Channel Adaptive Scheduling for Cooperative Relay Networks

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***Abstract*— We consider a wireless network consisting of several active source/destination pairs and several idle nodes. All nodes are equipped with one antenna. The communication between source/destination pairs takes place over two-hop links. Idle nodes are used as amplify-and-forward relays assisting the communication. We investigate the benefits and potential of joint cooperative diversity and channel adaptive scheduling in such a network. We show that this scheme is able to exploit multi-user diversity as well as cooperative diversity. It even makes a fair scheduling possible. Performance analysis is done analytically and by means of simulation.**

I. Introduction

The use of diversity in the spatial and temporal dimension to mitigate the effects of fading and therefore to increase the reliability of radio links in wireless networks is a well known technique for systems with co-located antennas (space-time coding). Recently a new form of realizing spatial diversity has been introduced in [1] and [2] called *cooperative diversity* or *user cooperation diversity*. The main idea is to use multiple nodes as a virtual macro antenna array, realizing spatial diversity in a distributed fashion. In such a network several nodes serve typically as relays for an active source/destination pair. Relays can be classified as either *decode-and-forward* (DF) or *amplify-and-forward* (AF) relays. AF relays, which are considered in this work, only retransmit an amplified version of their received signals. This leads to low-complexity relay transceivers, lower power consumption since there is no signal processing for decoding procedures. Moreover, AF relays are transparent to adaptive modulation techniques which may be employed by the source.

In a centralized wireless access system a scheduler at the access point schedules the medium access of the wireless nodes. A channel adaptive scheduler incorporates channel (link) state information (CSI) in this process [3]. As the achievable per-link throughput depends on the link quality, channel adaptive scheduling may improve the aggregate throughput. It thus exploits multi-user diversity [4].

Channel adaptive scheduling is particularly efficient in the high mobility regime, because the channel state varies sufficiently within the latency time scale of interest. Consequently, each node has a fair chance to see a good link in this time interval. However, WLANs typically operate in the low mobility regime. In this case a channel adaptive scheduler, which optimizes the aggregate throughput, essentially would only serve the source/destination pair with the best link. For quality of service (QoS) reasons, a realistic channel adaptive scheduling scheme in this case has to operate away from the aggregate throughput optimum.

If the access point has multiple co-located antennas, we can introduce time variations into a quasi-static fading environment by applying time-variant weights at the transmit antennas. This *opportunistic beamforming* scheme [5] essentially probes the weight vector space with random realizations of the antenna weights of the co-located transmit antenna array.

In [6] we propose a simple cooperative diversity scheme with AF relays. Essentially the block fading time-invariant channel is translated into a time-variant channel by introducing time-variant phase offsets at the relays. The main motivation for the work presented in [7] is the observation, that this time variance could be exploited by considering several source/destination pairs jointly for scheduling purposes. We refer to this approach as *joint cooperative diversity and scheduling*. The scheme considerably improves the utilization of the physical resources without compromising the QoS in a block fading low mobility environment. In this work we concentrate on the aggregate throughput of the system, which reflects the utilization of the physical resources. We derive analytical performance bounds for joint cooperative diversity and scheduling and give further insights by means of simulation. For QoS issues of our proposed scheme the reader is referred to [7].

The remainder of the paper is organized as follows: in section II we describe the system model and the traffic pattern of the used relaying scheme. In section III we summarize our approach of joint cooperative diversity and scheduling. An analytical performance analysis is presented in section IV. In section V we give further insights by means of simulation results. We conclude that joint cooperative diversity and scheduling extends the benefits of adaptive scheduling to the low mobility regime.

Notation: We shall use bold uppercase letters to denote matrices and bold lowercase letters to denote vectors. Further $(\cdot)^T$, $(\cdot)^\dagger$ stand for transpose and Hermitian transpose of a matrix, respectively. $\mathrm{diag}[a, \ldots, z]$ denotes a diagonal matrix with the elements $a, \ldots, z$ on its main diagonal, $\mathbf{I}$ is an identity matrix and $\mathbf{0}$ a matrix with all elements equal to zero. $\mathrm{E}[\cdot]$ is the expectation operator. Given a random variable X the functions $F_{\mathrm{X}}(x)$ and $f_{\mathrm{X}}(x)$ denote the corresponding cumulative density function (CDF) and the probability density function (PDF), respectively.

