Low complexity decoding of a class of linear space-time block codes by subspace partitioning

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***Abstract*— Future broadband wireless communication systems will be characterized by heterogeneity with regard to node capabilities, link level requirements, cell structures etc. High carrier frequencies and large system bandwidth will make extreme diversity factors feasible. We consider a recently proposed class of linear space-time block codes (and associated decoders), which meet the requirements of future communication systems. In the case of these codes an increase of the block length can lead to higher diversity factors but the required decoder complexity increases too. It is not always possible for the transmitter to adapt the used block length to the decoder complexity of every receiver (e.g. for a broadcast transmission or if the transmitter has no knowledge about the receiver complexity), especially when high diversity gains are favorable.**

In this paper we present an approach to low complexity decoding by subspace partitioning. So, if a transmitter uses large block lengths, low complexity nodes can handle these blocks and achieve moderate diversity gains while nodes with higher capabilities can profit from high diversity gains.

I. Introduction

The statistical changes of the communication channel (fading) often found during the transmission of digital information can affect the average reliability of the information transmission for a given signal power. Space-time codes combat these fading effects by utilizing the diversity of the communication channel given for example by the use of an antenna array at the transmitter and / or at the receiver [1], [2]. In this paper we consider a special class of linear space-time block codes according to [1], which are highly flexible and adaptive; a priori channel knowledge at the transmitter can be used, but is not required. In addition, the joint usage of transmit diversity and of orthogonal 'spatial subchannels' of Multiple Input Multiple Output (MIMO) channels[1] is possible. These codes are independent from the used modulation alphabet and can easily be adapted to the requests of the transmission, for example to different node complexities, subnet structures or transmission channels. Furthermore the use of a large block length is possible and leads to high diversity factors.

Fig. 1 a) shows a system block diagram of these codes. They consist of two concatenated but decoupled linear block codes, the *inner code* and the *outer code*. The inner code is used for an adaption to the applied number of transmit and receive antennas. In [1] efficient code matrices are given for transmit diversity and for joint usage of transmit diversity and 'spatial subchannels' of MIMO channels, which are available in rich diversity [1], [5]. The outer code is optimized for diversity performance and achieves a high diversity gain and an excellent performance in a fading environment. Due to the code concatenation the diversity performance optimization and channel conditioning (adaption to number of transmit and receive antennas, transmit diversity, use of a MIMO channel) are decoupled.

One of the tasks of the decoder for these codes is the compensation of intersymbol interference (ISI), which can result from an optimized diversity performance and from interfering spatial subchannels [1], [5], [6] ; because otherwise the performance could be affected by the ISI.

In the following a baseband representation according to Fig. 1 b) is used because the inner code is not in the focus of this paper. The inner code and the MIMO channel are modeled as a Rayleigh (fast) fading channel $\vec{x}$. The outer code is repre-

[1] Since the work of Foschini, Telatar and others [3], [4] it is well known that the capacity of wireless systems can be increased using antenna arrays at transmitter and receiver to create MIMO channels.

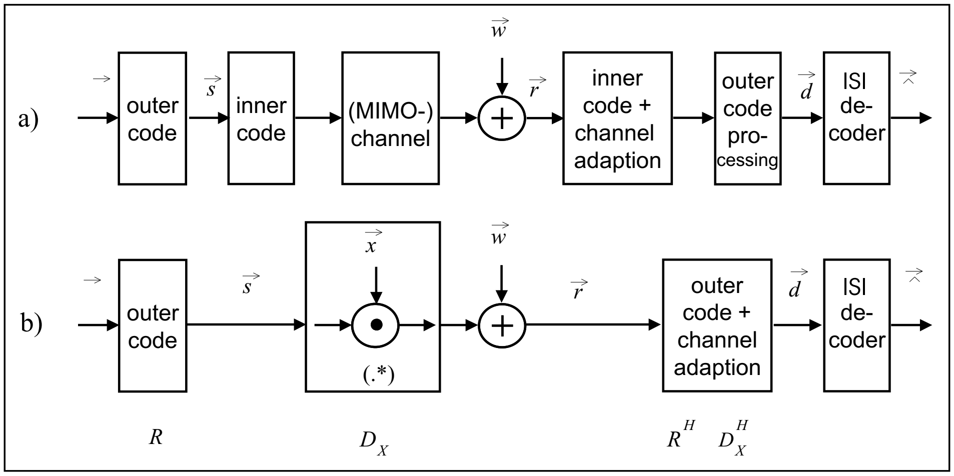

Figure 1. Linear space time block code

sented by the code matrix R which is unitary

$$R^H \cdot R = I \tag{1}$$

where I is the unit matrix. The vector $\vec{\alpha}$ is the transmitted symbol vector of length N_I, $\vec{w}$ contains the samples of the noise (modeled as AWGN). The matrix D_x is a diagonal matrix of $\vec{x}$. The received symbol vector $\vec{d}$ after channel adaption and outer code processing can be derived as follows:

