A Theoretical Analysis of Multiuser Zero Forcing Relaying with Noisy Channel State Information

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Abstract— **We consider a wireless ad hoc network with single antenna nodes under a two-hop relay traffic pattern. Some of the nodes in the network form source/destination pairs, while the other nodes serve as amplify and forward relays. The relay gains are assigned such, that the interference between different source/destination links is nulled (multiuser zero forcing relaying) [7,8]. This essentially realizes a distributed spatial multiplexing gain with single antenna nodes. The main contribution of this paper is an analysis of multiuser ZF relaying, which takes noisy channel state information, phase noise and quantization noise into account. Our analytical expressions are tight for the parameter space of practical interest. The system imperfections essentially introduce an additive noise component at the destinations. We show, that the variance of this component is proportional to the number of source/destination pairs. Finally we observe, that for perfect channel state information the sum rate of the system grows beyond any limit as the number of nodes tends to infinity. For noisy CSI however this asymptotic sum rate becomes finite.**

I. Introduction

Spatial multiplexing is mandatory to achieve the extreme bandwidth efficiency of future Gigabit/sec WLANs. These Multiple Input/Multiple Output (MIMO) systems achieve an unprecedented spectral efficiency in a rich scattering environment. As opposed to conventional MIMO systems, distributed antenna systems (DAS) employ multiple antennas, which are not co-located at one site [3]. Recently cooperative relaying schemes have been proposed to improve wireless communication in multi-node networks. To date cooperative relaying schemes have primarily been proposed to achieve diversity [1,2]. In [4,5] we propose distributed antenna systems and linear relaying to relax the rich scattering requirement of conventional MIMO signaling. In [6] upper and lower bounds on the capacity of MIMO wireless networks under are given. In [7,8] we have proposed a multiuser relaying scheme, which nulls the interference between different source/destination pairs by appropriate gain allocation at the amplify&forward relays. We refer to this scheme as multiuser zero forcing (ZF) relaying. The main contribution of this paper is an analysis of this scheme with consideration of practical system imperfections.

II. Multiuser Zero Forcing Relaying

We consider a wireless network with N_{node} *single antenna* nodes. All nodes are within radio range of each other. N_a source/destination pairs communicate concurrently on the same physical channel. The communication follow a two-hop relay traffic pattern, i.e. each transmission cycle includes two channel uses: one for the uplink transmission from the sources to all relays and one for the downlink transmission from the relays to the destinations. The relays multiply the received signal with a complex gain prior to retransmission (amplify&forward). In this application we utilize the relays to achieve diversity and to orthogonalize the individual source/destination links. We refer to this scenario as multiuser ZF relaying. A configuration with N_a source/destination pairs and $N_r = N_a \cdot (N_a - 1) + 1$ relays is called a *minimum relay configuration*, as it employs the minimum number of relays required for zero forcing with N_a concurrent links. Fig. 1 shows the compound signal model. The transmit symbols of the N_a sources are stacked in the transmit symbol vector $\vec{s}$, which is i.i.d. complex normal: $CN\left(0, I_{N_a} \cdot \sigma_s^2\right)$. This vector is transmitted through the uplink channel matrix H_{Sr} to the relay nodes. The vector $\vec{m} = CN\left(0, \sigma_m^2 I_{N_r}\right)$ comprises the AWGN

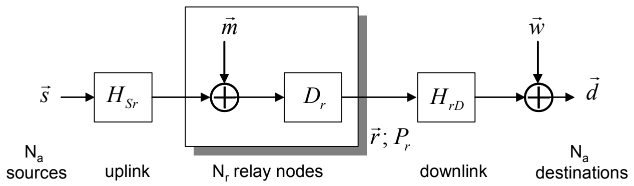

Fig. 1: Signal model

contributions at the relay nodes. The received signal at the relay nodes is multiplied with the *diagonal* gain matrix D_r . We impose an average sum power constraint on the relays. The total average sum power of sources and relay is proportional to the number of source/destination pairs. The relay node transmit signal $\vec{r}$ is passed through the downlink channel matrix H_{rD} to the N_a single antenna destinations. Throughout the paper the elements of uplink and downlink channel matrix are i.i.d. complex normal with unit variance. The vector $\vec{w} = CN\left(0, \sigma_w^2 I_{N_a}\right)$ comprises the AWGN contribution at the destinations. For all numerical results we let $\sigma_w^2 = \sigma_m^2$. As the signal model in Fig. 1 is linear, we obtain the equivalent $\left(N_a \times N_a\right)$ signal model in Fig. 2.

