Abstract—Placing relays around the base station (BS) to assist wireless communication is an effective way of extending coverage in cellular systems. This paper provides a quantitative analysis of coverage extension by using decode-and-forward (DF) relays. To describe the relation between the number of relays and the coverage range extension, we introduce the concept of coverage angle and coverage range. We provide analytical upper and lower bounds for the coverage range in a cellular system for any given coverage angle. By means of simulations, we show the tightness of our analytical approach.

I. INTRODUCTION

The goal of future wireless communication systems is to provide much higher transmission rate with the transmission power that is in the order of today’s 3G systems. One consequence is that the transmitted power per bit will be just a small fraction of that in our current wireless systems. Furthermore 4G systems will move to higher center frequency bands, hence they are also subject to much higher pathloss. Those facts make it impossible for 4G systems to cover the same range with the same infrastructure as today’s. One effective way of solving the coverage problem is to place relay stations (RS) in the system to assist communication [1], [2].

The topic of relaying in wireless networks has become an active and vital area of research recently. In [3] and [4], several relaying strategies for wireless multihop networks were outlined. The authors of [5] analyzed the spatial diversity performance of various protocols and found that full spatial diversity can be achieved by certain protocols provided that appropriate power control is employed. A frequency channel reuse scheme in a cellular relaying network was proposed in [6], which uses a pre-configured relaying channel selection algorithm to minimize co-channel interference in the network. Network aspects in relaying were analyzed in e.g. [7].

This paper is intended to analyze the coverage extension by using fixed decode-and-forward (DF) relay stations with multiple antennas in a cellular downlink environment. We consider dedicated relay stations that have sufficient power supply, higher signal processing capabilities and better antennas than the mobile stations (MS). Also because such fixed relays are normally placed on selected positions where the channel with the base station (BS) is very good, the distance from the BS to the RS can be much larger than the distance between BS and any MS within the original cell. We consider the scenario that each relay is responsible for assisting transmission to one mobile station. This can be considered as a measurement campaign scenario where one mobile equipment moves around to determine the coverage and only receives data from its nearest relay. Due to space constraint, we only consider uniform power allocation at the transmitter antennas.

The remainder of the paper is organized as follows: In Section II, the maximum transmission rate for the DF relaying scheme is summarized. In Section III, we describe the idea and metric of coverage range and its relationship with coverage angle in detail. We present two analytical approximations of the coverage range with respect to a certain coverage angle in Section IV. The simulation setup and the performance results are presented in Section V. Conclusions are drawn in Section VI.

II. TRANSMISSION RATE FOR DF RELAYING SCHEMES

Fig. 1 shows a relaying system with one BS, one RS and one MS, all equipped with multiple antennas. The downlink transmission rate of such a system has been shown in [8]. We recapitulate the main results in this section.

We denote the number of antennas at BS, RS and MS as $M/R/N$ respectively. $H_0$, $H_1$ and $H_2$ are the direct (BS-MS) channel, the first hop (BS-RS) channel and the second hop (RS-MS) channel matrices, respectively. And the power constraint at BS and RS are $P_{BS}$ and $P_{RS}$. We assume uniform power allocation for transmission. The noise vectors at RS and MS are i.i.d. Gaussian distributed, with noise variance $\sigma_r^2$ and $\sigma_d^2$ at each antenna respectively. Since today’s relays normally cannot send and receive signals at the same time in the same frequency, we only consider half-duplex relays in this paper.

In the DF relaying scheme, the BS first transmits the data symbol $x_0$ in the first time slot. Let $y_{1\alpha}$ and $y_{1\alpha}^{(1)}$ represent the signals received at RS and MS in the first time slot. The
maximum mutual information of the first hop transmission can be expressed as

\[ I(x_{i}; y_{r}) = \log_2 \det \left( I_R + \frac{P_{BS}}{M\sigma_i^2} H_i H_i^H \right). \]  

(1)

The RS retransmits the received data to the MS if the decoding is successful. We denote the signal received at the MS in the second time slot to be \( y_d^{(2)} \). The maximum mutual information between the transmitted data symbols and the received signals at MS can be expressed as [8]

\[ I(x_{i}; y_d^{(1)}, y_d^{(2)}) = \log_2 \det \left( I_N + \frac{P_{BS}}{M\sigma_i^2} H_0 H_0^H + \frac{P_{RS}}{\rho\sigma_i^2} H_2 H_2^H \right). \]  

(2)

We require both the RS and MS to fully decode the received data signals. Thus the maximum transmission rate for the considered overall system can be calculated as

\[ C = \frac{1}{2} \min \{ I(x_{i}; y_{r}), I(x_{i}; y_d^{(1)}, y_d^{(2)}) \}. \]  

(3)

The factor 1/2 in front of (3) comes from the fact of two channel uses.