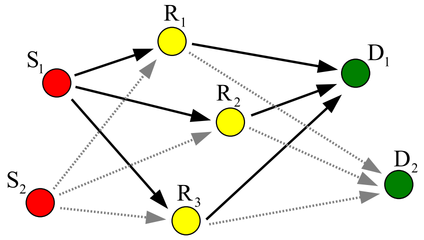

Fig. 1. Two-hop cooperative relaying network consisting of two active source/destination links and three relays. All nodes are single antenna nodes.

II. System Model

In the following we describe the system model for cooperative relaying. To highlight the effects of cooperative relaying, we assume that communication takes place only over two-hop links and there is no direct connection between source and destination. Such a system is depicted in Fig. 1 for $M = 2$ source/destination links.

In our scenario user mobility is low and the channel coefficients are constant over the latency time scale of interest. We assume that the channel is time-invariant over at least one *transmission cycle* (block fading). As a transmission cycle we denote a period of time (i.e. a burst of symbols) in which an active source/destination pair is communicating.

We assume L amplify-and-forward relays assisting the communication link. In such a link the transmission of one data packet from the source S to the destination D occupies two time slots which both together establish one transmission cycle. In the first slot the source transmits the data packet to the relay. During the second slot the relays retransmit an amplified version of the received signals to the destination.

We denote the channel between the source and the relays as *uplink*, and the channel between the relays and the destination as *downlink*. For one source/destination link the channel coefficients of the uplink are stacked in the vector $\mathbf{h}_\mathrm{u}$. The complex conjugates of the downlink channel coefficients are stacked in $\mathbf{h}_\mathrm{d}$. We consider frequency-flat fading, which usually include path loss, shadowing and small-scale fading. For clarity of exposition we neglect path loss and shadowing. Thus, all channel coefficients are i.i.d. complex normal random variables $\mathcal{CN}(0,1)$.

At time instance k in the first time slot the source S sends the symbol $s^{(k)}$, with average transmit power P. The received signal of all relays is given by the vector

$$\mathbf{y}^{(k)} = \mathbf{h}_\mathrm{u} s^{(k)} + \mathbf{m}^{(k)} \tag{1}$$

where $\mathbf{m}^{(k)} \sim \mathcal{CN}(\mathbf{0}, \sigma_\mathrm{R}^2 \mathbf{I})$ (i.i.d. for all k) contain the AWGN contributions at the relays.

Fig. 2 shows the system model of an AF relay. $\phi_{\mathrm{LO},l}$ represents the phase offset of the local oscillator (LO) at the relay R_l relative to a given reference phase. This phase offset is required in the system model, because the LOs of the relays may be free running. In this case $\{\phi_{\mathrm{LO},l}\}$ are i.i.d. random variables. Only if there is a *global phase reference*, i.e., all LOs are phase synchronized, $\phi_{\mathrm{LO},l}$ is equal to zero. The factor

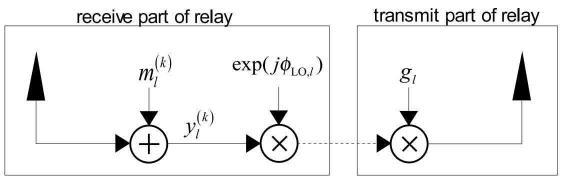

Fig. 2. System model of amplify-and-forward relay

g_l is the amplification gain at relay R_l. A popular choice of g_l (e.g. in [1]) is

$$g_l = \sqrt{\frac{P_\mathrm{R}}{P\,|h_{\mathrm{u},l}|^2 + \sigma_\mathrm{R}^2}}. \tag{2}$$