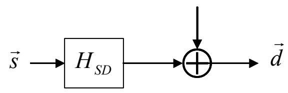

Fig. 2: Equivalent system model

The equivalent channel matrix is given by

$$H_{SD} = H_{rD} \cdot D_r \cdot H_{Sr}$$

and the additive complex normal noise vector $\vec{n}$ follows as

$$\vec{n} = H_{rD} \cdot D_r \cdot \vec{m} + \vec{w}$$

Multiuser zero forcing is characterized by a diagonal equivalent channel matrix. Let $\vec{h}_{Sr}^{(q)}$ be the vector of channel coefficients from the source (q) to all relays and $\vec{h}_{rD}^{(p)}$ the vector of channel coefficients from all relays to destination (p). We define the $\left(N_r \times N_a \cdot (N_a - 1)\right)$ matrix

$$H = \left[\cdots \vec{h}_{rD}^{(p)} \odot \vec{h}_{Sr}^{(q)} \cdots\right] \quad \forall p,q \in \{1 \cdots N_a\} \text{ and } p \neq q$$

where $\odot$ denotes the Hadamard (element-wise) product. The interference between different source/destination links is nulled, if the relay gain vector $\vec{u} = diag\left(D_r^*\right)$ satisfies [7]

$$\vec{u}^H \cdot H \equiv 0$$

The approach is very efficient to achieve a distributed spatial multiplexing gain. In Fig. 3 we show the average sum rate versus the number of nodes in the network. The performance gain of multiuser zero forcing relaying in comparison to the single user $N_a = 1$ reference is impressive: for $N_{node} = 100$ and SNR=20dB the sum rate is tripled, for SNR=40dB we even observe a fivefold increase. For further details refer to [8].

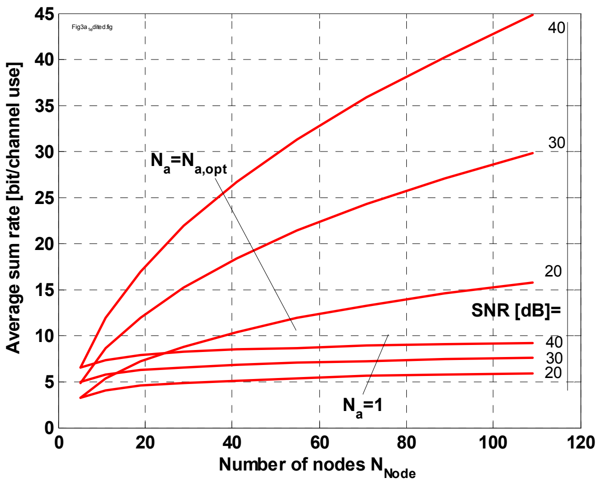

Fig. 3: Maximum average sum rate versus N_{node}, the total number of nodes in the network

III. Minimum Relay Configuration with Perfect CSI

The equivalent channel coefficient for source/destination pair (p) is given by

$$H_{SD}[p,p] = \vec{u}^H \cdot \left(\vec{h}_{Sr}^{(p)} \odot \vec{h}_{rD}^{(p)}\right)$$

If the elements of the gain vector and the compound channel vector were statistically independent, the compound channel coefficients would be complex normal for large N_r. The noise variance at destination (p) is given by

$$\Lambda_{nn}[p\ ,p\ ] = \sigma_w^2 + \sigma_m^2 \cdot \left(\vec{u} \odot \vec{h}_{rD}^{(p)}\right)^H \cdot \left(\vec{u} \odot \vec{h}_{rD}^{(p)}\right)$$