$$\begin{aligned} \vec{d} &= R^H \cdot D_x^H \cdot D_x \cdot R \cdot \vec{\alpha} \\ &\quad + R^H \cdot D_x^H \cdot \vec{w} && (2) \\ &= \Lambda_{ISI} \cdot \vec{\alpha} + \vec{n} && (3) \\ \Lambda_{ISI} &= R^H \cdot D_x^H \cdot D_x \cdot R && (4) \\ \vec{n} &= R^H \cdot D_x^H \cdot \vec{w} && (5) \end{aligned}$$

The fading in D_x introduces ISI because the orthogonality of R is destroyed (2). The ISI included in the received signal is linear and in (3) represented by the matrix Λ_{ISI} of dimension (N_I x N_I). In [7] a decoder (the *MAP–DFE*) using an efficient ISI cancelling method is presented, that achieves for 4-QAM almost the performance of a maximum likelihood decoder but with a much lower complexity. However this complexity increases disproportionately with an increasing block length[2] N_I. So a low complexity node using this decoder could not be able to decode a block in time, if the block length is large. But for other participant nodes a large block length could be necessary due to the high diversity gain, which is possible for large block lengths.

[2]In each iteration of the decoding process the MAP–DFE has to invert a matrix of dimension (N_I x N_I).

II. Low complexity decoding

In contrast to the in [1] and [7] proposed procedures regarding the ISI compensation we present in this paper a new approach. The block of received symbols is preprocessed by M linear block codes, i.e. the received symbol vector $\vec{d}$ is multiplied by M linear matrices to get M vectors $\vec{d_m}$. An ISI decoder processes each of these vectors. In the following we consider the case of $M = 2$ (Fig. 2) and N_I even:

$$\begin{aligned} \vec{d_1} &= C \cdot \vec{d} = C \cdot (\Lambda_{ISI} \cdot \vec{\alpha} + \vec{n}) && (6) \\ \vec{d_2} &= D \cdot \vec{d} = D \cdot (\Lambda_{ISI} \cdot \vec{\alpha} + \vec{n}) && (7) \end{aligned}$$

The matrices C and D are of dimension ($\frac{N_I}{2}$ x N_I) and chosen in such a way that the products with matrix Λ_{ISI} have a subarray of dimension ($\frac{N_I}{2}$ x $\frac{N_I}{2}$) consisting only of zeros:

$$C \cdot \Lambda_{ISI} = \begin{pmatrix} m_{11} & m_{12} & \dots & m_{1\frac{N_I}{2}} & 0 & \dots & 0 \\ m_{21} & m_{22} & \dots & m_{2\frac{N_I}{2}} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ m_{\frac{N_I}{2}1} & m_{\frac{N_I}{2}2} & \dots & m_{\frac{N_I}{2}\frac{N_I}{2}} & 0 & \dots & 0 \end{pmatrix}$$

$$D \cdot \Lambda_{ISI} = \begin{pmatrix} 0 & 0 & \dots & 0 & m_{1(\frac{N_I}{2}+1)} & \dots & m_{1N_I} \\ 0 & 0 & \dots & 0 & m_{2(\frac{N_I}{2}+1)} & \dots & m_{2N_I} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & m_{\frac{N_I}{2}(\frac{N_I}{2}+1)} & \dots & m_{\frac{N_I}{2}N_I} \end{pmatrix}$$