I.e. each relay transmits with power P_R. We impose a total power constraint P on all relays, i.e., $P_\mathrm{R} = P/L$. With the gain matrix $\mathbf{G} = \mathrm{diag}\,[g_1, \ldots, g_L]$ and the phase reference matrix $\mathbf{\Phi} = \mathrm{diag}\left[\exp(j\phi_{\mathrm{LO},1}), \ldots, \exp(j\phi_{\mathrm{LO},L})\right]$ the received signal at the destination is given by

$$r^{(k+1)} = \mathbf{h}_\mathrm{d}^\dagger \mathbf{G}\mathbf{\Phi}\mathbf{y}^{(k)} + w^{(k+1)} \tag{3}$$

$$= \mathbf{h}_\mathrm{d}^\dagger \mathbf{G}\mathbf{\Phi}\mathbf{h}_\mathrm{u} s^{(k)} + \mathbf{h}_\mathrm{d}^\dagger \mathbf{G}\mathbf{\Phi}\mathbf{m}^{(k)} + w^{(k+1)}, \tag{4}$$

where $w^{(k+1)} \sim \mathcal{CN}(0, \sigma^2)$ denotes the AWGN contribution at the destination.

The instantaneous capacity (per complex dimension) for a link described in (4) is given by

$$C_\mathrm{I} = \frac{1}{2}\log_2\left(1 + \frac{P\left|\mathbf{h}_\mathrm{d}^\dagger \mathbf{G}\mathbf{\Phi}\mathbf{h}_\mathrm{u}\right|^2}{\sigma^2 + \sigma_\mathrm{R}^2 \mathbf{h}_\mathrm{d}^\dagger \mathbf{G}\mathbf{G}^\dagger \mathbf{h}_\mathrm{d}}\right), \tag{5}$$

where σ^2 and σ_R^2 is the noise variance at the destination and the relays, respectively. The factor $1/2$ accounts for the two channel uses required by the relay traffic pattern.

A. Impact of local phase reference on coherent combining

If perfect up- and downlink CSI and a global phase reference are available at the relays the optimal gain coefficients g_l result in a coherent combining of the signal contributions at the destination [8].
A substantial signaling overhead is required to phase-lock all LOs of all nodes (i.e., to establish a global phase reference). We consider this overhead prohibitive, i.p., if the number of relays is large. For this reason we assume in the sequel that $\{\phi_{\mathrm{LO},l}\}$ are i.i.d. random variables with uniform distribution in the interval $[0, 2\pi)$.

III. Joint Cooperative Diversity and Scheduling

Our starting point is the simple cooperative diversity scheme which we proposed in [6]. It requires only very limited uplink CSI and no downlink CSI. For clarity of exposition we briefly summarize the approach: the linear processing at the relays is time-variant, which results in a time-variant equivalent source/destination channel coefficient (i.e., a time-variant SNR at the destination). In a simple embodiment the relays use a time-invariant gain and a relay-specific time-variant phase

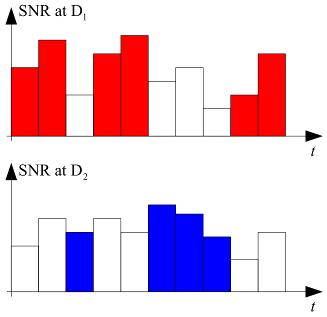

Fig. 3. Adaptive scheduling depending on the SNR at the destinations

offset (phase signature sequence). Thus, the amplification gain of relay l at time instance k is now for example given as

$$g_l^{(k)} = \underbrace{\sqrt{\frac{P_{\mathrm{R}}}{P\left|h_{\mathrm{u},l}\right|^2 + \sigma_{\mathrm{R}}^2}}}_{\text{time-invariant}} \cdot \underbrace{\exp\left(j\varphi_l^{(k)}\right)}_{\text{time-variant}} \quad (6)$$

where $\varphi_l^{(k)}$ denotes the relay-specific and time-dependent phase offset.