Due to the relay noise contribution the noise variance is a random variable. It involves the sum of N_r random variables. Thus we expect, that for large N_r the noise variance is quite independent of the actual realization of the channel matrices. These considerations suggest, that asymptotically the cumulative distribution function (CDF) of the destination SNR of multiuser ZF relaying resembles the single hop Rayleigh fading case. To this end in Fig. 4 we show the CDF of the normalized destination SNR. We have normalized the SNR such, that for all curves the ratio $\overline{SNR}$ of the average signal power to the average noise power is equal to one. Parameter of the curves is the number N_a of source/destination pairs. For reference the dashed black line show the CDF for complex normal fading coefficients (exponential probability density function). For $N_a \geq 4$ the SNR at the destination is very well approximated by an exponential distribution.

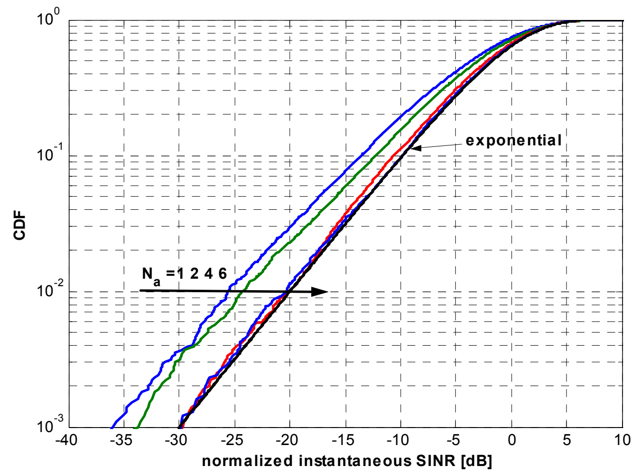

Fig. 4: CDF of the normalized instantaneous SNR

Based on this observation we obtain a very tight approximation of the ergodic sum rate $\bar{R}_{sum}\left(\overline{SNR}, N_a\right)$ of multiuser zero forcing relaying (with complex normal transmit symbols) by invoking the well known formula for the ergodic capacity in Rayleigh fading:

$$\begin{aligned}\bar{R}_{sum}\left(\overline{SNR}, N_a\right) &\approx \frac{1}{2} \cdot \frac{N_a}{\ln 2} \cdot \exp\left(1/\overline{SNR}\right) \cdot Ei\left(1/\overline{SNR}\right) \\ &\leq \frac{N_a}{2} \cdot \log_2\left(1 + \overline{SNR}\right)\end{aligned} \quad (1)$$

$Ei(x)$ is the exponential integral function. The factor ½ accounts for the relay traffic pattern, which requires 2 channel uses for each transmission from a source to its destination.

IV. Average SINR for Noisy Channel State Information

If the channel matrices are perfectly known, ZF relaying eliminates the interference between different source/destination links. In reality however the channel matrices have to be estimated and are subject to channel estimation errors. In this section we analyze the average signal to interference and noise ratio (SINR) at destination node (p) in the presence of estimation errors. Without loss of generality we determine the signal, interference and noise power for a gain vector with unit length. The average relay sum transmit power and the fraction of the total sum transmit power, which is allocated to the relays, enters as a parameter in the final SINR expression. In the analysis we assume, that (i) the channel coefficients are complex normal i.i.d with unit variance, (ii) all sources use the same transmit power and (iii) the total sum transmit power is proportional to the number of source/destination pairs. Furthermore we only consider the minimum relay configuration, i.e. $N_r = N_a \cdot (N_a - 1) + 1$. The analysis is based on the small perturbation approach, i.e. we neglect higher order noise terms.

Average interference power: Let $\hat{H} = H + \Delta H$ denote the estimate of the matrix H (Section II) and ΔH the estimation error. Furthermore let $\vec{d}$ be the ZF gain vector for $\hat{H}$. We define an orthogonal basis V for H, i.e. $V^H \cdot V = I_{N_r - 1}$ and $H = V \cdot V^H \cdot H$. Due to the minimum relay configuration the optimum ZF gain vector $\vec{u} = null(V^H)$. Note that V and $\vec{u}$ are jointly an orthonormal basis for the space $\mathbb{C}^{N_r \times N_r}$. We express the estimation error $\Delta H = \Delta H_u + \Delta H_V$ in terms of the components $\Delta H_u = \vec{u} \cdot \vec{u}^H \cdot \Delta H$ in the dimension $\vec{u}$ and $\Delta H_V = V \cdot V^H \cdot \Delta H$ in the subspace V. In the same way we decompose the actual gain vector $\vec{d} = \vec{d}_u + \vec{d}_V$.