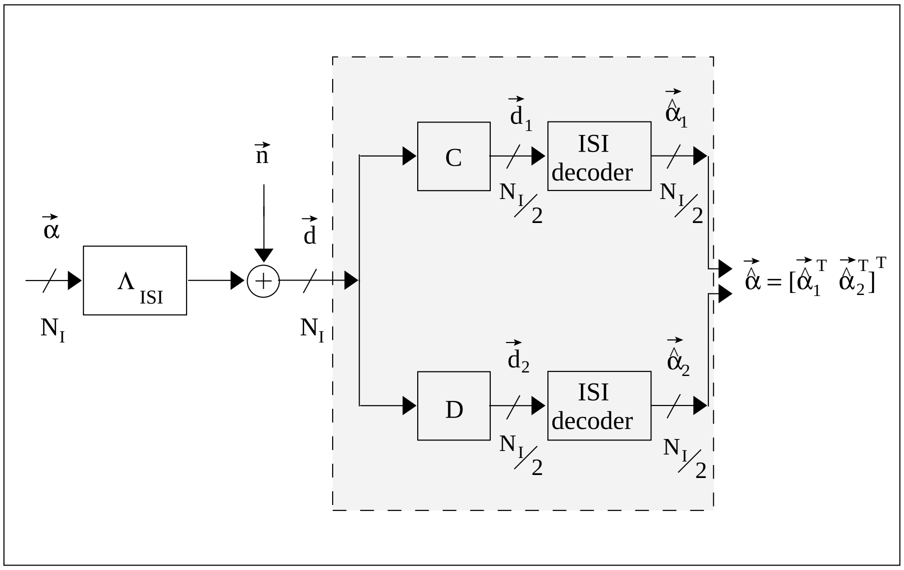

Figure 2. Low complexity decoder, $M = 2$

As a result each ISI decoder of the two branches in Fig. 2 has to deal with an ISI matrix of dimension ($\frac{N_I}{2}$ x $\frac{N_I}{2}$) instead of (N_I x N_I). This leads to a lower complexity because the complexity increases disproportionately with the block length. The two ISI decoders produce the two vectors $\hat{\vec{\alpha}}_1$ and $\hat{\vec{\alpha}}_2$; the estimate of $\vec{\alpha}$, the vector $\hat{\vec{\alpha}}$, is given by:

$$\hat{\vec{\alpha}} = [\hat{\vec{\alpha}}_1^T, \ \hat{\vec{\alpha}}_2^T]^T \tag{8}$$

The row vectors $\vec{c}_i^{\,T}$ forming the matrix C are found using a basis of the orthogonal complement (*null space*) of the subspace given by the last $\frac{N_I}{2}$ column vectors of the matrix $\Lambda_{ISI} = [\vec{\lambda}_1, \ldots, \vec{\lambda}_i, \ldots, \vec{\lambda}_{N_I}]$:

$$L = [\vec{\lambda}_{(\frac{N_I}{2}+1)}, \vec{\lambda}_{(\frac{N_I}{2}+2)}, \ldots, \vec{\lambda}_{N_I}]. \tag{9}$$

This means, a matrix C is required with

$$C \cdot L = \underline{0} \tag{10}$$

where $\underline{0}$ is the zero matrix.

Using singular value decomposition [8] of the matrix

$$A = L^T \tag{11}$$

we get

$$A = U \cdot S \cdot V^H \tag{12}$$

where U and V are unitary matrices with

$$U \cdot U^H = U^H \cdot U = I \tag{13}$$

$$V \cdot V^H = V^H \cdot V = I \tag{14}$$

and the dimensions

$$\dim(U) = (\frac{N_I}{2} \text{ x } \frac{N_I}{2}) \tag{15}$$

$$\dim(V) = (N_I \text{ x } N_I). \tag{16}$$

The matrix S is a diagonal matrix of the singular values of A; the dimension of S is ($\frac{N_I}{2}$ x N_I).

If the matrix V_1 is given by the last $\frac{N_I}{2}$ column vectors of the matrix V

$$V_1 = [\vec{v}_{(\frac{N_I}{2}+1)}, \ \vec{v}_{(\frac{N_I}{2}+2)}, \cdots, \vec{v}_{N_I}] \tag{17}$$

than the matrix C can be expressed as

$$C = V_1^T. \tag{18}$$

With this choice of C follows

$$\begin{aligned} C \cdot L &= C \cdot A^T = C \cdot (USV^H)^T \\ &= C \cdot (V^H)^T \cdot S^T \cdot U^T \\ &= V_1^T \cdot (V^H)^T \cdot S^T \cdot U^T \\ &= (V^H \cdot V_1)^T \cdot S^T \cdot U^T \\ &= \underline{0} \cdot U^T = \underline{0} \end{aligned} \tag{19}$$

because

$$(V^H \cdot V_1)^T = \begin{pmatrix} \underline{0}_{(\frac{N_I}{2} \text{ x } \frac{N_I}{2})} \\ I_{(\frac{N_I}{2} \text{ x } \frac{N_I}{2})} \end{pmatrix}^T \tag{20}$$

and because of the structure of S.