We consider all $\varphi_l^{(k)}$ as independent realizations of a random variable φ_l. If we assume an infinite number of phase realizations in each transmission cycle the throughput for a given channel realization is the expectation of the instantaneous capacity with respect to $\{\varphi_l\}$

$$C_{\mathrm{I}} = \frac{1}{2}\,\mathrm{E}_{\{\varphi_l\}}\left[\log_2\left(1 + \frac{P\left|\mathbf{h}_{\mathrm{d}}^{\dagger}\mathbf{G}^{(k)}\mathbf{\Phi}\mathbf{h}_{\mathrm{u}}\right|^2}{\sigma^2 + \sigma_{\mathrm{R}}^2\mathbf{h}_{\mathrm{d}}^{\dagger}\mathbf{G}^{(k)}\mathbf{G}^{(k)\dagger}\mathbf{h}_{\mathrm{d}}}\right)\right]. \quad (7)$$

The key idea of the work presented in [7] is to further exploit the time-variance by joint consideration of several source/destination pairs to achieve multi-user diversity by scheduling. For illustration we assume an idealized scheduler, which perfectly knows the time-variant destination SNRs.

In [7] we defined two figures of merit to quantify the performance of our joint cooperative diversity and scheduling approach. We use the *aggregate throughput* as a measure, how efficient the network utilizes the physical resources. The aggregate throughput is the throughput in the given physical resources (averaged over the transmission cycle). Due to the block-fading channel the aggregate throughput is a random variable. In contrast we use the *per-link throughput* as QoS measure. The per-link throughput is the throughput the user experiences in a given transmission cycle. It is again a random variable. It is is closely related to QoS parameters like data rate and delay.

Fig. 3 shows a typical time-variant destination SNR for $M = 2$ links over one transmission cycle. The phase signature sequence consists of 10 segments. To optimize the aggregate throughput, our scheduler selects the best link in each time segment. This is indicated by the shaded boxes in Fig. 3. Here $\mathrm{S}_1/\mathrm{D}_1$ is scheduled in six time segments, whereas $\mathrm{S}_2/\mathrm{D}_2$ is scheduled in four. Note that in this scenario channel adaptive scheduling is reasonably fair, even if the channel itself is not time-variant (cf. [7]). We explicitly benefit from the time-variance, which is introduced by the time-variant processing at the relays.

IV. Performance Analysis

For the performance analysis we restrict ourself to the case where the relays operate in the high-SNR region. Thus, we can neglect the noise contribution to the retransmitted signals, i.e., $\sigma_{\mathrm{R}}^2 = 0$. Therefore, with $\nu_l^{(k)} = \phi_{\mathrm{LO},l} + \varphi_l^{(k)}$ (7) reduces to

$$C_{\mathrm{I}} = \frac{1}{2}\mathrm{E}_{\{\varphi_l\}}\left[\log_2\left(1 + \frac{P_{\mathrm{R}}\left|\sum_{l=1}^{L} h_{\mathrm{d},l}\cdot e^{j\nu_l^{(k)}}\right|^2}{\sigma^2}\right)\right] \quad (8)$$

$$= \frac{1}{2}\mathrm{E}_{\{\varphi_l\}}\left[\log_2\left(1 + \frac{P_{\mathrm{R}}\left|h^{(k)}\right|^2}{\sigma^2}\right)\right]. \quad (9)$$

Furthermore, we assume $h_{\mathrm{d},l} \sim \mathcal{CN}(0,1)$ are i.i.d. with respect to the spatial and temporal domain.

A. No Phase Rotations

This case corresponds to a block fading channel, where the channel is constant over the whole transmission cycle. Therefore, only one out of M active source/destination pairs is scheduled over the whole transmission cycle. This can lead to high delays at the other source/destination pairs, especially in low mobility environments (cf. [7]).

In the case where no phase rotations at the relays are introduced the SNR-term in the logarithm of (9) is independent of k and is exponential distributed with respect to the downlink channel coefficients as

$$f_{\mathrm{X}}(x) = \alpha \cdot e^{-\alpha x}, \; x \geq 0 \quad (10)$$

with $1/\alpha = P_{\mathrm{R}}L/\sigma^2 = P/\sigma^2$. Therefore the distribution of the throughput with $y = \frac{1}{2}\log_2(1+x)$ is given by [9]

$$f_{\mathrm{Y}}(y) = \beta \cdot 4^y \cdot e^{-\alpha(4^y - 1)}, \; y \geq 0 \quad (11)$$

with $\beta = 2\log(2)\alpha$.