The actual gain vector with a noisy channel estimate solves

$$\begin{aligned} &\left(\vec{d}_u^H + \vec{d}_V^H\right)\left(H + \Delta H_V + \Delta H_u\right) = 0 \\ &\Rightarrow \vec{d}_u^H \cdot \left(\Delta H_u\right) + \vec{d}_V^H \cdot \left(H + \Delta H_V\right) = 0 \end{aligned} \tag{2}$$

The off-diagonal (ISI) elements of the equivalent channel matrix H_{SD} are the elements of the vector

$$\vec{h}_{ISI}^H = \vec{d}^H \cdot H = \vec{d}_V^H \cdot H \tag{3}$$

With (2) and (3) we obtain

$$\vec{h}_{ISI}^H = -\left(\vec{d}_u^H \cdot \Delta H_u + \vec{d}_V^H \cdot \Delta H_V\right) \tag{4}$$

The squared length of the ISI vector follows as

$$\vec{h}_{ISI}^H \vec{h}_{ISI} = \vec{d}_u^H \cdot \Delta H_u \Delta H_u^H \cdot \vec{d}_u + \vec{d}_V^H \cdot \Delta H_V \Delta H_V^H \cdot \vec{d}_V \tag{5}$$

because $\vec{u}$ and V are orthogonal.

Let us assume for the moment, that the estimation error ΔH has i.i.d. elements with variance $\sigma_{\Delta H}^2$ and that it is independent of the ZF gain vector $\vec{u}$. As $\Delta H_u = \vec{u} \cdot \vec{u}^H \cdot \Delta H$ and $\Delta H_V = V \cdot V^H \cdot \Delta H$, with (5) we obtain

$$E_{\Delta H}\left[\vec{h}_{ISI}^H \vec{h}_{ISI}\right] = \sigma_{\Delta H}^2 (N_r - 1) \cdot \left(\vec{d}_u^H \cdot \vec{d}_u\right) + \sigma_{\Delta H}^2 (N_r - 1) \cdot \left(\vec{d}_V^H \cdot \vec{d}_V\right) \tag{6}$$

If the estimation error is very small, $\left|\vec{d}_u\right|^2 \to |\vec{u}|^2 = 1$ and $\vec{d}_V^H \cdot \vec{d}_V \to 0$. Thus the second term in (6) is asymptotically negligible under this assumption.

We adopt this conclusion for the problem at hand and obtain with (4) the approximate ISI vector

$$\vec{h}_{ISI}^H = -\vec{d}_u^H \cdot \Delta H_u \tag{7}$$

For small estimation errors, the component $\vec{d}_u$ of $\vec{d}$ is approximately equal to the optimal gain vector and we have

$$\vec{h}_{ISI} \approx -\vec{u}^H \cdot \Delta H_u$$

With $\Delta H_u = \vec{u} \cdot \vec{u}^H \cdot \Delta H$ (7) simplifies to

$$\vec{h}_{ISI} \approx -\vec{u}^H \cdot \Delta H$$

We model the noisy CSI as additive i.i.d. noise on the actual up- and downlink coefficient vectors

$$\hat{\vec{h}}_{Sr}^{(q)} = \vec{h}_{Sr}^{(q)} + \vec{x}_{Sr}^{(q)}$$

Note, that this in general does not lead to i.i.d elements of the estimation error ΔH. The column of $\hat{H}$ for the link from source (q) to destination (p) is given by

$$\hat{\vec{h}}_{Sr}^{(q)} \odot \hat{\vec{h}}_{rD}^{(p)} = \left(\vec{h}_{Sr}^{(q)} + \vec{x}_{Sr}^{(q)}\right) \odot \left(\vec{h}_{rD}^{(p)} + \vec{x}_{rD}^{(p)}\right)$$