III. COVERAGE RANGE EXTENSION IN CELLULAR SYSTEMS

In the following, we explain the concepts of coverage angle and coverage range in detail. Throughout the paper, we assume different relays use different channels (e.g. frequency bands) to serve users within their coverage, and the relays do not cooperate in transmission, i.e. each relay transmits data independently. Each MS does not combine signals from different relays. Therefore, we do not consider intracell interference in this paper.

Upcoming 4G systems are expected to move to much higher carrier frequencies (> 5 GHz) than today’s systems (< 2.2 GHz); this leads to higher signal attenuations. In addition, the bandwidth of those new systems will be much broader than the bandwidth of today’s cellular systems, but the transmit power will probably not increase. Therefore we expect that the coverage range of a 4G base station will be small compared to today’s cellular systems. In this paper we investigate the coverage range and propose methods to increase it. We neglect the influence of intercell interference on the coverage range by investigating a single cell scenario; this refers to a case where neighboring cells use orthogonal channels (e.g. OFDMA).

A. Coverage Angle vs. Coverage Range

In this paper, we assume isotropic channel conditions. Thus the original cell (without relays) is circular. In accordance with most other papers on cellular relaying (e.g. [6]), we consider the case that the relays are placed uniformly surrounding the BS. After placing relays, we still require the new cell to be a circular cell, in which a certain data-rate QoS requirement is fulfilled. We refer to the maximum cell radius achieved by the BS and the uniformly placed relays as coverage range \( r_{cov} \).

Note that we always require a circular shape for the cell. If the border of the area in which the QoS requirement is fulfilled is not a circle, the circle which has the maximum radius within this shape determines \( r_{cov} \).

To clarify our definition of coverage range, Fig. 2 shows the coverage region by placing eight relays. The radius \( r_0 \) is the coverage range for the BS only. The maximum circular area is defined by the radius \( r_2 \) which is the new \( r_{cov} \) for the system of the BS and relays. In Fig. 2 the angle \( \alpha \) that determines the size of the circle sector supported by one specific relay is also shown. We refer to this angle as coverage angle \( \alpha_{cov} \). The coverage range can be extended by placing more relays around the BS. For example, in Fig. 2, the coverage range of the system can approximate \( r_1 \) by placing infinite relays uniformly surrounding the BS. In the following, when we say the coverage angle is \( \alpha_{cov} \), we mean that \( N_r = \lfloor 360^\circ / \alpha_{cov} \rfloor \) relays are placed uniformly in the cell, and they divide the cell into equal sized sectors with angle \( \alpha_{cov} \). The coverage range is the maximum radius of circular area achieved by placing those \( N_r \) relays. Due to symmetry, we only depict one relay in the following if the coverage angle is specified.

B. QoS Requirements and SNR Regime

The QoS requirement determines the coverage area, and thus highly influences the achievable coverage range \( r_{cov} \). The QoS requirement can be related either to the ergodic or to the outage capacity. In this paper we require the 1%-outage capacity for each point in the coverage area to be higher than 1 bps/Hz. This means the MS at the border of the cell is in the low SNR regime, where no spatial multiplexing gain is available and the capacity scales linearly with the SNR. This makes diversity and array gain techniques much more attractive compared to spatial multiplexing.

