Monostatic Indoor Localization: Bounds and Limits

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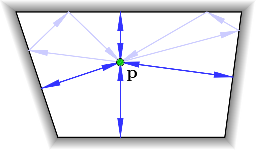
Monostatic Indoor Localization

Agent transmits an UWB pulse and listens to the echoes. With a known floorplan, the echoes yield information about $\mathbf{p}$.



Monostatic Applications:

- Multipath-assisted localization with cooperation and/or anchors[1]
- Cooperative Anchor-less UWB Tracking and Room Mapping[2]
- Bat-like localization and mapping with zero infrastructure.

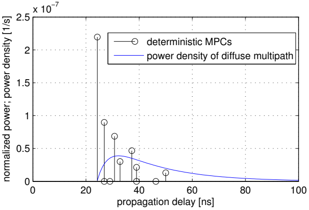
This Paper: CRLB for position estimation from monostatic measurements to quantify the contained position information.

[1] Leitinger and Witrisal et al., "Evaluation of Position-related Information in Multipath...", *JSAC 2014*

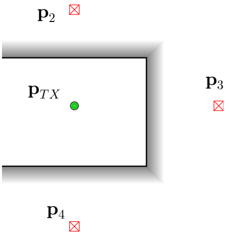
[2] Noes and Denis, "...Cooperation in Simultaneous Anchor-less Tracking and Room Mapping...," *IWSSIP 2012*

Signal Model & Virtual Anchor Concept

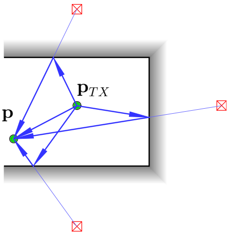
$$r(t) = \underbrace{\sum_{k=1}^{K} \alpha_k s(t - \tau_k)}_{\text{Deterministic MPCs}} + \underbrace{(s * \nu)(t)}_{\text{Diffuse MP}} + \underbrace{n(t)}_{\text{AWGN}}$$



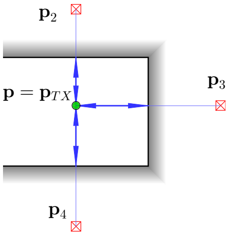
Delays relate to geometry: $\tau_k = \frac{1}{c}||\mathbf{p} - \mathbf{p}_k||$



Virtual Anchors



Bistatic MPCs



Monostatic MPCs

CRLB for Monostatic Position Estimation

Equivalent Fisher Inf. Matrix

$$\boldsymbol{\mathcal{I}}_{\mathbf{p}} = \frac{\partial \boldsymbol{\tau}}{\partial \mathbf{p}}^T \boldsymbol{\Lambda} \frac{\partial \boldsymbol{\tau}}{\partial \mathbf{p}}$$

Cramér-Rao Lower Bound

$$\text{var}\{\hat{\mathbf{p}}\} \geq \text{tr}\{\boldsymbol{\mathcal{I}}_{\mathbf{p}}^{-1}\}$$

$\boldsymbol{\Lambda}$ ...... Detectability of path delays in the received signal: pulse shape, bandwidth, overlap, diffuse MP, noise. Known Quantity[1,2]

$\frac{\partial \boldsymbol{\tau}}{\partial \mathbf{p}}$ ...... Spatial sensitivity of delays subject to indoor geometry. Monostatic Case: Involved → Scope of this Paper

$$\frac{\partial \boldsymbol{\tau}}{\partial \mathbf{p}}^{\mathrm{T}} = \left[\frac{\partial \tau_1}{\partial \mathbf{p}} \cdots \frac{\partial \tau_k}{\partial \mathbf{p}} \cdots \frac{\partial \tau_K}{\partial \mathbf{p}} \right]$$

Spatial delay gradient of the k-th multipath component

[1] Shen and Win, "Fundamental Limits of Wideband Localization - Part I: ...", *Trans.IT 2010*

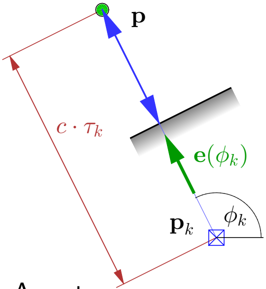
[2] Witrisal and Meissner, "Performance Bounds for Multipath-assisted Indoor Navigation...", *ICC 2012*