The estimation error is

$$\Delta \vec{h}^{(p,q)} = \vec{h}_{Sr}^{(q)} \odot \vec{x}_{rD}^{(p)} + \vec{x}_{Sr}^{(q)} \odot \vec{h}_{rD}^{(p)} + \vec{x}_{Sr}^{(q)} \odot \vec{x}_{rD}^{(p)}$$

Note, that this vector is a column of the estimation error matrix ΔH. The approximate ISI coefficient from source (q) to destination (p) follows as

$$h_{ISI}^{(p,q)} = \tilde{H}_{SD}[p,q] \approx -\vec{u}^H \left(\vec{h}_{Sr}^{(q)} \odot \vec{x}_{rD}^{(p)} + \vec{x}_{Sr}^{(q)} \odot \vec{h}_{rD}^{(p)} + \vec{x}_{Sr}^{(q)} \odot \vec{x}_{rD}^{(p)}\right)$$

The second moment of this ISI coefficient follows as

$$\begin{aligned} &E_{\vec{x}_{rD}^{(p)}, \vec{x}_{Sr}^{(q)}}\left[\left|h_{ISI}^{(p,q)}\right|^2\right] \\ &\approx \vec{u}^H \cdot \left[\sigma_x^2 \cdot diag\left(\vec{h}_{Sr}^{(q)} \odot \vec{h}_{Sr}^{(q)*}\right) + \sigma_x^2 \cdot diag\left(\vec{h}_{rD}^{(p)} \odot \vec{h}_{rD}^{(p)*}\right) + \sigma_x^4 I_{N_r}\right] \cdot \vec{u} \end{aligned}$$

The sum of the ISI from all sources to destination (p) thus is given by

$$\begin{aligned} &\bar{P}_{ISI}^{(p)} = \sigma_s^2 \cdot \sum_{q \neq p} E_{\vec{x}_{rD}^{(p)}, \vec{x}_{Sr}^{(q)}}\left[\left|h_{ISI}^{(p,q)}\right|^2\right] \\ &\approx \sigma_s^2 \cdot \sigma_x^2 \cdot \left[(N_a - 1)\left(\left|\vec{u} \odot \vec{h}_{rD}^{(p)}\right|^2 + \sigma_x^2\right) + \sum_{q \neq p}\left|\vec{u} \odot \vec{h}_{Sr}^{(q)}\right|^2\right] \end{aligned}$$

The channel average ISI power at destination (p) follows directly

$$\begin{aligned} &\bar{P}_{ISI}^{(p)} = E_H\left[\sigma_s^2 \cdot \sum_{q \neq p} E_{\vec{x}_{rD}^{(p)}, \vec{x}_{Sr}^{(q)}}\left[\left|h_{ISI}^{(p,q)}\right|^2\right]\right] \\ &\approx \sigma_s^2 \cdot \sigma_x^2 \cdot \left[(N_a - 1)\left(E_H\left[\left|\vec{u} \odot \vec{h}_{rD}^{(p)}\right|^2\right] + \sigma_x^2\right) + \sum_{q \neq p} E_H\left[\left|\vec{u} \odot \vec{h}_{Sr}^{(q)}\right|^2\right]\right] \end{aligned}$$

For symmetry reasons the expectations are independent of (p) and (q) (an interchange of uplink and downlink has no effect on H) and we obtain finally

$$\bar{P}_{ISI}^{(p)} \approx \sigma_s^2 \cdot \sigma_x^2 \cdot \left[(N_a - 1)\left(2 g_r(N_a) + \sigma_x^2\right)\right] \tag{8}$$

with $g_r(N_a) \equiv E_H\left[\left|\vec{u} \odot \vec{h}_{Sr}^{(q)}\right|^2\right]$.