Let $\mathrm{Y}_{(M)}$ denote the maximum out of a set of M random variables (i.e., throughput of different source/destination pairs) with probability density function (PDF) $f_{\mathrm{Y}}(y)$ and cumulative density function (CDF) $F_{\mathrm{Y}}(y)$.
The PDF $f_{\mathrm{Y}_{(M)}}(y)$ is then given by [10]

$$f_{\mathrm{Y}_{(M)}}(y) = M f_{\mathrm{Y}}(y)\left[F_{\mathrm{Y}}(y)\right]^{M-1} \quad (12)$$

$$= \beta \cdot M \cdot 4^y \cdot e^{-\alpha(4^y-1)} \cdot \left(1 - e^{-\alpha(4^y-1)}\right)^{M-1}, \quad (13)$$

and the CDF by

$$F_{\mathrm{Y}_{(M)}}(y) = \left[F_{\mathrm{Y}}(y)\right]^M = \left(1 - e^{-\alpha(4^y-1)}\right)^M. \quad (14)$$

In Fig. 4 the CDFs of the aggregate throughput for a different number of source/destination pairs are depicted for $1/\alpha = 20\mathrm{dB}$. The multi-user diversity gain is clearly visible. At 1%-

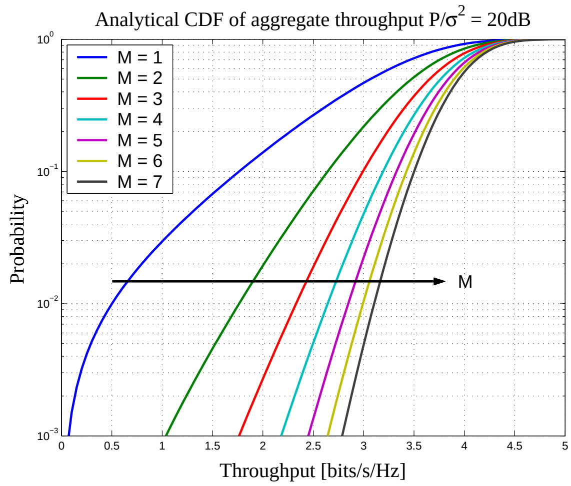

Fig. 4. CDFs of aggregate throughput for different number of source/destination pairs; $1/\alpha = 20$dB

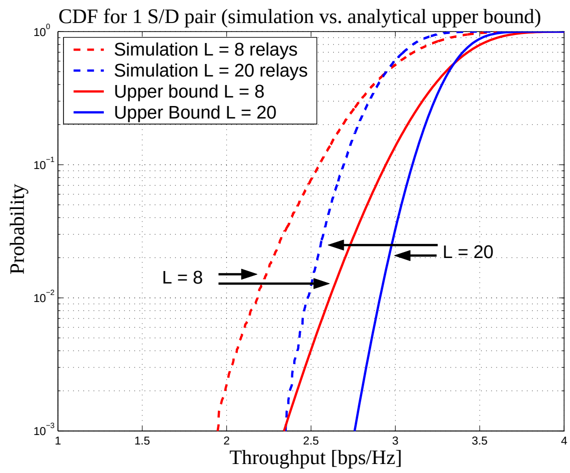

Fig. 5. CDF of (15) (dashed lines) and (16) (solid lines) for $L = 8$ and $L = 20$ relays at $P/\sigma^2 = 20$dB

outage probability the aggregate throughput is improved by a factor of six if $M = 6$ source/destinations are considered jointly.

B. With Phase Rotations

In the following we illustrate the diversity gain of the proposed time-variant relay processing scheme. For that purpose we restrict ourself to the case of one source/destination pair $M = 1$ operating in a link with L relays. Note that $\log_2(1 + \rho x)$ is a concave function in x for $\rho > 0$. Thus, using Jensen's inequality [10] the instantaneous capacity can be upper bounded as

$$C_{\mathrm{I}} = \frac{1}{2}\mathrm{E}_{\{\varphi_l\}}\left[\log_2\left(1 + \frac{P_{\mathrm{R}}\left|h^{(k)}\right|^2}{\sigma^2}\right)\right] \quad (15)$$

$$\leq \frac{1}{2}\log_2\left(1 + \underbrace{\mathrm{E}_{\{\varphi_l\}}\left[\frac{P_{\mathrm{R}}\left|h^{(k)}\right|^2}{\sigma^2}\right]}_{\mathrm{A}}\right). \quad (16)$$