Spatial Delay Gradient: Approach

$\frac{\partial \tau_k}{\partial \mathbf{p}}$ required in CRLB $\longrightarrow$ Differentiate $\tau_k = \frac{1}{c}\|\mathbf{p} - \mathbf{p}_k\|$ $\longrightarrow$

Intermediate Result

$$\frac{\partial \tau_k}{\partial \mathbf{p}} = \frac{1}{c}\left(\mathbf{I} - \frac{\partial \mathbf{p}_k}{\partial \mathbf{p}}\right)^{\mathrm{T}} \mathbf{e}(\phi_k)$$



c .... Speed of Light

$\phi_k = \sphericalangle(\mathbf{p} - \mathbf{p}_k)$ .... Angle from VA to Agent

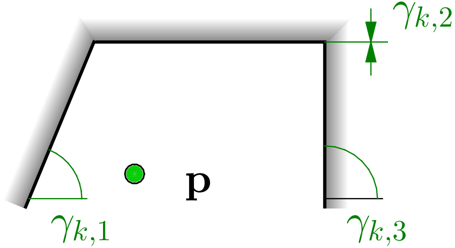
$\mathbf{e}(\phi_k) = \frac{\mathbf{p}-\mathbf{p}_k}{\|\mathbf{p}-\mathbf{p}_k\|}$ .... Unit Vector from VA to Agent

$\frac{1}{c}\mathbf{e}(\phi_k)$ .... Bistatic Gradient (fixed Anchors[1], Radar[2])

[1] Shen and Win, "Fundamental Limits of Wideband Localization - Part I: ...", *Trans.IT 2010*

[2] Godrich et al., "...Accuracy Gain in MIMO Radar-Based Systems", *Trans.IT 2010*

Example: Virtual Anchor of Order $Q_k = 3$



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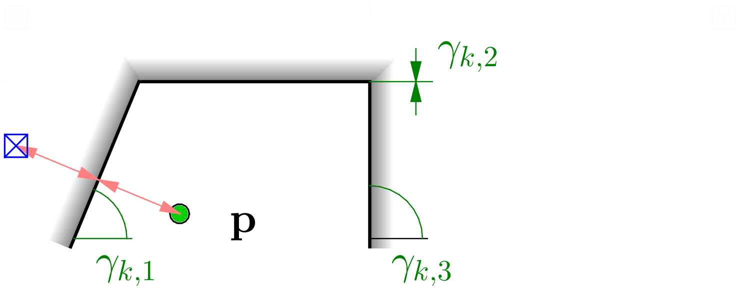

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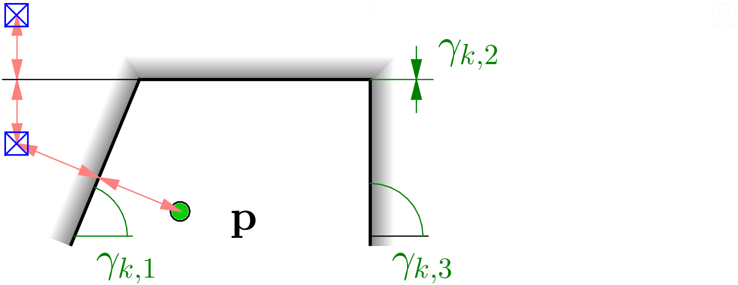

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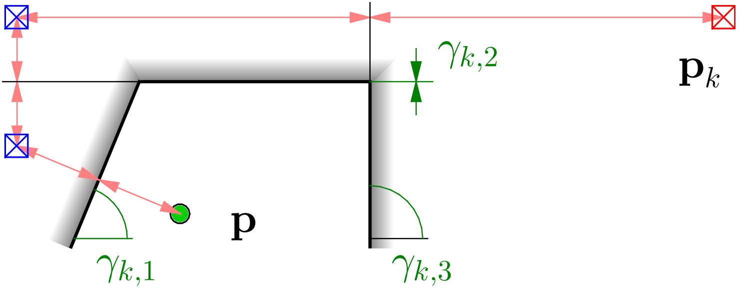

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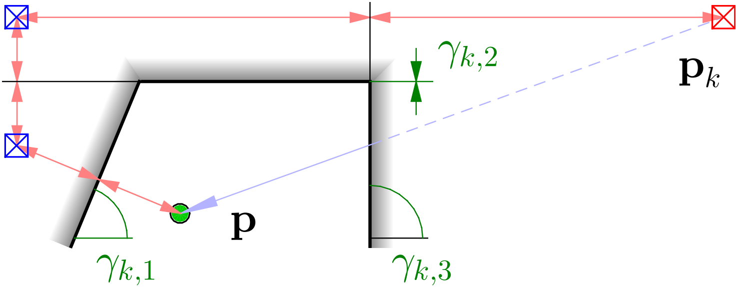

