

An equivalent system model with i.i.d. noise is shown in Fig. 4.

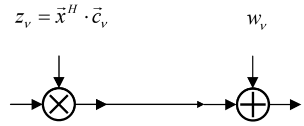

Fig. 4: Equivalent system model.

If B_ν is known at the transmitter (known channel case), the optimal inner coding vector follows as

$$\vec{c}_\nu = \frac{\vec{x}}{\left\|\vec{x}\right\|} \quad \Rightarrow \quad z_\nu = \left\|\vec{x}\right\|$$

Here z_ν is time-invariant. A diversity gain is obtained because z_ν is not normally distributed. The outer code is not required to utilize the diversity gain. Note that the norm $\left\|\vec{x}\right\|$ rather than the individual elements of the channel vector determine the system performance.

In the unknown channel case the inner code vector is independent of the current realization of the channel vector. When viewed over all channel realizations, z_ν is a complex normal random variable for any ν. If $\vec{c}_\nu$ is time-invariant, i.e. $\vec{c}_\nu = \vec{c}$, no TX-diversity is achieved, because all transmit symbol s_ν are affected by the same normal fading variable z.

$$\tilde{s}_\nu = z \cdot s_\nu + w_\nu$$

If $\vec{c}_\nu$ is time-variant, different transmit symbols are affected by different fading variables. A diversity gain is possible but requires an appropriate outer code. If the coefficients of the channel vector are i.i.d. we obtain a sequence of N_{TX} statistically independent equivalent fading coefficients z_ν by choosing a set of N_{TX} orthonormal inner code vectors $\left\{\vec{c}_\nu \,\middle|\, 1 \le \nu \le N_{TX}\right\}$. A simple choice is a sequence of unit vectors. For $N_{TX} = 4;\quad N_{dim} = 8$:

$$\left\{\vec{c}_\nu\right\} = \left\{ \begin{bmatrix}1\\0\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \begin{bmatrix}0\\0\\0\\1\end{bmatrix} \begin{bmatrix}1\\0\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \begin{bmatrix}0\\0\\0\\1\end{bmatrix} \right\} \tag{1}$$

In this case the TX-antennas are switched and the equivalent fading coefficients are time-variant. As a result the outer code has to be optimized for diversity gain in time-variant fading. The (unknown channel) TX-diversity transforms the system with multiple TX-antennas into an equivalent system with time-variant fading z_ν.

In the known channel case the system performance is determined by $\|\vec{x}\|$. In contrast in the unknown channel case for a given $\|\vec{x}\|$ there are

- critical channel realizations, e.g. (TX-antenna switching)

  $\vec{x}^H = [1,0,0,0] \quad \Rightarrow \quad z_\nu = \ldots,1,0,0,0,1\ldots$

  The outer code has to cope with $3/4$ erasures.
- uncritical channel realizations, e.g.

  $\vec{x}^H = \left[\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2}\right] \quad \Rightarrow \quad z_\nu = \ldots,\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},\ldots$

  The outer code perceives an AWGN channel.

This observation is more precisely reflected in the instantaneous capacity per complex dimension of the equivalent system in normalized fading (i.e. $\|\vec{x}\| = const.$). Fig. 5 shows the cumulative density function (CDF) of the instantaneous capacity for the parameters: $\|\vec{x}\|^2 = 4;\ \sigma_w^2 = 0.1;\ N_{TX} = 4;\ N_{dim} = 8$.

In the known channel case the capacity is independent of the current realization of the channel vector and the CDF is given by the right vertical line in Fig. 5. The same capacity results in the unknown channel case for a channel vector $\vec{x}^H = [1 \quad 1 \quad 1 \quad 1]$. For $\vec{x}^H = [2 \quad 0 \quad 0 \quad 0]$ however we obtain the worst case capacity (left vertical line) within the set of channel vectors with equal norm.

The capacity for the inner code vector sequence (1) - antenna switching - exhibits considerable variations. It appears beneficial to reduce these variations in order to decouple the design of the inner and outer code and with respect to the known channel case. On this basis it is possible to optimize $\left\{\vec{c}_\nu\right\}$. We observed that for most values of N_{dim} and N_{TX} a good set of inner coding vectors is given by the first N_{TX} columns of the $\left(N_{dim} \times N_{dim}\right)$ Fourier matrix F.

$$\vec{c}_\nu = F\left[\nu, 1:N_{TX}\right]^T \tag{2}$$

with $F[n,m] = 1\big/\sqrt{N_{dim}} \cdot \exp\left(-j \cdot 2\pi \cdot (n-1) \cdot (m-1) / N_{dim}\right)$.