In (16) the expectation with respect to the phases A is a random variable which depends on the channel coefficients and the phaseshifts. To derive the distribution functions of A we define the sample mean

$$\tilde{\mathrm{A}} = \frac{1}{N}\sum_{i=1}^{N}\frac{P\left|U_i\right|^2}{\sigma^2}, \quad (17)$$

where $U_i \sim \mathcal{CN}(0,1)$ are i.i.d. auxiliary random variables. Due to the fact that the downlink $h^{(k)}$ consists of L i.i.d random variables, the degrees of freedom in $h^{(k)}$ is limited by the number of relays L. Therefore, averaging with respect to the phases is the same as averaging L i.i.d. variates as it is done in (17). Thus, the distributions of A and $\tilde{\mathrm{A}}$ are the same if $N = L$.

Each addend of the sum in (17) has an exponential distribution as in (10) but with different parameter $1/\alpha \rightarrow 1/\gamma = P/(N\sigma^2)$. A variate that is the sum of $N = L$ exponential distributed variates is chi-square distributed with $2N$ degrees of freedom [9]. Therefore, the CDF $F_{\mathrm{I}}(z)$ of the instantaneous capacity (15) can be upper bounded by

$$F_{\mathrm{Z}}(z) = 1 - e^{(-\gamma(4^z - 1))}\sum_{k=1}^{L}\frac{1}{k!}\left(\gamma\left(4^z - 1\right)\right)^k, \; z \geq 0. \quad (18)$$

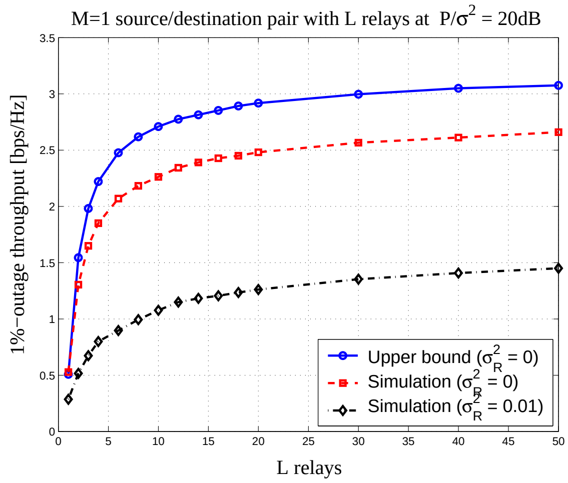

Fig. 6. 1%-outage throughput vs. the number of relays L of (15) (dashed), of the upper bound (16) (solid), and of (7) (dashed dotted) with $\sigma_{\mathrm{R}}^2 = \sigma^2$

In Fig. 5 the CDFs of (15) (dashed lines) and the upper bound of (16) (solid lines) for $L = 8$ and $L = 20$ relays are shown. It can be seen that the analytically evaluated upper bound has exactly the same shape as the empirical CDF. Furthermore, the steeper slope of the CDF for $L = 20$ relays illustrates the achieved diversity gain.

Fig. 6 depicts the 1%-outage throughput vs. the number of relays L of (15) (dashed), of the upper bound (16) (solid), and of (7) (dashed dotted) with $\sigma_{\mathrm{R}}^2 = \sigma^2$. It can be seen that outgoing from a reasonable number of relays (e.g. $L = 10$) the gain in outage throughput achieved by adding further relays is