The matrix D can be found in a similar way.

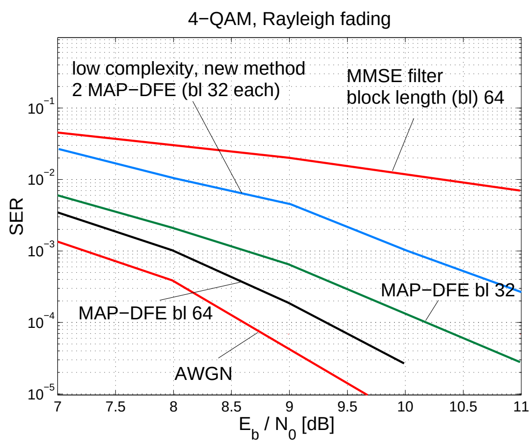

Figure 3. SER performance comparison of different decoders (block length 64 and 32)

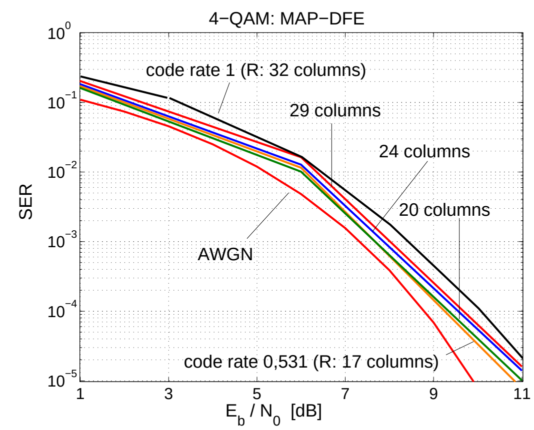

Figure 4. SER performance comparison of different code rates between 1 and 0.5 and MAP–DFE; no subspace partitioning

The two ISI decoder compensate the remaining ISI in the two vectors $\vec{d_1} = C \cdot \vec{d}$ and $\vec{d_2} = D \cdot \vec{d}$. The ISI can be described by two ISI matrices of dimension ($\frac{N_I}{2}$ x $\frac{N_I}{2}$).

III. Results

Fig. 3 shows the symbol error rate (SER) of 4-QAM versus $\frac{E_b}{N_0}$ at the receiver for a Rayleigh fading channel ($\vec{x}$) and block length $N_I = 64$ ($\vec{d}$ consists of 64 elements) and 32 respectively. A linear space-time block code according to [1] and different decoders are used: a linear MMSE filter in combination with block length 64, a MAP-DFE decoder according to [7] (for block length 32 and 64) and a *low complexity decoder using subspace partitioning* with a MAP-DFE as ISI decoder in each of its two branches (see Fig. 2). The MAP-DFE in each branch decodes an ISI matrix of dimension (32x32), that means $\vec{d}$ consists of 64 elements in this simulation, the code rate is $r_C = 1$, i.e. the dimension of R is (N_I x N_I).

The performance of the new *low complexity decoding method* shows a clearly better performance (a lower error probability) than the MMSE filter, that decodes the received symbol vector of length 64. The new approach achieves a higher diversity gain; the slope of the SER curve is steeper (even at the lower block length). However the MAP-DFE for block length 32 performs better, and the MAP-DFE for block length 64 achieves in the Rayleigh fading environment almost the performance of 4-QAM on an AWGN-channel; this shows the high diversity gains that are feasible for the considered class of space-time codes.

A simple way to improve the performance of suboptimal decoders is to reduce the code rate of the considered space-time codes. In the following this is investigated in view of a combination with *low complexity decoding using subspace partitioning*.