Average noise power: The noise variance at destination (p) consists of the local noise contribution σ_w^2 and the contribution $\sigma_r^{(p)2}$ of the relay noise. This is given by

$$\sigma_r^{(p)2} = \sigma_m^2 \cdot \left|\vec{d} \odot \vec{h}_{rD}^{(p)}\right|^2 \approx \sigma_m^2 \cdot \left|\vec{u} \odot \vec{h}_{rD}^{(p)}\right|^2$$

The approximation holds in the small perturbation region, because $\vec{d} \odot \vec{d}^* \approx \vec{u} \odot \vec{u}^*$. The channel averaged relay noise contribution follows as

$$\bar{\sigma}_r^{(p)2} \approx \sigma_m^2 \cdot E_H\left[\left|\vec{u} \odot \vec{h}_{rD}^{(p)}\right|^2\right] \equiv \sigma_m^2 \cdot g_r\left(N_a\right)$$

Average signal power: In the small perturbation region the average signal power at destination (p) is only marginally affected by the error in the gain vector. Thus we use the perfect gain vector in the signal power expression and obtain

$$\bar{S}^{(p)} = \sigma_s^2 \cdot E_H\left[\left|H_{SD}[p,p]\right|^2\right]$$
$$= \sigma_s^2 \cdot E_H\left[\vec{u}^H \cdot \left(\vec{h}_{Sr}^{(p)} \odot \vec{h}_{rD}^{(p)}\right) \cdot \left(\vec{h}_{Sr}^{(p)H} \odot \vec{h}_{rD}^{(p)H}\right) \cdot \vec{u}\right]$$
$$\equiv \sigma_s^2 \cdot g_s\left(N_a\right)$$

Relay sum transmit power: In the small perturbation region the average relay sum transmit power is only marginally affected by the error in the gain vector. Thus we consider the average sum transmit power of all relays for a gain vector $\vec{u}_0 = \vec{u} \cdot c$ with arbitrary length. Furthermore we let $\sigma_m^2 = 0$ (noiseless relays), i.e. we consider the transmit power in the large SNR regime. The contribution of source (q) to the sum output power of the relays is

$$\sigma_s^2 \cdot \left|\vec{h}_{Sr}^{(q)} \odot \vec{u}_0\right|^2$$

As the signals from the different sources are statistically independent, we obtain the sum relay transmit power as the sum of the contributions of all sources:

$$P_{r,0} = \sigma_s^2 \cdot \sum_q \left|\vec{h}_{Sr}^{(q)} \odot \vec{u}_0\right|^2$$

Thus the channel averaged relay sum transmit power follows as

$$\bar{P}_{r,0} = \sigma_s^2 \cdot \sum_q E_H\left[\left|\vec{h}_{Sr}^{(q)} \odot \vec{u}\right|^2\right] \cdot c^2 = N_a \cdot \sigma_s^2 \cdot c^2 \cdot E_H\left[\left|\vec{h}_{Sr}^{(q)} \odot \vec{u}\right|^2\right]$$
$$= N_a \cdot \sigma_s^2 \cdot c^2 \cdot g_r\left(N_a\right)$$

Note that $P_S \equiv N_a \cdot \sigma_s^2$ is the source sum transmit power. Thus the scaling factor c is a function of (i) the number of source/destination pairs and (ii) the fraction of the total sum power, which is allocated to the relays:

$$c^2 = \bar{P}_{r,0} / \left(P_s \cdot g_r\left(N_a\right)\right)$$

SINR: The signal power, the interference power and the relay noise contribution at the destinations are proportional to the gain scaling factor c^2. This is obviously not true for the local noise at the destinations. Thus we obtain the average SINR

$$\overline{SINR}^{(p)} = \frac{c^2 \cdot \bar{S}^{(p)}}{c^2 \cdot \left(\bar{\sigma}_r^{(p)2} + \bar{P}_{ISI}^{(p)}\right) + \sigma_w^2}$$
$$= \frac{g_N\left(N_a\right)}{\left(SNR_u^{-1} + SNR_{est}^{-1} \cdot \left(N_a - 1\right)\left(2 + SNR_{est}^{-1} / g_r\left(N_a\right)\right) + SNR_d^{-1} \dfrac{P_s}{\bar{P}_{r,0}}\right)} \qquad (9)$$

where $g_N\left(N_a\right) \equiv g_s\left(N_a\right) / g_r\left(N_a\right)$. The variable $SNR_u = \sigma_s^2 / \sigma_m^2$ is the average SNR at the relay in a system with one source/destination pair and one relay; $SNR_d = \sigma_s^2 / \sigma_w^2$ is the average SNR at the destination, if this relay is noiseless and $SNR_{est} = 1/\sigma_x^2$ is the SNR of the channel estimator (recall, that the channel coefficients have unit variance).