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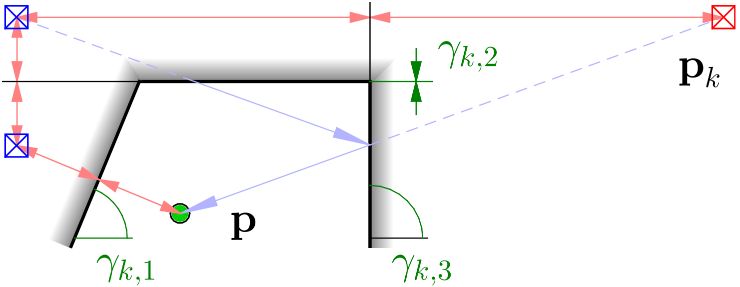

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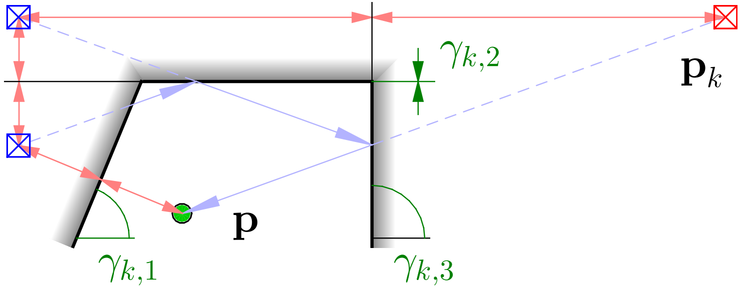

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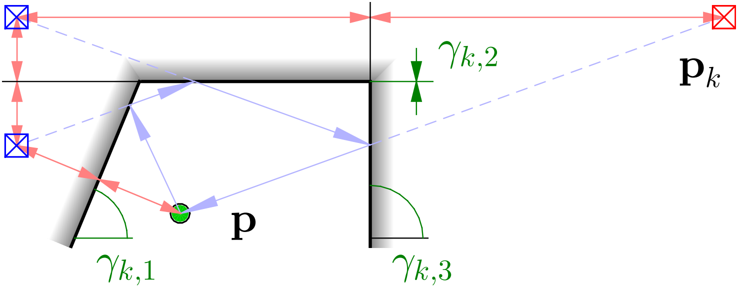
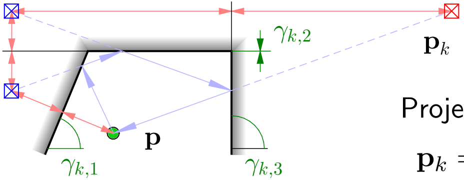


Jacobian for an Order Q_k Reflection



$$\bar{\gamma}_k = \gamma_{k,1} - \gamma_{k,2} + \gamma_{k,3} = \frac{3\pi}{8} - 0 + \frac{\pi}{2}$$

Mirror Matrix Property

$$\mathbf{M}(\gamma)\mathbf{M}(\beta) = \mathbf{M}(\gamma-\beta)\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \mathbf{Rot}(2(\gamma-\beta))$$

Projective geometry yields VA position

$$\mathbf{p}_k = f(\mathbf{p}, \gamma_{k,1}, \ldots, \gamma_{k,Q_k}, \text{wall offsets})$$

Jacobian is a composition of mirrorings

$$\left(\frac{\partial \mathbf{p}_k}{\partial \mathbf{p}}\right)^T = \prod_{q=1}^{Q_k} \mathbf{M}(\gamma_{k,q}) = \mathbf{Rot}(2\bar{\gamma}_k)\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}^{Q_k}$$

Effective wall angle of the k-th MPC

Spatial Delay Gradient: Result

Effective Wall Angle

$$\bar{\gamma}_k = \sum_{q=1}^{Q_k} (-1)^{q-1} \cdot \gamma_{k,q}$$

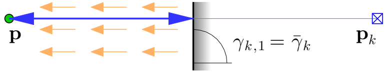
$$\frac{\partial \tau_k}{\partial \mathbf{p}} = \frac{1}{c}\Big(\mathbf{e}(\phi_k) \ - \ \mathbf{e}\big((-1)^{Q_k}\phi_k + 2\bar{\gamma}_k\big) \Big)$$

Depending on multipath geometry, terms add up or cancel.