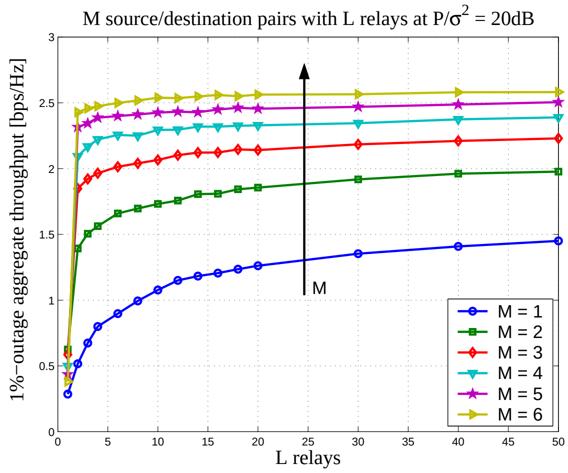

Fig. 7. 1%-outage aggregate throughput vs. number of relays L at $P/\sigma^2 =$ 20dB; M active source/destination pairs considered jointly

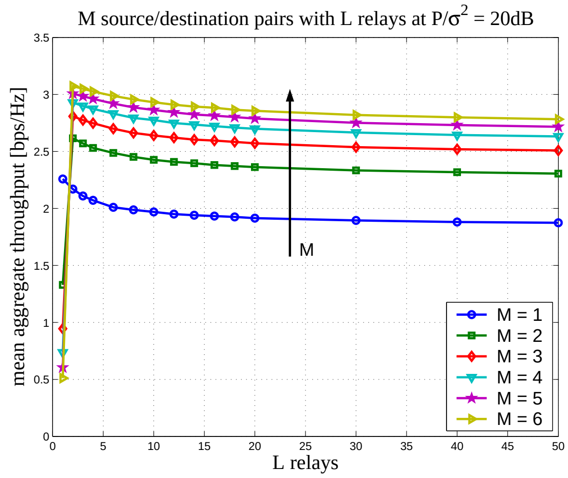

Fig. 8. Mean aggregate throughput vs. number of relays L at $P/\sigma^2 = $ 20dB; M active source/destination pairs considered jointly

the same for all curves. Therefore, the analytical upper bound can be used to estimate this gain even for realistic assumptions as in (7).

V. Simulation Results

In this section we present the performance of the proposed joint cooperative diversity and scheduling scheme by means of simulation considering also the noise contribution of the relays, i.e., $\sigma_{\mathrm{R}}^2 = \sigma^2$.

In Fig. 7 the 1%-outage aggregate throughput vs. the number of relays L for a different number of active source/destination pairs M is shown. A signal to noise ratio (SNR) of $P/\sigma^2 = 20$dB is considered. In contrast, Fig. 8 shows the mean aggregate throughput for the same parameters.

First we concentrate on the 1%-outage aggregate throughput. It can be seen that the effect of an increasing number of relays L for a lower number of active source/destination pairs (e.g., $M = 1$ or $M = 2$) is higher than for $M > 2$.

For $M \geq 4$ no further gain is achieved by using a large number of relays which is due to the fact that in this scenario the multi-user diversity gain is sufficiently large compared to the increasing cooperative diversity gain.

For $L = 1$ no temporal diversity is available which can be exploited by the scheduler for $M \geq 2$ active source/destination pairs (see the degradation in outage throughput for $L = 1$).

In Fig. 8 it can be seen that the mean throughput for a fixed M decreases with increasing number of used relays. This is due the used gain allocation (2). With increasing number of relays the probability increases that the uplink of at least one relay is in a deep fade. Thus, this relay would mainly amplify noise which dominates the SNR at the destination (cf. [8]).

Therefore, there is a trade-off between outage aggregate throughput, which translates into diversity gain, and mean aggregate throughput. From Table I is can be seen that for $M = \{1, 2, 3, 4\}$ source/destination pairs the percentage increase in outage is larger than the loss in mean aggregate throughput, while for $M \geq 5$ it is less. Hence, only a reasonable number of relays should be used in a joint cooperative diversity and scheduling system

TABLE I

Percentage increase in aggregate throughput from $L = 2 \rightarrow 50$

M	**1**	**2**	**3**	**4**	**5**	**6**
1%-outage	180.8%	42%	20.7%	14.1%	8.2%	6.3%
mean	-13.6%	-11.8%	-10.7%	-10.1%	-9.6%	-9.4%

VI. Conclusions

We investigated the benefits and potential of joint cooperative diversity and channel adaptive scheduling in a cooperative wireless network. We showed analytically and by means of simulation that multi-user as well as cooperative diversity can be achieved by this scheme.

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