IV. ANALYSIS OF COVERAGE RANGE IN RELAYING SYSTEMS

If \( N_r \) relays are placed uniformly on a circle surrounding the BS, each relay covers a sector of the coverage angle \( 360^\circ / N_r \). For a certain coverage angle (or the corresponding number of relays), there exists an optimal position for the relay placement where the maximum circular coverage range can be achieved. Normally, relays are placed where good connection with the BS can be established. It is reasonable to assume that the first hop channel \( H_1 \) is much better than \( H_0 \) and \( H_2 \), at least
for the users on the border. Thus for those border users that determine the coverage range, we have
\[ I(x_{a};y_{t}) \gg I(x_{a};y_{d}^{(1)},y_{d}^{(2)}), \]
where \( I(x_{a};y_{t}) \) and \( I(x_{a};y_{d}^{(1)},y_{d}^{(2)}) \) are the mutual information expressed in (1) and (2) respectively. Thus for border users, the capacity expression for the whole DF relaying system can be written as
\[
C \approx \frac{1}{2} \log_{2} \det \left( I_{N} + \frac{P_{BS}}{\sigma_{d}^{2}} H_{0} H_{0}^{H} + \frac{P_{RS}}{\sigma_{d}^{2}} H_{2} H_{2}^{H} \right).
\]
The coverage range of the whole relaying system can be approximated by considering the circular coverage range of the BS and that of the RS individually. In the following analysis, we just consider the cases when each BS and RS has the same number of antennas and equal power allocation at BS and RS, i.e. \( M = R \) and \( P_{BS} = P_{RS} \). Please note that relays can be placed outside the original cell of the BS because the BS-RS channel is much better than the BS-MS channel.

In the following, we provide two ways of approximating the coverage range in a cellular network. The optimistic approximation assumes that the BS has the same coverage as that of the original BS. This is true when the border users receive equally strong signals from the BS and the RS. But for some other border users, who are much farther away from the BS than from the RS, the signal from the RS is much stronger than the BS signal. Thus we also propose the pessimistic approximation to calculate the coverage of such a relaying network. The BS-MS and RS-MS channels are assumed to be i.i.d. Rayleigh fading in our analysis.

A. “Optimistic” Approximation for Coverage Range

Let us first consider the border user \( A \) that is depicted in Fig. 3. Its distances to the BS and RS are approximately the same. Since \( M = R \) and \( P_{BS} = P_{RS} \), the signals from BS and RS are approximately equally strong, which results in the SNR improvement at MS. For the best case, we have \( H_{0} H_{0}^{H} = H_{2} H_{2}^{H} \). Thus the transmission rate of user \( A \) can be approximated as
\[
C \leq C_{u} = \frac{1}{2} \log_{2} \det \left( I_{N} + \frac{2P_{RS}}{\sigma_{d}^{2}} H_{2} H_{2}^{H} \right) = \frac{1}{2} \sum_{i=1}^{N} \log_{2} \left( 1 + \frac{2P_{RS}}{\sigma_{d}^{2}} \lambda_{i} \right),
\]
where \( \lambda_{i} \) are the eigenvalues of \( H_{2} H_{2}^{H} \). Since the users on the border of the cell is in the low SNR regime, we use \( \log_{2}(1 + x) \approx x \log_{2}(e) \) for small \( x \) and get
\[
C_{u} \approx \frac{P_{RS}}{\sigma_{d}^{2}} \log_{2}(e) \cdot \sum_{i=1}^{N} \lambda_{i}. \]
In this best case, the capacity loss due to two hop transmission is compensated for by the combination of BS and RS signals. The maximum transmission rate expression of (7) is the same as that of the one-hop transmission border users. So the optimistic approximation assumes the radius of the RS coverage to be the same as that of the original BS coverage.

For the i.i.d. Rayleigh channel, \( \sum_{i=1}^{N} \lambda_{i} \) is a Chi-squared random variable with \( 2RN \) degrees of freedom [9]. Without pathloss, the PDF of \( \sum_{i=1}^{N} \lambda_{i} \) can be written as
\[
f_{1}(x) = \frac{x^{RN-1}}{(RN-1)!} e^{-x} u(x),
\]
where \( u(x) \) is the unit step function. And its CDF expression can be found in [10].

If the pathloss from RS to user A is \( PL_{2} \) dB, the PDF of the maximum coverage range is a scaled version of \( f_{1}(x) \) in (8). The scaling factor is
\[
\eta = \frac{P_{RS}}{\sigma_{d}^{2}} \log_{2}(e) \cdot 10^{\frac{PL_{2}}{10}},
\]
Since the pathloss is a function of the distance between the transmitter and receiver and we require the 1% outage transmission rate to be larger than 1 bps/Hz, the distance from RS to the border user A (i.e. the radius of RS coverage in Fig. 3) can be calculated by solving the equation on the CDF of \( C_{u} \)
\[
F_{2}(y) = \int_{0}^{y} f_{1}(x) dx = 0.01.
\]
where \( y = 1 \) bps/Hz.