Magnitude-times-Direction Form

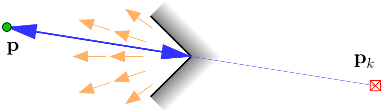
$$\frac{\partial \tau_k}{\partial \mathbf{p}} = \frac{2}{c} \begin{cases} \sin(\bar{\gamma}_k)\ \mathbf{e}(\bar{\gamma}_k + \phi_k - \frac{\pi}{2}) & \text{If } Q_k \text{ is even} \\ \sin(\bar{\gamma}_k - \phi_k)\ \mathbf{e}(\bar{\gamma}_k - \frac{\pi}{2}) & \text{If } Q_k \text{ is odd} \end{cases}$$

Examples: Common Monostatic MPC Geometries



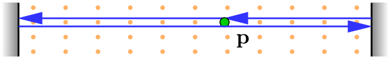
Single Reflection

$$\bar{\gamma}_k = \phi_k \pm \frac{\pi}{2} \quad \Rightarrow \quad \frac{\partial \tau_k}{\partial \mathbf{p}} = \frac{2}{c}\mathbf{e}(\phi_k)$$



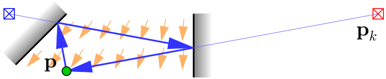
⊥-Corner Reflection

$$\bar{\gamma}_k = \pm \frac{\pi}{2} \quad \Rightarrow \quad \frac{\partial \tau_k}{\partial \mathbf{p}} = \frac{2}{c}\mathbf{e}(\phi_k)$$



Parallel Walls Reflection

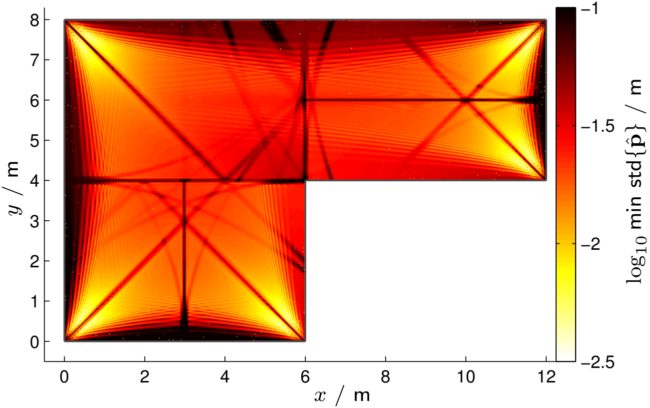
$$\bar{\gamma}_k = 0 \quad \Rightarrow \quad \frac{\partial \tau_k}{\partial \mathbf{p}} = \mathbf{0}$$



Crooked Walls Reflection

$$\frac{\partial \tau_k}{\partial \mathbf{p}} = \frac{2}{c}\sin\big(\bar{\gamma}_k\big)\mathbf{e}\big(\phi_k + \bar{\gamma}_k - \frac{\pi}{2}\big)$$