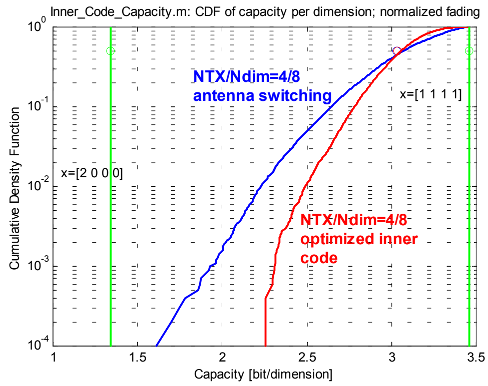

Fig. 5: CDF of the instantaneous capacity.

It is evident from Fig. 5 that these inner code vectors considerable reduces the variations of the instantaneous capacity in normalized fading. Note that the mean capacity is not affected by the optimization.

B. *Spatial Subchannel Mode*

In the spatial subchannel mode $N_U > 1$ and the inner code matrices have N_U columns: $C_\nu = \left(N_{TX} \times N_U\right)$. Some requirements for the optimization of C_ν are

- for $N_{TX} > N_U$ we want to benefit from the additional TX-diversity
- easy adaptation for different N_U

Our approach involves a time invariant spatial subchannel matrix M to expand $\left\{\vec{c}_\nu\right\}$ from the pure TX-diversity mode

$$C_\nu = diag\left(\vec{c}_\nu\right) \cdot M \tag{3}$$

$diag\left(\vec{c}_\nu\right)$ is a $\left(N_{TX} \times N_{TX}\right)$ diagonal matrix of the elements of $\vec{c}_\nu$ .This way the adaptation to a different number N_U of subchannels is simply obtained by adding/deleting columns of M. The inner code vector $\vec{c}_\nu$ is optimized for pure TX-diversity (Section III.A).

Our choice of M is a $\left(N_{TX} \times N_U\right)$ complex orthonormal matrix $M^H \cdot M = I_{N_U}$. The elements of M have approximately constant magnitude $\left|M\left[k_1,k_2\right]\right| \approx$ const. and the columns of M are obtained by cyclic shifts of the first column $M[:,1]$. Specifically consider an $\left(N_{TX} \times 1\right)$ vector $\vec{h}$ with the elements

$$\vec{h}[n] = \exp\left(j \cdot 2\pi \cdot x_{cc} \cdot (n-1)^2 / N_{TX}^2\right) \tag{4}$$

The first column of the spatial subchannel matrix M is given by the FFT of $\vec{h}$. The parameter x_{cc} is optimized a given N_{TX}. In this paper we use the $(N_{TX}; x_{cc})$ pairs (2;1) and (4;14).

The efficiency of this approach is illustrated by the CDF of the instantaneous capacity per temporal dimension (Fig. 6). Because we are comparing different choices of $N_{TX} / N_{RX} / N_U$ the fading is not normalized in this graph. The left-hand set of curves indicates the CDF for 1 receive antenna and no spatial subchanneling ($N_U = 1$). Parameter of the curves is the number of TX antennas (1, 2, 4, 6, 8). The right-hand set of curves depicts the CDF for 2 RX antennas. We utilize 2 spatial subchannels in this case. Parameter of these curves is the number of TX antennas (2, 4, 6, 8). In all cases the noise variance is $\sigma_w^2 = 0.1$ and we have $N_{dim} = 8$.

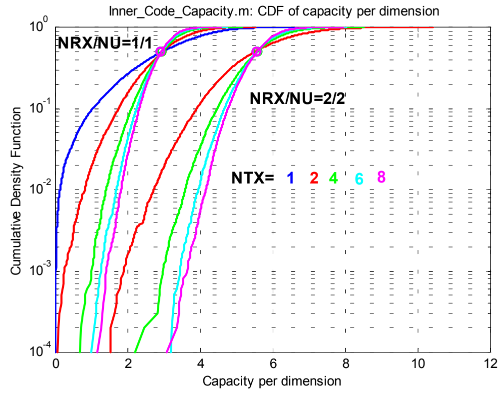

Fig. 6: CDF of the capacity with spatial subchanneling

The increase in capacity for $N_U = N_{RX} = 2$ is possible, because 2 TX-symbols are multiplexed on 1 temporal dimension by the matrix M. The CDF curves clearly indicate the TX-diversity gain for $N_U = 2$ and $N_{TX} > 2$ (a diversity gain results in less variations of the instantaneous capacity).