A. Reduction of the code rate

For the codes according to [1] a simple reduction of the code rate is possible by deleting columns of the outer code matrix, the resulting dimension of R is (N_1 x N_2) with $N_1 > N_2$. As a result of the code concatenation this measure is independent from the used (MIMO) channel because the inner code is decoupled of the outer code. For a reduced code rate the ISI is decreased, because the number of columns of the code matrix is reduced. This leads to an ISI matrix Λ_{ISI} of a smaller dimension according to (4) while the minimum pairwise product distance of the outer code is maintained (for details see [1]). So the performance of suboptimal decoders is improved. In Fig. 3 the dimension of R is (N_I x N_I) (i.e. code rate $r_C = 1$) and $\vec{\alpha}$ has N_I elements. For a reduced code rate the dimension of R is (N_C x N_K) and $\vec{\alpha}$ has N_K elements with $N_I > N_K$ and $N_C = N_I$; the resulting code rate is $r_C = \frac{N_C}{N_K}$.

Fig. 4 and Fig. 5 show the symbol error rate for 4–QAM versus $\frac{E_b}{N_0}$ for a rayleigh fading channel $\vec{x}$ and block length $N_C = 32$ ($\vec{d}$ consists of 32 elements); no *low complexity decoding by subspace partitioning* is used; the number of columns N_K of the matrix R varies. With decreasing code rate the performance improves for both decoders.

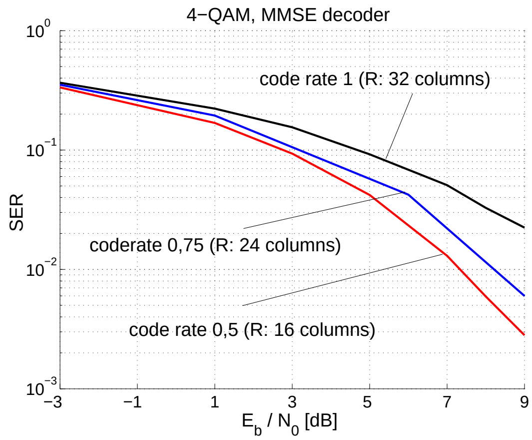

Figure 5. SER performance comparison of different code rates between 1 and 0.5 and MMSE decoder; no subspace partitioning

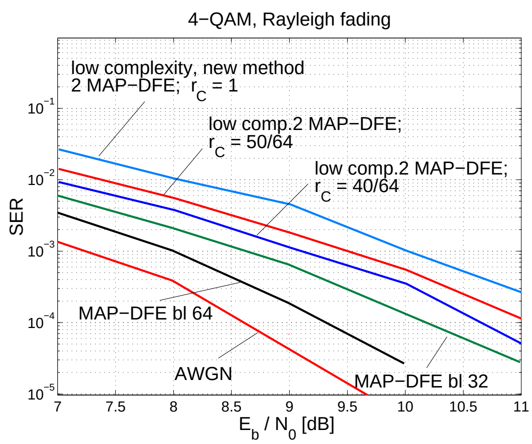

Figure 6. SER performance comparison of different decoders and different code rates

In Fig. 6 the performance improvement by a reduction of the code rate is shown for a *low complexity decoder using subspace partitioning* with a MAP–DFE as ISI decoder in each of its two branches.

IV. Conclusions

In heterogenous networks the use of *low complexity decoding by subspace partitioning* can be favorable. If the transmitter has the knowledge of the capabilities of the receiving node and can adapt the block length, it is better to use a lower block length (Fig. 3). But if this is not possible, then *low complexity decoding using subspace partitioning* is an interesting option. In heterogeneous environments decoders with a high complexity can attain the high diversity gain that is feasible for large block length (e.g. block length 64). Meanwhile low complexity decoders, that have not to cope with severe fading effects, can demodulate the same blocks using subspace partitioning (e.g. two blocks of length 32 instead of one block of length 64).

Using the coding scheme according to [1] a moderate reduction of the code rate is an efficient and simple measure to either reduce the needed node complexity while maintaining the performance or to improve the performance for a given node complexity. The results for the combination of a reduced code rate with low complexity decoders show that by using the considered space-time codes the needed decoder complexity and the feasible diversity gains can be adapted in a wide range. That means there is a rich trade off between complexity and performance. Furthermore, these codes achieve very high diversity factors (Fig. 3, Fig. 4, Fig. 6, [1], [7]) and are capable of exploiting the high capacities of MIMO channels. So, this coding scheme and the associated decoders are promising candidates for heterogeneous future mobile communication systems.

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