Of course, not all border users in RS coverage can have the same channel condition as user A. But the coverage range of the whole system is upper bounded by assuming the coverage area of each RS to be the same as that of the BS. And we denote their radiiuses as \( a \). As depicted in Fig. 3, the line \( OT \) is tangent to the BS coverage circle and RS coverage circle at \( O \) and \( T \), respectively. For a coverage angle \( \theta > 60^\circ \), the maximum coverage range is achieved when the point \( T \) is on the angle \( \theta \), which corresponds to the coverage range of \( a/\sin(\theta/2) \). And for the coverage angle \( \theta \leq 60^\circ \) as depicted in Fig. 4, the maximum coverage range equals the distance from BS to the point \( A' \), which crosses the crossing point \( A \) of the
two circles. This is because the tangent point $T$ cannot be on the angle $\theta$, otherwise the coverage range is not continuous. So the expression for coverage range vs. coverage angle with optimistic approximation is as follows:

$$r_{cov} = \begin{cases} a + 2a \cos \theta, & \text{if } \theta < 60^\circ; \\ \frac{a}{\sin(\theta/2)}, & \text{if } \theta \geq 60^\circ. \end{cases} \quad (11)$$

where $a$ is calculated according to (10).

B. “Pessimistic” Approximation for Coverage Range

As mentioned before, not all border users can receive equally strong signals from BS and RS. As depicted in Fig. 5, the border user $B$ on the other side of the RS is much farther away from BS than from RS. So $\|H_0\|_2^2 \ll \|H_2\|_2^2$. We ignore the signal contribution from BS and get the transmission rate of user $B$ as

$$C \geq C_1 = \frac{1}{2} \log_2 \det \left( I_N + \frac{P_{RS}}{R^2} H_2 H_2^H \right) = \frac{1}{2} \sum_{i=1}^{N} \log_2 \left( 1 + \frac{P_{RS}}{R^2} \lambda_i \right), \quad (12)$$

where we also assume Rayleigh fading on the second hop channel, and $\lambda_i$, $i = 1 \ldots N$ are the eigenvalues of $H_2 H_2^H$. The coverage range lower bound is achieved by calculating the radius of RS coverage according to (12), which is much smaller than the radius of the BS coverage.

We denote the radius of the BS covered area as $a_1$ and the radius of the RS covered area as $a_2$ respectively as depicted in Fig. 5. After some calculations, the expression for coverage range vs. coverage angle in this case can be expressed as

$$r_{cov} = \begin{cases} a_1 + 2a_2 \cos \varphi, & \text{if } \theta < \theta_1; \\ \frac{a_2}{\sin(\theta/2)}, & \text{if } \theta_1 \leq \theta < \theta_2; \\ a_1, & \text{if } \theta \geq \theta_2, \end{cases}$$

where $\theta_1 = 2 \arcsin \left( \sqrt{p^2 + 8 - p}/4 \right)$, $\theta_2 = 2 \arcsin(1/p)$ and $\cos \varphi = \cos(\theta/2) \sqrt{1 - p^2 \sin^2(\theta/2) - p \sin^2(\theta/2)}$ with $p = a_1/a_2$. $a_1$ and $a_2$ are calculated according to (10) and (12) respectively.

V. SIMULATION RESULTS

In the following, we present the simulation setup and the performance results of our coverage range vs. coverage angle analysis. DF relaying schemes are compared with single-hop transmission (direct transmission). The channel knowledge at the receiver is always assumed to be perfectly available. Different antenna configurations are displayed as $M/R/N$ in the figures.

A. Reference Channel Model

In this paper, we assume the BS to be placed at above rooftop level, i.e. more than 30m high. The RS is placed on rooftop level, so we can assume good channel conditions between the BS and the RS. The MS is located at street level, i.e. at the height of a person. The transmit power at BS and RS is 1W each, which is much lower than current wireless systems. We choose the same transmit power at RS as at BS because the transmit power at BS is already very low.

We use the following channel models, whose parameters were proposed in the IST WINNER project [11]. We consider a wideband channel, but we constrict ourselves to frequency-flat fading. This restriction gives us lower bounds on the coverage range performance by relaying, because frequency diversity can also be exploited to enhance the coverage range in frequency-selective fading environments. The first hop channel $H_1$ is a Ricean channel with $K$-factor of 10 dB, while the direct path channel $H_0$ and second hop channel $H_2$ are pure Rayleigh channels. The pathloss model is as follows:

$$PL_0 = 35.0 \log_{10}(d_0) + 38.4; \quad (13)$$
$$PL_1 = 36.5 + 20 \log_{10}(f_c/2.5) + 23.5 \log_{10}(d_1); \quad (14)$$
$$PL_2 = 50 \log_{10}(d_2) + 38.4. \quad (15)$$

where $PL_0$, $PL_1$ and $PL_2$ are the pathloss in dB for the direct channel (BS-MS), first hop channel (BS-RS) and second hop channel (RS-MS) respectively. $d_0$, $d_1$ and $d_2$ are the distance between the transmitter and receiver for the corresponding channels. $f_c$ represents the center frequency in GHz. In our simulations, $f_c = 5$ GHz.