IV. CONSTRUCTON OF OUTER CODE

According to our design paradigm the outer code is optimized for diversity performance. Fig. 7 shows an appropriate signal model. The input symbol vector $\vec{\alpha}$ has N_I elements. The outer code matrix R is $N_I \times N_c$. For uncoded transmission R is the $(N_I \times N_I)$ identity matrix. The transmit symbol vector $\vec{s} = R \cdot \vec{\alpha}$ comprises N_c elements. For $N_I = 1$ the outer code is a rate $1/N_c$ repetition code. For $N_I = N_c$ the outer code has rate 1. This case is assumed throughout the paper. The transmit symbol vector is subject to multiplicative fading, i.e. the fading vector $\vec{z}$ is multiplied element per element with the transmit symbol vector. In this section we assume that the elements of $\vec{z}$ are statistically independent complex normal random variables. The received vector is perturbed by AWGN. The decoder generates an estimate $\hat{\vec{\alpha}}$ of the input symbol vector.

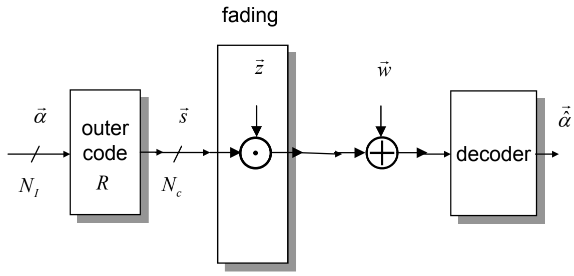

Fig. 7: Signal model for outer code construction

There are a number of sensible cost functions for the optimization of the code matrix. In this paper we use the maximal fading averaged pairwise probability of message error (MaxPairMER): let M be the size of the input symbol alphabet. Then there are $(M^{N_I} - 1) \cdot M^{N_I}$ pairs of input symbol vectors. MaxPairMER is the largest fading averaged error probability of all pairs. Note that this cost function depends on the input symbol alphabet.

With respect to the AWGN performance ($\vec{z} = [1, 1, \cdots]^T$) we require R to be orthonormal: $R^H \cdot R = I_{N_I}$

- an orthonormal transformation preserves the Euclidean distance and thus the error performance on an AWGN channel.
- the resulting lack of intersymbol interference under AWGN conditions yields a low complexity decoder

For symmetry and complexity reasons we furthermore prefer a cyclic matrix R. To reduce the complexity of the optimization problem we employ a parameterized approach. Let $\vec{h} = R[:,1]$. Excellent result are obtained by

$$\vec{h}[n] = \frac{1}{\sqrt{N_c}} \cdot \sum_{k=1}^{N_c} \exp\left(j \cdot 2\pi \cdot a_{cc} \cdot \frac{(k-1)^2}{N_c^2}\right) \cdot \exp\left(j \cdot 2\pi \cdot \frac{(k-1) \cdot (n-1)}{N_c}\right)$$

The parameter a_{cc} is determined such that the cost function is minimized for given N_I, N_c and input symbol alphabet. For code R4C ($N_c = 4$) the optimal value is $a_{cc} = 3.4$. Note that $\vec{h}$ is the cyclic impulse response of a chirp filter. Fig. 8 provides further insight into code R4C. It depicts the log of the maximum fading averaged pairwise probability MER for each of the $4^4 = 256$ input symbol vectors. For comparison the squares indicate the performance of a repetition code with 1, 2, 3 and 4 transmissions. R4C is symmetric in the sense that all input (i.e. transmit symbol) vectors have a neighbor with the same maximum pairwise MER. The code performs only slightly worse than the rate 1/4 repetition code. The lower vertical line gives the minimum Euclidean distance of R4C for all transmit symbol vectors. Due to the orthonormality of R the ED of the input symbol vector is preserved. A particularly simple orthonormal code matrix is the $(N_c \times N_c)$ Hadamard matrix with elements ± 1. For comparison with R4C we give the maximum pairwise MER of the (4×4) Hadamard matrix. Two observations are typical also for other choices of code matrices:

- there is a number of „reasonable" input symbol vectors.
- for some input symbol vectors the maximum pairwise MER drops considerably. The Hadamard matrix is particularly poor in this respect, because some input difference vectors

$\vec{\alpha}^{(n)} - \vec{\alpha}^{(m)}$ are orthogonal to all but one row of R. As a result the associated transmit difference vector has only one nonzero element.