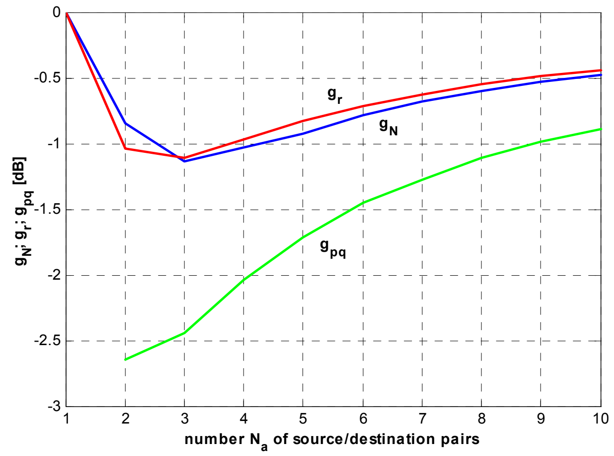

Fig. 5: The functions $g_N\left(N_a\right)$, $g_r\left(N_a\right)$ and $g_{pq}\left(N_a\right)$ versus the number of source/destination pairs.

It is not straightforward to determine $g_N\left(N_a\right)$ and $g_r\left(N_a\right)$ analytically. For this reason in Fig. 5 simulation results are given. Both functions are very similar. Asymptotically for large N_a we obtain $g_N\left(N_a \to \infty\right) = g_r\left(N_a \to \infty\right) = 0dB$.

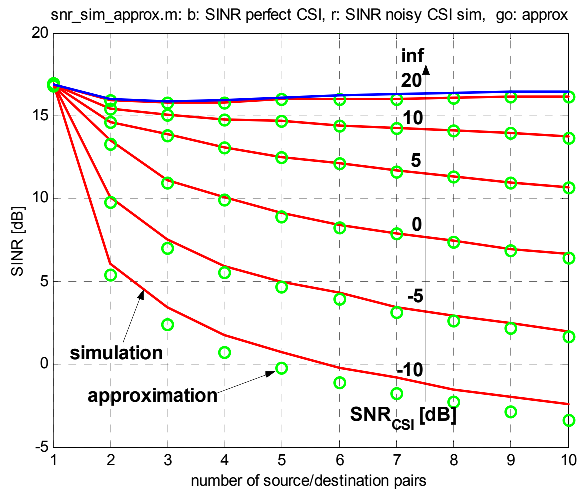

Fig. 6: SINR versus the number of source/destination pairs. Parameter of the curves is the channel estimation error. The circles indicate the analytical results.

In Fig. 6 we compare the approximation (9) of the average SINR with simulation results. Parameter of the curves is SNR_{CSI}, which is defined as the difference of uplink SNR and estimator SNR: $SNR_{CSI} \equiv SNR_{est} - SNR_u\,[dB]$. The simulations were performed for $SNR_u = SNR_d = 20dB$ and we have used the values from Fig. 5 for the approximation. The approximation is very tight for $SNR_{CSI} \geq -5\,dB$. But even for -10dB the approximation holds surprisingly well.