The simulation parameters are summarized in Table I.

<table>
<thead>
<tr>
<th>TABLE I SIMULATION PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
</tr>
<tr>
<td>Center frequency</td>
</tr>
<tr>
<td>Transmit power at BS and RS</td>
</tr>
<tr>
<td>Noise figure of RS and MS</td>
</tr>
<tr>
<td>QoS requirement (1%-outage capacity)</td>
</tr>
</tbody>
</table>

B. Analytical Approximations for Coverage Range

The simulation results and theoretical approximation of coverage range vs. coverage angle diagram for $M/R/N = 1/1/1$, $2/2/2$ and $4/4/4$ antenna configurations are depicted in Fig. 6. The optimistic and pessimistic approximations both have the same starting point at coverage angle $\alpha_{cov} = 180^\circ$. This is because at $\alpha_{cov} = 180^\circ$, the covered area of the RS merges into the area covered by the BS. So the coverage range at $\alpha_{cov} = 180^\circ$ equals the radius of BS covered area. For $2/2/2$ and $4/4/4$ antenna configurations, the pessimistic approximation is quite tight for coverage angle below $60^\circ$. For $1/1/1$ antenna configuration, the simulation and pessimistic approximation results gradually approach the same coverage range as $\alpha_{cov} \rightarrow 0^\circ$. 

Fig. 5. Pessimistic approximation for coverage range

\[ \text{Fig. 5. Pessimistic approximation for coverage range} \]
C. Coverage Range Improvement for Relaying Schemes

Fig. 7 shows the coverage range of DF relaying schemes compared to the coverage range of direct transmission for different types of antenna configurations. The coverage ranges of the relaying systems are better than their corresponding one-hop transmission systems for the coverage angle $\alpha_{cov} < 120^\circ$. This means $N_r = 3$ relays per cell can guarantee the coverage range extension. As shown in Fig. 7, placing more relays (lowering coverage angle) will achieve higher coverage range extension. For example, for a $(4/4/4)$ configuration and coverage angle $\alpha_{cov} = 60^\circ$, which corresponds to 6 relays per cell, the DF relay achieves coverage range of $r_{cov} \approx 300$m. Compared to the coverage range of direct transmission ($r_{cov} \approx 200$m), this corresponds to an 50% increase in coverage range.

Using multiple antennas is another effective means for coverage extension. As shown in Fig. 7, the coverage range of $2/2/2$ antenna configuration more than doubles the coverage range of $1/1/1$ antenna configuration for any coverage angle. And the coverage range of $4/4/4$ relaying system nearly doubles that of the $2/2/2$ system. Furthermore we conclude that the coverage range does not improve by placing more antennas at BS if we have fixed antenna configurations ($R/N$) at RS and MS. This can be seen from the simulation result of $8/4/2$ antenna configuration as compared to that of $4/4/2$ antenna configuration. Their coverage ranges are nearly identical. The same is also true for the simulation result of $8/4/4$ antenna configuration as compared to that of $4/4/4$ antenna configuration. This is because the first hop channel between BS and RS is already very good and the overall transmission rate is just limited by the second hop. Thus placing more antennas at BS provides no more advantages.

VI. CONCLUSIONS

We propose the concepts of coverage angle and coverage range in this paper. And we propose two analytical approximations of calculating the coverage range for any given coverage angle based on the outage capacity criterion. The simulation results show that these two methods provide the upper and lower bounds for the cellular relaying coverage range. Furthermore, simulations show that the coverage range extension for MIMO relaying systems is significant as compared to direct transmission, even though we have a rather high QoS requirement and require a circular cell. Huge improvement can be obtained by using multiple antennas. We can conclude that the use of multiple antennas and multiple relays is promising for coverage extension for 4G systems. If the first hop channel is already very good, as is the case for most relaying systems, placing more antennas at BS does not provide additional coverage extension.

REFERENCES