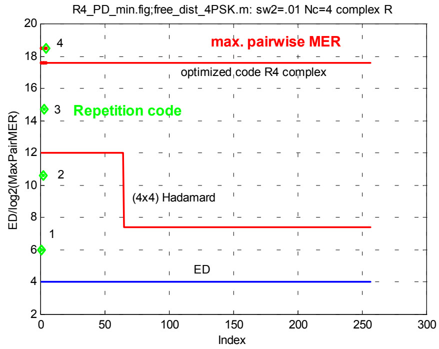

Fig. 8: Maximum mean pairwise MER

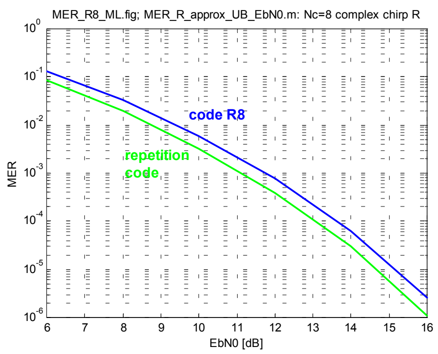

Fig. 9: Mean MER of code R8C with ML-decoder

Fig. 9 shows the mean MER performance of code R8C ($N_c = 8$) with Maximum Likelihood (ML-) decoder and 4-PSK input symbol alphabet. For comparison the mean MER of the rate 1/8 repetition code is given. It is evident that R8C achieves almost the full diversity gain. On the other hand the bandwidth efficiency of the repetition code is 1/4 bit per dimension as compared to 2 bit per dimension of the code R8C

V. PERFORMANCE

The complexity of the ML-decoder increases exponentially with N_I and the input symbol alphabet size M . For this reason it is only feasible for small N_I and M . In a companion paper [3] a suboptimal reduced complexity decoder is presented (MAP-MMSE-DFE). This decoder is feasible for large N_I and M . In this section we give some performance results of the concatenated code with the MAP-MMSE-DFE decoder. Note that the linearity of our codes renders a variety of known intersymbol interference cancellation techniques applicable. Thus a wide trade-off between complexity and performance is opened up.

Fig. 10 depicts the mean MER performance in pure TX diversity. Parameter of the curves is the number of transmit antennas. The outer code is R8C with 4-PSK symbol alphabet, the inner code vectors are obtained from the (8×8) Fourier matrix (2).

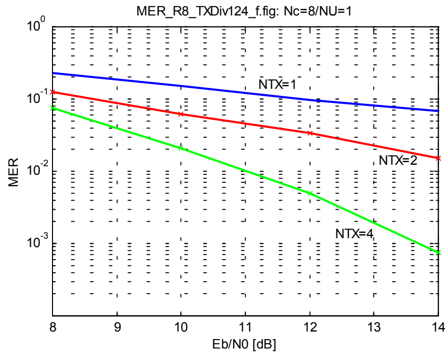

Fig. 10: Mean MER in pure TX diversity

Fig. 11 addresses a joint TX diversity and spatial subchanneling application. The outer code is R16C with 4-PSK symbol alphabet. The upper curve relates to 2 TX and 2 RX antennas. The inner code is constructed from the first 2 columns of the (16×16) Fourier matrix (2) and the (2×2) matrix M (3). In this setup the code realizes 2 spatial subchannels and the bandwidth efficiency is 4 bits/complex dimension. The lower curve results as we increase the number of TX antennas to 4. The TX diversity component of the inner code is expanded to 4 TX antennas by using 2 more columns of the Fourier matrix. The new spatial subchannel matrix M is (4×2). It is evident, that the code is able to jointly utilize TX diversity and spatial subchanneling.

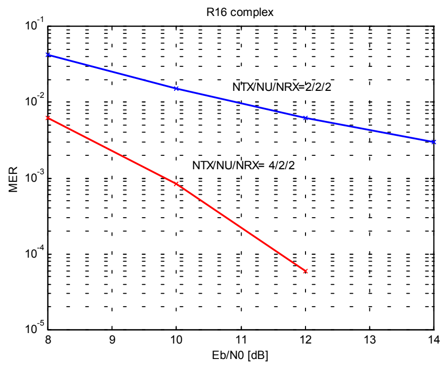

Fig. 11: Mean MER in joint TX diversity/spatial subchanneling

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