V. Further Sources of Impairment

Phase noise: It is straightforward to apply our analysis to phase noise. Here we assume, that the relays had been perfectly phase synchronous at the time instant, when the channel matrices had been measured. Thus the initial gain vector is correct. The phase noise of the local oscillator in each relay introduces a phase uncertainty in the complex gain coefficient. We define a diagonal matrix $D_{\Delta\varphi} \equiv diag\left(\exp\left(j\Delta\varphi_k\right)\right)$, which contains the random phase errors $\Delta\varphi_k$ and obtain the actual gain vector as $\vec{d} = D_{\Delta\varphi} \cdot \vec{u}$. Let $\Delta\varphi_k$ be a small phase perturbation with zero mean and variance $\sigma_{\Delta\varphi}^2 \ll 1$. For the minimum relay configuration we obtain the average interference power at any one destination as

$$\overline{P}_{ISI}^{(p)} = \sigma_s^2 \cdot \sigma_{\Delta\varphi}^2 \cdot \left(N_a - 1\right) \cdot g_{pq}\left(N_a\right) \qquad (10)$$

where $g_{qp}\left(N_a\right) \equiv E\left[\left|\vec{h}_{Sr}^{(q)} \odot \vec{h}_{rD}^{(p)} \odot \vec{u}\right|^2\right]$. In Fig. 5 the function $g_{qp}\left(N_a\right)$ is plotted.

Quantization noise: To evaluate the impact of quantization noise, we assume, that the real and the imaginary part of the elements of the perfect gain vector $\vec{u}$ are quantized with quantization steps Δq_n. Following the usual assumption of uniformly distributed quantization noise, the quantization noise variance is given by $\sigma_{qn}^2 = \Delta q_n^2 / 12$. The actual gain vector is subject to additive quantization noise $\vec{q}_n$ and we obtain the ISI vector $\vec{h}_{ISI} = \vec{q}_n^H \cdot H$. For the minimum relay configuration the average interference power at any one destination is thus

$$P_{ISI}^{(p)} = \sigma_s^2 \cdot \sigma_{qn}^2 \cdot \left(N_a - 1\right) \qquad (11)$$

VI. Discussion of Results

We have presented an analysis of the average destination SINR of multiuser ZF relaying for different impairments at the relays: (i) noisy channel state information, (ii) phase noise of the local oscillator and (iii) quantization noise of the gain vector. All impairments introduce interference at the destinations. The interference power is proportional to the number N_a of source/destination pairs.

If the channel estimation error is due to a lag time between measurement and transmission, the estimator error variance $\sigma_x^2 = SNR_{est}^{-1}$ is independent of the uplink/downlink SNR $SNR_{u/d}$. According to (9), for a given N_a the destination SINR saturates in the high SNR regime around the value

$$\overline{SINR}^{(p)} \xrightarrow{SNR_{u/d} \to \infty} \frac{SNR_{est}}{2 \cdot \left(N_a - 1\right)}$$

and the sum rate is dominated by the quality of the channel estimation rather than by $SNR_{u/d}$. Phase and quantization noise have a similar effect.

If the channel estimation error is due to a finite energy training sequence, $SNR_{CSI} = const$, i.e. $SNR_{est} \sim SNR_{u/d}$. For a given N_a the noisy CSI results in a constant SNR loss. Nevertheless for realistic N_a multiuser ZF relaying is quite robust to channel estimation errors. From Fig. 6 we see, that for 10 source/destination pairs there is almost no SNR penalty, if $SNR_{CSI} = 20dB$. For $SNR_{CSI} = 10dB$ we observe an SNR loss of only 2dB.

From a theoretical point of view the asymptotic sum rate in the large system case is interesting. For $N_a > 10; SNR_{est} > 10$ the destination SINR is well approximated by (9)

$$\overline{SINR}^{(p)} \approx \frac{SNR}{2\left(1 + SNR_{CSI}^{-1} \cdot N_a\right)}$$

where we have assumed $SNR_u = SNR_d$ and $P_s = \overline{P}_{r,0}$. Clearly for large N_a the system is operated in the low SNR regime, if SNR_{CSI} is finite. Note, that this large system consideration does not violate the small perturbation assumption ($SNR_{est} \gg 1$). As $\log_2\left(1+x\right) \approx x / \ln 2 \quad \forall x \ll 1$ with (1) we obtain the a large N_a approximation of the sum rate

$$\lim_{N_a \to \infty} \overline{R}_{sum} \approx \frac{SNR_{est}}{4 \cdot \ln 2}$$

This is quite a fundamental result, as with perfect CSI the average sum rate would grow beyond any limit (1).

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