Example: Monostatic CRLB over 2D L-Room

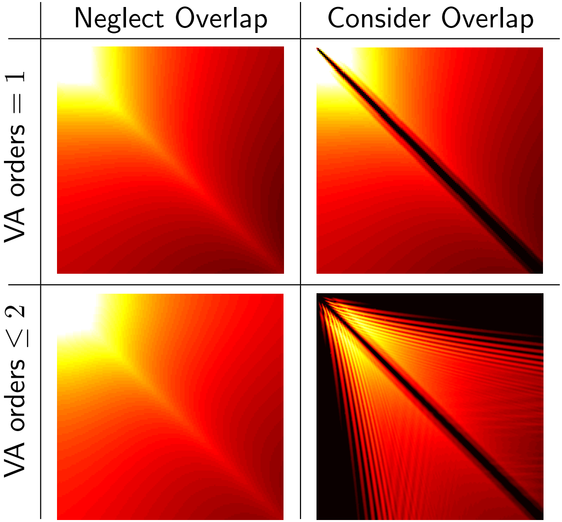


RRC Pulse of 1GHz BW, Reflection Orders ≤ 2,

-3dB / Reflection, Path overlap considered,

Diffuse Multipath: $\gamma_{\text{rise}} = 5\text{ns}, \gamma_1 = 20\text{ns}$

Reflection Order and Overlap

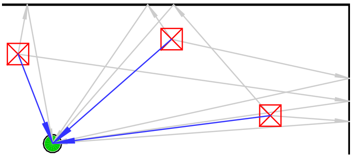


Summary

- The CRLB on position error is composed of spatial gradients of multipath delays and the FIM of signal model parameters.
- The spatial gradients are involved functions of room geometry, which was tackled by means of projective geometry.
- The spatial sensitivity is double compared to anchor schemes.
- Path overlap can be a major problem close to walls and on the room's symmetry axes.
- Monostatic (bat-like) localization is intrisically non-robust but has great potential for supporting other localization schemes.

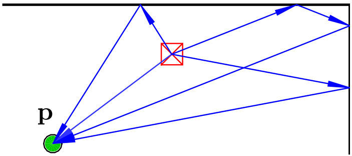
Backup: Multipath-Assisted Localization

Conventional Time-of-Arrival



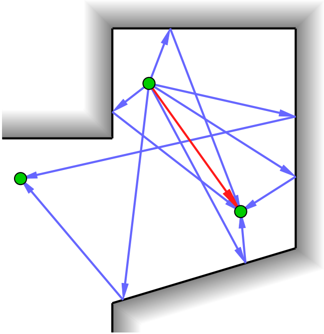
- Requires ≥ 3 fixed anchors $\rightarrow$ expensive, inflexible
- Multipath poses interference (unless prior channel knowledge is available)

Multipath-Assisted

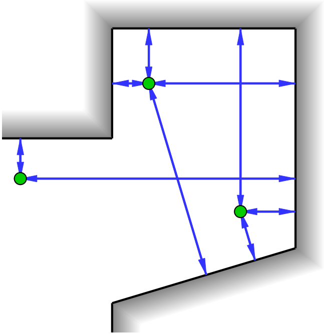


- Assumed prior knowledge: Known floorplan
- Utilizes reflected paths
- Enables trilateration with just a single fixed anchor

Backup: Cooperative MP-Assisted Localization



Bistatic



Monostatic

Backup: CRLB (Single Transmission)

Parameter vector

$$\boldsymbol{\psi} := \left(\boldsymbol{\tau}^T,\ \Re\boldsymbol{\alpha}^T,\ \Im\boldsymbol{\alpha}^T\right)^T$$

Fisher Information Matrix (FIM) $\boldsymbol{\mathcal{I}}_{\boldsymbol{\psi}}$

Parameter Transformation

$$\boldsymbol{\theta} := \left(\mathbf{p}^T,\ \Re\boldsymbol{\alpha}^T,\ \Im\boldsymbol{\alpha}^T\right)^T$$

$$\boldsymbol{\mathcal{I}}_{\boldsymbol{\theta}} = \frac{\partial \boldsymbol{\psi}}{\partial \boldsymbol{\theta}}^T \cdot \boldsymbol{\mathcal{I}}_{\boldsymbol{\psi}} \cdot \frac{\partial \boldsymbol{\psi}}{\partial \boldsymbol{\theta}}$$

Parameter Jacobian

$$\frac{\partial \boldsymbol{\psi}}{\partial \boldsymbol{\theta}} = \begin{pmatrix} \partial\boldsymbol{\tau}/\partial\mathbf{p} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}$$

Equivalent FIM (EFIM)

$$\boldsymbol{\mathcal{I}}_{\mathbf{p}}^{-1} = [\boldsymbol{\mathcal{I}}_{\boldsymbol{\theta}}^{-1}]_{2\times 2}$$

Position Error Bound

$$\mathsf{PEB}(\mathbf{p}) = \sqrt{\mathsf{tr}\{\boldsymbol{\mathcal{I}}_{\mathbf{p}}^{-1}\}}$$

$$\mathsf{PEB}(\mathbf{p}) \leq \sqrt{\mathsf{var}\{\hat{p}_x\} + \mathsf{var}\{\hat{p}_y\}}$$

Impact of Geometry and Channel

$$\boldsymbol{\mathcal{I}}_{\mathbf{p}} = \frac{\partial \boldsymbol{\tau}}{\partial \mathbf{p}}^T \cdot \boldsymbol{\Lambda} \cdot \frac{\partial \boldsymbol{\tau}}{\partial \mathbf{p}}$$